Session 1P9
Numerical Method

RCS Prediction of Large Cavities on a Distributed Memory Parallel Computer

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Abstract—This paper describes the implementation and results of a finite element based radar cross section (RCS) prediction method on a distributed memory parallel computer. This method has been specifically developed for the analysis of large cavities with model reduction of rotationally periodic and mirrored geometries. Realistic propulsion system components have been modeled with this method at X-band on shared-memory parallel computers [1]. This paper describes the extension of this method to distributed memory parallel computers and the inherent communication process required. The paper also discusses timing results and parallel efficiency.

1. Introduction

Engine system inlet and exhaust ducts are among the most difficult areas to reduce the RCS of a military aircraft. Methods of predicting the performance of such devices are important to achieving optimal designs in a timely and cost efficient manner.

For radar frequencies of interest, an engine cavity is considered to be electromagnetically large —where the physical dimension is much larger than the wavelength. The most widely used methods for modeling large objects are based on asymptotic techniques such as ray tracing, diffraction theory, and physical optics. However, for cavity structures in particular, the limited accuracy of asymptotic methods makes them suitable only for first-order engineering approximations.

Compounding the challenge of modeling the electromagnetic large aspect of a military engine cavity is the requirement of modeling complex-shaped geometries, such as turbine blades, cooling holes, flame holders, etc., and the requirement of modeling radar absorbing materials in both bulk and composite configurations.

GE Aircraft Engines has for a number of years been developing techniques based on the finite element method (FEM). FEM has shown its robustness in modeling the complex material and geometry configurations at the accuracy levels necessary for low observable designs. Methods presented previously [1] and reviewed here, incorporate the use of special transforms for model reduction of the rotationally periodic engine geometry. These transforms, coupled with the use of specialized sparse matrix solution techniques, have allowed our FEM to model cavities at the higher frequencies of interest.

Results presented previously were performed on parallel computers utilizing a shared memory facility. These computers are limited to a relatively low number of processors that can be efficiently run in parallel (approximately ten processors). To extend this, the present computer architecture of choice is a distributed memory system where processors maintain their own computer memory and information is passed between them by a message passing system. Preliminary results in using this type of parallel computer for our FEM approach are described here.

2. Formulation

2.1. Basics

The mathematical frequency domain finite element formulation used here is of a standard type using a curl-curl type wave equation for the electric field:

$$\nabla \times \frac{1}{j\omega\mu} \nabla \times E + j\omega\varepsilon E = 0 \tag{1}$$

A dual formulation for the magnetic field could also be utilized, however, because resistive sheets would have to be "gapped" [2] in the magnetic field formulation, the electric field formulation is preferred for not having this cumbersome modeling step.

Standard types of finite elements, with hexahedron, wedge, and tetrahedral shapes are used (Fig. 1). The order of the edge-type element basis function used is commonly referred as H1 type—where the field behavior

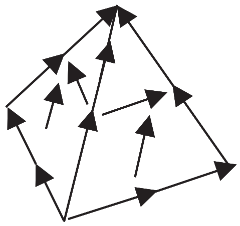

Figure 1: Edge element tetrahedral, wedge, and hexahedral shapes.

is modeled as linear along the edge direction and quadratic in the orthogonal direction. The element types used here are also curvilinear in construction for better modeling of curved surfaces.

Applying the Galerkin weighted residual method to Eq. 1 results in a sparse set of matrix equations, A, with a forcing function, b, representing the incident electromagnetic field, the field solution at each finite element unknown is represented by the vector x in Eq. 2.

$$Ax = b \tag{2}$$

For geometries of interest and for discretization levels of four elements per wavelength or better, this sparse matrix may result in the tens of millions of unknowns for the higher frequencies of interest. However, methods can be employed to reduce the model for solution in a timely manner on a not-so massive parallel computer. For rotationally periodic structures such as the engine front frame shown in Fig. 2(a), the resulting matrix would have a repeatable block pattern, as shown in Fig. 2(b). This matrix type is known as a block circulant matrix [3].

Figure 2: (a) Engine front frame.

