Session 2A9
Space-Time Dynamics of Pulsed Beam Fields in Complex Media

Uniform Signal Contribution of the Step Function Modulated Sine Wave

N. A. Cartwright (University of Vermont, USA); K. E. Oughstun (University of Vermont, USA); 542

Fast Time Domain Integral Equation Solver for Simulation of Propagation of Wide-band Pulses through Dispersive Media

E. Bleszynski (Monopole Research, Thousand Oaks, USA); M. Bleszynski (Monopole Research, Thousand Oaks, USA); T. Jaroszewicz (Monopole Research, Thousand Oaks, USA); 543

Observation of Precursor-like Behaviour of Ultra-fast Pulses Propagating in Water

A. Fox (Dartmouth College, USA); U. Österberg (Dartmouth College, USA); X. Gu (Georgia Institute of Technology, USA); R. Trebino (Georgia Institute of Technology, USA); 544

Optimal Waveform Design for Imaging with an Active Array

L. Borcea (Rice University, USA); G. Papanicolaou (Stanford University, USA); C. Tsogka (University of Chicago, USA); 545

Analytic Pulsed-beam Communication Channels

G. Kaiser (Center for Signals and Waves, USA); 546

Weak Lacunae of Electromagnetic Waves in Dilute Plasma with Anisotropy

S. V. Tsynkov (North Carolina State University, USA); 547

Qualitative Methods in Inverse Electromagnetic Scattering

F. Cakoni (University of Delaware, USA); 548

Uniform Signal Contribution of the Step Function Modulated Sine Wave

N. A. Cartwright and K. E. Oughstun
University of Vermont, USA

A canonical problem of central importance in the theory of ultrawideband pulse propagation through temporally dispersive, absorptive materials is the propagation of a step function modulated signal through a dielectric material that exhibits anomalous dispersion. One method in which a closed-form approximation to the propagated pulse may be obtained is application of asymptotic expansion techniques to the Fourier integral representation. For a linearly-polarized plane-wave pulse traveling in the positive z-direction with the temporal behavior on the plane $z = 0$ given by

$$E(0, t) = u(t) \sin(\omega_c t), \tag{1}$$

the propagated pulse on the plane $z > 0$ is given exactly by the Fourier-Laplace integral

$$E(z, t) = \frac{1}{2\pi} \Re \left\{ i \int_{ia-\infty}^{ia+\infty} \frac{i}{\omega - \omega_c} \exp\left[\frac{z}{c}\phi(\omega, \theta)\right] d\omega \right\}. \tag{2}$$

Here, $u(t)$ denotes the step function, ω_c is the *carrier frequency* of the input pulse, $\phi(\omega, \theta)$ is the *complex phase function*

$$\phi(\omega, \theta) \equiv i\frac{c}{z}\left(\tilde{k}(\omega)z - \omega t\right) = i\omega\left[n(\omega) - \theta\right], \tag{3}$$

where $\tilde{k}(\omega) = (\omega/c)n(\omega)$ is the *complex wavenumber*, $\theta = ct/z$ is a dimensionless space-time parameter, and a is a real constant greater than the abscissa of absolute convergence for the initial field $E(0, t)$. The problem then is to evalute the contour integral (2) for all $\theta > 1$, the field given by (2) vanishing identically for all $\theta \leq 1$.

Asymptotic expansion techniques were first applied to (2) by Brillouin in 1914 [Brilluoin, *Ann. Phys.* **44** (1914)]. His analysis showed that the propagated field is comprised of two precursors and the main signal as

$$E(z, t) = E_S(z, t) + E_B(z, t) + E_c(z, t), \tag{4}$$

where $E_S(z, t)$ is the Sommerfeld precursor, $E_B(z, t)$ is the Brillouin precursor and $E_c(z, t)$ is the signal contribution. Later analysis by Oughstun and Sherman [*Pulse Propagation in Causal Dielectrics*, Springer-Verlag (1984)] using uniform expansion techniques showed that Brillouin's results were quantitatively incorrect in several important space-time regions. However, their expansion exhibited several discontinuities in its description of the propagated signal. Here, corrections to this previous research (in particular, to the signal contribution) are presented which results in a completely uniform asymptotic description of the propagated pulse.

Fast Time Domain Integral Equation Solver for Simulation of Propagation of Wide-band Pulses through Dispersive Media

E. Bleszynski, M. Bleszynski, and T. Jaroszewicz
Monopole Research, Thousand Oaks, USA

We present recent results on the analytical formulation and representative applications of the fast time domain integral equations specially tailored to problems interaction of wideband pulses with dispersive media. The algorithm takes advantage of the the block-triangular and block Toeplitz structures of impedance matrix in temporal indices, and of the Toeplitz structure of far-field component of the impedance matrix in spatial indices, hence allowing for the simultaneous compression in space and time. Furthermore, since the algorithm relies only on the translational invariance of the Green function, its computational cost is independent of the degree of dispersion of medium. An additional advantage of the method is its applicability to problems involving Green functions given in either tabulated or analytic forms.