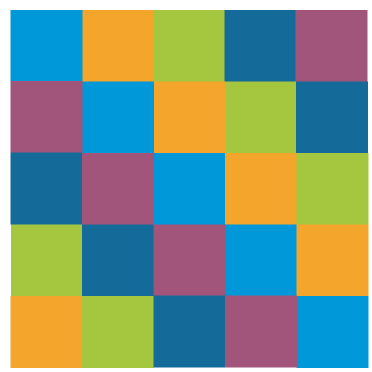

Figure 2: (b) Matrix with repeating block structure.

The number of blocks in a row/column of the matrix in Fig. 2(b) corresponds to the number of periodic structures "p" within the device. Also, the order of the block would be equal to the number of finite element unknowns one periodic "pie slice" volume of the structure —see Fig. 3(a).

The repeated pattern matrix of Fig. 2(b) can be reduced to a block diagonal matrix, as shown in Fig. 3(b), by applying a discrete body of revolution Fourier transform [4]. This transform can be represented by matrices P and P^{-1} that left and right multiplies the system matrix A of Eq. 2, respectively:

$$PAP^{-1}Px = Pb \tag{3}$$

Although this discrete Fourier transform is for rotationally symmetric structures, similar transforms have been constructed for geometries with mirror plane symmetries.

This block diagonal form has multiple advantages over solving the overall system as in Fig. 2(b). First, each block can be solved independently and in parallel simultaneously. Next, it dramatically reduces the "bandwidth" of a sparse matrix factorization scheme leading to a geometric decrease in the number of floating point operations. And lastly, the total solution is reconstructed from these independent sets without loss of accuracy.

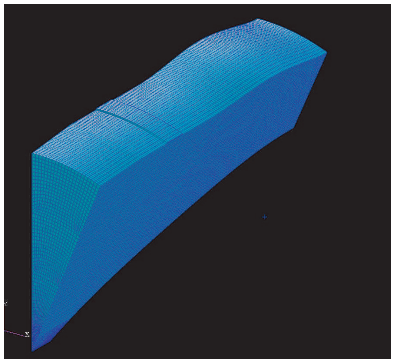

Figure 3: (a) Finite element model of one periodic section.

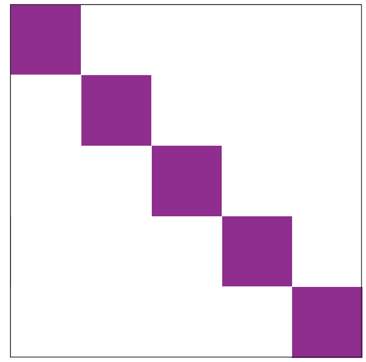

Figure 3: (b) Block diagonal matrix.

However, for the size of problems required for realistic propulsion systems, the individuals block themselves must also be solved in parallel. The necessity of this is two-fold: first the speed increase of parallel system is required to incorporate the analysis into a timely design iteration process, and second, the computer memory requirements of a block would exceed an individual processor and must be spread over multiple nodes of a parallel system.

2.2. Parallel Matrix Solution

A matrix factorization method is used that takes advantage of the aperture nature of this cavity problem. This solution method is similar to the one presented in [5] and is applied in both parallel and serial versions of the analysis code. This matrix factorization method takes advantage that the forcing function of the system is applied only to the front surface aperture of the cavity. Also, the calculation of the RCS requires the field solution only over this same aperture surface.

Factorization schemes, by themselves, are attractive over alternative iterative schemes because of the need to solve for multiple look-angles and polarizations. The total sum of these solutions, and again in particular for higher frequency problems where the RCS vs. look-angle curves may have high scintillation patterns, may order into the one-thousand or better range.

This frontal-factorization scheme takes advantage of the cavity/aperture geometry by factoring from the opposite end of the cavity (opposite from the aperture surface) to the aperture surface in a wave-front fashion. Because back-substitution is only required over the aperture surface to calculate RCS, the memory for the factored matrix behind the "wave-front" is released and reused. This keeps the total memory requirement to a minimal amount. Also, the order for the number of floating point operations is equal to

$$O(N_w^3 N_\ell) \tag{4}$$

where N_w is equal to the number of unknowns in the wave-front and N_ℓ is the number of unknown along the length of the cavity. As seen from Eq. 4, the reduction of N_w by the periodic decomposition scheme by a factor of $1/p$ where p is the number of periods, drastically reduces the total number of floating point operations.

For serial computers or parallel computers with shared memory architectures, this "wave-front" banded factorization scheme is a straightforward procedure. For distributed memory cluster computers, the algorithm is somewhat challenging to construct and implement with efficient parallelism. In our method, a block method of factorization [6] with a skyline profile is used. Here, the individual factorization blocks are assigned to separate processors on the cluster computer. Data communication between processors is performed with the Message Passing Interface (MPI) library.

3. Results

The computer used for the following two example problems is a Dell 2850 cluster. Each node of this computer consists of dual Intel Xeon$^{\text{TM}}$ processors running at 2.8 GHz (512 kilobyte cache) with 4 gigabytes of memory. The operating system is Red Hat Linux 9.0, Intel Fortran and C compilers were used, and the message-passing library implementation is LAM MPI.

The first example is a test body representative geometry of an exhaust duct (see Fig. 4). This test geometry has a length of 35 inches, a diameter of 38 inches, and has a rotational periodicity of 16. The geometry was meshed for a frequency of 10 GHz. This mesh has a mixture of hexahedral and wedge shaped elements. The total number of cubic wavelengths of the cavity without model reduction is approximately 24000. The number of finite element unknowns generated from the model-reduced mesh is approximately 1.57 million and the number of non-zeros in the resulting matrix is 129.2 million. The total amount of memory used across all processors is approximately 2.2 gigabytes.

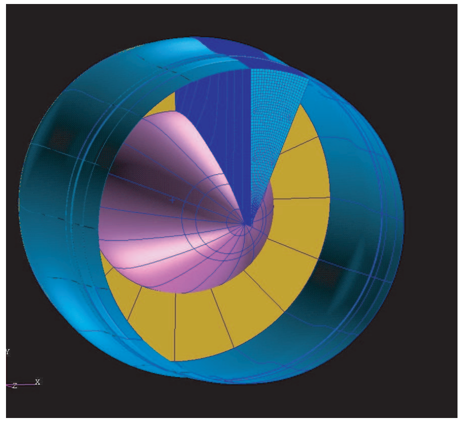

Figure 4: Example exhaust duct test case.

This problem was run with a modest number of processors so that all harmonics of the discrete Fourier decomposition could be run simultaneously on a cluster numbering less than twenty-five nodes.

Runs with two and three nodes (four and six processors, respectively) were performed. The timings for matrix factorization are: approximately 25 hours for two-nodes and 23 hours for three-nodes. The total number of floating point operations for the factorization is 332×10^{12} and the floating point rate is 0.925 gigaflops per processor (3.7 Gigaflops total) for the two node case and 0.671 gigaflops per processor (4.03 gigaflops total) for the three node case. The parallel efficiency for the two-code case is 66 percent and the three-node case is 50 percent. 1644 solutions (822 look-angles with both polarizations) were solved for; the total solution and RCS integration times were 516 seconds for two nodes and 504 seconds for three nodes.

The second example is another exhaust duct of greater internal geometric complexity and slightly larger in size. The approximate length and diameter are 40 inches and 38 inches, respectively. It also has a rotational periodicity of 16 but includes more internal structures that lead to a higher floating-point operation count. A frequency of 10 GHz is used and the total number of cubic wavelengths without model reduction is approximately 27500.

The total number of finite element unknowns is approximately 2.5 million after model reduction (hexahedral and wedge shaped elements were again used) and the number of non-zeros in the matrix is 203.5 million. The total number of unknowns on the reduced model aperture surface is 8936.

The matrix factorization time is approximately 72 hours on five processors. The total number of floating point operations for this factorization is 1.2×10^{15}, which results in a rate of 0.94 gigaflops per processor (4.7 gigaflops total). The total amount of memory across the five processors is approximately 6.0 gigabytes for the wave-front factorization. The parallel efficiency is estimated at 68 percent.

4. Conclusions

This paper demonstrated the application of finite element analysis with the combination of model reduction by rotational decomposition and the use of distributed memory parallel computers. The results presented here are our first attempt at using a distributed memory parallel computer for this analysis method. The authors believe that further parallel efficiencies can be obtained with added effort on methods of parallel matrix factorization.