An important practical element of the underlying time-domain integral formulation is that instead of using the customary integral equation operators involving the Green function and its derivatives, we construct effective integral equation operators equal to (i) the Fourier transform of the dispersive medium Green function, (ii) the Fourier transform of the product of the dispersive medium Green function with the inverse of dielectric permittivity.

Some of the most recent enhancements of our time-domain capabilities include numerically efficient modeling of specific time-localized waveforms, such as linear combinations of hermite polynomials. There is a need for such particular solver numerical capability in the context of several current and future medical and military potential applications. We derived compact analytical expressions for the projections (on spatial and temporal basis functions) of the incident wave represented as a superposition of hermite polynomials. A particular example of practical interest belonging to this class of waveforms is the "Mexican hat" wave-form, expressible as a linear superposition of Hermite polynomials up to the second order. We implemented the resulting formulation in the code module generating the incident wave projection on Rao-Glisson-Wilton (RWG) basis functions defined on triangular patches, and on pulse or band-limited (approximate prolate spheroidal) temporal basis functions.

We present numerical results for scattering on an arbitrarily sharped homogeneous dielectric body of a "conductive Debye" material, in which the electric permittivity is given by the Debye formula with an added conductivity term. The computations are carried out using a surface-integral equation formulation, involving matrix elements of free-space and dispersive dielectric Green functions. We evaluate the matrix elements of the dispersive medium Green function by means of integration in the complex frequency plane. We discuss some aspects of numerical quadrature in evaluation of integrals along branch cuts with singular (but integrable) behavior of branch-cut discontinuities.

We show validation results for a dielectric sphere, for which we compare scattered pulse shapes obtained from our time-domain integral equations with those synthesized using Mie solutions.

*This work is supported by the AFOSR grant F49620-01-0504.

Observation of Precursor-like Behaviour of Ultra-fast Pulses Propagating in Water

A. Fox[1], **U. Österberg**[1], **X. Gu**[2], **and R. Trebino**[2]

[1]Dartmouth College, USA
[2]Georgia Institute of Technology, USA

We have measured temporal, spectral and absorptive characteristics of broadband optical pulses around 800 nm propagating through pure water. The bandwidth of our pulses varied between 10 nm to 100 nm and the temporal pulsewidth varied between 60 fs to 2 ps and the pulse repetition rates were either 1 kHz or 80 MHz. The distances the pulses propagated through water varied between 0.3 meters to 6.1 meters. All measurements were performed under strictly linear conditions.

Our absorption measurements showed non-exponential decay as a function of path length. Pulses of varying temporal widths, bandwidths, chirps, and repetition rates were compared with simulated classical absorption predictions for statistically significant deviations. Deviations occurred for low repetition rates and pulse lengths shorter than approximately 500 fs. For the 60 fs pulses we observed 2 orders of magnitude less absorption after approximately 6 meters of propagation through water compared to 2 ps long pulses which absorbed according to Beer's law.

The temporal and spectral measurements were performed using cross-correlation frequency-resolved optical gating (XFROG). The XFROG technique records a spectrogram which enables us to extract both amplitude and phase information of the short optical pulses exiting from the water tube. These spectrograms clearly showed the breakup of a Gaussian pulse into three distinct pulses with different arrival times. The pulses were also centered at different carrier-frequencies and they had developed different types of chirp.

Using the theory developed for calculating the temporal energy velocity of propagation in an absorbing and dispersive medium [1] in conjunction with proven experimental data for both the real and imaginary part of the dielectric function of water [2] we were able to qualitatively relate our pulse breakup with that of points of stationary phase.

These observations are believed to be the manifestation of precursor activity.

REFERENCES

1. Lisak, M., "Energy expressions and energy velocity for wave packets in an absorptive and dispersive medium," *J. Phys. A: Math. Gen.*, Vol. 9, 1145–1158, 1976.
2. Segelstein, D. J., "The complex refractive index of water," M. Sc. Thesis, Department of Physics, University of Missouri-Kansas City, 1981.

Optimal Waveform Design for Imaging with an Active Array

L. Borcea
Rice University, USA
G. Papanicolaou
Stanford University, USA
C. Tsogka
University of Chicago, USA

We consider in this paper broadband array imaging of distributed reflectors by sending probing signals from one or more sources at the array and then recording the scattered echoes at all array elements.

We address the question of optimally choosing the waveform send by the array in order to construct the best possible image of the target for a given signal to noise ratio of the data. This is different from the problem of selecting the optimal waveform so as to maximize the received power at the array. The solution to this problem is known and its main drawback is that it corresponds to sending a narrowband waveform that peaks at the resonance frequency of the reflectors. That is because maximizing the power is equivalent to iterative time reversal, or, the singular value decomposition of the impulse response matrix. The resulting narrow-band waveform gives strong scattered echoes, but it is bad for imaging because lack of bandwidth means no range resolution and no statistical stability in clutter.