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An Efficient *hp* Adaptive Finite Element Solver for Time-harmonic Electromagnetic Fields

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Thanks to its great flexibility in modelling geometry and material properties, the finite element (FE) method is a widely used tool for the numerical analysis of electromagnetic devices. With the FE method, there are two different ways of improving the accuracy of numerical solutions. In case of p enrichment, the degrees of the basis functions are increased whereas, in case of h refinement, the element sizes are decreased. When the fields are smooth, p enrichment yields exponential convergence, whereas h refinement is always limited to algebraic rates of convergence. On the other hand, when singularities are present, the performance of p schemes is poor whereas nonuniform h methods succeed in keeping the rate of convergence unchanged. Since real world configurations typically involve both regions of smooth fields and localized areas of rapid field variations or even singularities, p enrichment and h refinement should be viewed as complementary rather than competing techniques.

The FE method we propose in this paper combines hp adaptivity with fast solution techniques. As for h refinement, we construct sequences of nested tetrahedral meshes which allow for subregions of greatly varying refinement levels, and impose special restriction operators to make the FE basis functions maintain proper continuity conditions. The resulting FE spaces are perfectly nested, which makes them very well-suited for advanced geometrical multi-grid solvers exploiting local sub-meshing techniques.

Regarding p enrichment, a set of hierarchical $\mathcal{H}(\mathrm{curl})$ conforming basis functions developed by one of the authors is employed. It is of the incomplete order type, features explicit basis functions for higher order gradients as well as increased sparsity within the stiffness matrix, and possesses interpolation properties that greatly simplify h refinement and hence bridge the gap to the before mentioned method. Our basic building block for a fast solver in the p domain is a multiplicative Schwarz method.

One challenge with hp schemes is that the aspects of multi-grid schemes for h refinement and Schwarz methods for p enhancement can no longer be considered separately. In fact, there are many ways to cycle back and forth between a low-order FE space over a coarse mesh and a high-order space over a fine discretization, and it turns out that the corresponding algorithms differ greatly in their computational complexity.

In our talk we will give the details of the proposed hp adaptive FE solver. We will demonstrate the efficiency of the new approach by a number of numerical examples.

Challenges for Computational Electromagnetics for Low Frequencies

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Full wave electromagnetic simulation of circuits in computer technology is a challenging problem, as it is in the domain where wave physics meets circuit physics, namely, it is the "twilight zone". In this regime, electromagnetic field does not behave fully as waves, but meanwhile, simple circuit theory such as KCL and KVL cannot fully capture the physics of the electromagnetic interaction. In this regime, two kinds of breakdown occur for computational electromagnetics: the low-frequency breakdown due to the inaccuracy in the integral equation, and the low-frequency breakdown in accelerators such as the fast multipole algorithm.

When the size of a geometry structure is much smaller than a wavelength, it is necessary to use a quasi-Helmholtz decomposition of the surface current to preclude the breakdown of the integral equation. Such a decomposition is achieved by using either the loop-tree basis or the loop-star basis for the quasi-Helmholtz decomposition. In this manner, the physics that corresponds the world of the capacitors, and that that corresponds to the world of the inductors can be correctly captured.

When the size of the geometry structure is comparable to wavelength, Rao-Wilton-Glisson (RWG) functions can be used to expand the current on the structure to capture the wave physics. Low-frequency breakdown problem can be delayed by using higher precision calculations when RWG functions are used.

As for the solution accelerator, we have recently proposed the mixed-form fast multipole algorithm that can work seamlessly from static to the microwave regime. It is both accurate and error controllable, as well as being memory efficient. However, there exist structures where both wave physics and circuit physics are important. This could be a large structure with many excruciating details as happens in a computer circuit, but the overall platform size is not small. In that case, it is more expedient to put Huygens' equivalence boxes around each region with fine details, and decouple the exterior problem from the interior problem. This can be regarded as having replaced a region with fine details with an N-port representation. Inside the Huygens' boxes, low-frequency techniques can be used to solve the problem so that low-frequency physics is correctly captured, with the ensuing geometry details. Outside the Huygens' boxes, when wave-like interactions are computed, less number of unknowns is needed to capture the wave physics, but meanwhile, the ability to model fine details is not foregone.