We propose instead to determine the source power allocation and waveforms with an optimality criterion based on the quality of the image. The main idea is to determine the waveform by solving an optimization problem using an appropriate measure that quantifies the quality of the image. The optimization problem is then solved subject to constraints such as limiting the power at the array and asking for an acceptable signal to instrument noise ratio at the receivers.

Analytic Pulsed-beam Communication Channels

G. Kaiser
Center for Signals and Waves, USA

Let $G(x_r - x_e)$ be the causal Green function for the wave equation in four space-time dimensions, representing the signal received at the space-time point x_r due to an impulse emitted at the space-time point x_e. Such emission and reception processes are highly idealized since no signal can be emitted or received at a precise point in space and at a precise time. We present a simple model for *extended* emitters and receivers by continuing G analytically to a function $\tilde{G}(z_r - z_e)$, where $z_e = x_e + iy_e$ is a complex space-time point representing a circular pulsed-beam emitting antenna dish centered at x_e and radiating in the direction of y_e and $z_r = x_r - iy_r$ is a complex space-time point representing a circular pulsed-beam receiving antenna dish centered at x_r and receiving from the direction of y_r. The analytic Green function $\tilde{G}(z_r - z_e)$ represents the *coupling amplitude* between the emission and reception dishes. The space components of y_e and y_r give the spatial orientations and radii of the dishes, while their time components determine the duration and collimation of the emission and reception processes. Causality requires that the orientation vectors y_e and y_r must belong to the future cone V_+ in space-time. The directivity D of the communication channel is a non-negative convex function on V_+, *i.e.,* $0 \le D(y_r + y_e) \le D(y_r) + D(y_e)$. That is, the directivity of the channel can be no better than the sum of its emission and reception components.

Weak Lacunae of Electromagnetic Waves in Dilute Plasma with Anisotropy

S. V. Tsynkov
North Carolina State University, USA

As has been shown in our previous work, the Maxwell equations of electromagnetic field do not, generally speaking, satisfy the Huygens' principle, unless the electromagnetic waves propagate through a very simple medium, such as the vacuum or a dielectric with static response. Accordingly, lacunae and the sharp aft fronts of the waves will not, generally speaking, exist in the corresponding solutions because of the propagation after-effects. However, for an important case of the high-frequency transverse electromagnetic waves in dilute plasma, the governing equations reduce to the Klein-Gordon equation. The latter is not Huygens' per se, but it turns out that lacunae can still be identified in its solutions, although in an approximate sense. The aft fronts for these "weak lacunae" can be clearly observed, but they may not be as sharp as in the pure Huygens' case. Moreover, it can be shown that the "depth" of a weak lacuna, i.e., the magnitude of its residual field relative to the magnitude of the field inside the primary wave packet, is controlled by the dimensionless ratio ω_{pe}/ω, where ω_{pe} is the Langmuir frequency and ω is the dominant carrying frequency of the waves, $\omega_{pe} \ll \omega$.

The aforementioned study has been carried out for the isotropic dilute plasma with the parameters close to those of the Earth's ionosphere. It is known, however, that the actual ionospheric propagation may be noticeably affected by the magnetic field of the Earth. This field makes the plasma anisotropic and also introduces an additional temporal scale into the model given by the electron cyclotron frequency Ω_{ee}. The cyclotron frequency is typically about an order of magnitude lower than the Langmuir frequency. In the current work we show that the additional effect of the Earth's magnetic field on lacunae for the case of high-frequency transverse propagation is small. Quantitatively, it is about Ω_{ee}/ω times smaller than the previously studied effect of the primary wave dispersion in plasma, where $\Omega_{ee} < \omega_{pe} \ll \omega$.

Qualitative Methods in Inverse Electromagnetic Scattering

F. Cakoni
University of Delaware, USA

Since the invention of radar, scientists and engineers have striven not only to detect but also to identify unknown objects through the use of electromagnetic waves. A significant step forward in the resolution of this problem was the use of synthetic aperture radar (SAR). However, (SAR) suffers from limitations arising from the incorrect model assumptions which ignore both multiple scattering and polarization effects. In recent years, in an effort to overcome the limitations of such an incorrect model, considerable effort has been put into the development of nonlinear optimization techniques which avoids incorrect modeling assumptions. The success of such an approach is based on strong a priori knowledge of the scattering object and hence is inappropriate for many, if not most, practical applications. In view of the problems inherent in the weak scattering and nonlinear optimization approaches to target identification, a new class of methods has been developed in the past few years loosely called qualitative methods in inverse scattering theory. The main theme of this talk is the use of one such qualitative method, the linear sampling method, to solve inverse electromagnetic scattering problems. In particular, we will discuss the imagining of objects imbedded in a known inhomogeneous media using electromagnetic radiation at a fixed frequency.