Multi-level Multiplicative Schwarz Preconditioner for Solving Matrix Equations from DD-FE-BEM Formulation

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A symmetric coupling between finite and boundary elements for solving electromagnetic wave radiation and scattering problems in $\mathbb{R}^3$ has been recently proposed by Vouvakis et al., [1]. The new formulation results in a symmetric complex matrix equation which is free of internal resonances, and the spectral radius of the couplings between finite and boundary elements is bounded by 1. Moreover, the formulation is also allowing non-conformal couplings between FEM and BEM, and thus, offers great modeling versatility for solving real-life complex problems. By non-conformity, we mean that the surface triangulations of the FEM and the BEM do not need to be the same, as well as the flexibility of choosing different order of basis functions for FEM and BEM portions separately.

This paper addresses the practical issue of how to solve the resulting symmetric complex matrix equations efficiently. As is well known that in order to solve large sparse matrix equations (or even dense matrix equations, for that matter) efficiently using iterative solution, such as Conjugate Gradient (CG) methods, the most critical ingredient is the preconditioner. In the authors' group, we have developed over the years a robust preconditioner, termed p-MUS (p-type MUltiplicative Schwarz), for preconditioning the matrix equations from the application of the vector finite elements [2]. We extend the p-MUS to the current DD-FE-BEM formulation, and treating the BEM block as another "abstract" domain, and construct three possible preconditioners: an inner-outer loop domain decomposition preconditioner, an additive p-type Schwarz method, and a Multi-level MUltiplicative Schwarz (M-MUS) preconditioner for solving the resulting DD-FE-BEM matrix equations. Various numerical examples, including both radiation and scattering problems, will be presented and comparisons of the three preconditioners will be discussed as well in the presentation.

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Higher Order Hierarchical FEM Solutions with Enhanced Efficiency and Practicality

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Recently, computational techniques based on using electrically large curved elements for geometrical modeling (large domains) and higher order basis functions for field modeling have been employed within the framework of the finite element method (FEM), with an objective to significantly reduce the number of unknowns and computational resources for a given (high) accuracy when compared to low-order small-domain solutions. However, these advantages become evident and convincing only if the large-domain FEM approach is carefully planned and implemented. This paper addresses several numerical aspects of the higher order implementation and presents some new advancements in the context of hierarchical curl-conforming polynomial vector basis functions on generalized hexahedral finite elements [1], which are all crucial for making this approach an efficient and practical analysis and design tool for engineers.

Hierarchical curl-conforming vector basis functions enable using different orders of field approximation in different elements for efficient selective discretization of the solution domain. We demonstrate very effective higher order FEM models of complex structures consisting of both very large and very small elements of very different shapes. We also discuss some of the algorithms for the higher order hexahedral mesh generation. Although hierarchical polynomials are inherently ideal for p-refinement of solutions, for general structures it must be combined with an h-refinement. We show excellent convergence properties for several hp-refined meshes and discuss possible further improvements of the technique.

Hierarchical basis functions generally have poor orthogonality properties, which results in FEM matrices with large condition numbers. The ill-conditioning is principally caused by a strong mutual coupling between the pairs of higher-order functions defined on the same (electrically large) generalized hexahedron, which become increasingly similar to one another as the polynomial degrees increase. In order to reduce this coupling, basis functions with better orthogonality properties have to be utilized. We show that higher order large-domain hierarchical curl-conforming FEM vector basis functions constructed from standard orthogonal polynomials and their modifications on generalized curvilinear hexahedral elements exhibit a very slow increase of the condition number of the FEM matrix with increasing the field-approximation orders and a very dramatic reduction of the condition number for high orders as compared to the technique in [1] using field expansions based on simple power functions (the reduction is as large as fourteen orders of magnitude in some cases).

To ensure that the CPU time per unknown in higher order solutions is comparable to that in low-order solutions, rapid and accurate recursive procedures are needed for evaluation of elements of FEM matrices. We show how important for the efficiency of the solution is that computation algorithms avoid redundant operations related to the indices for basis and testing functions and for geometrical representations within all of the interactions in the FEM solution, as well as the summation indices in the Gauss-Legendre integration formulas. In addition, it is crucial that the topological analysis of the problem and assembly of the connectivity matrix are also done in an optimal way that minimizes the total number of nonzero elements and ensures a similar level of sparsity of system matrices as for low-order solutions.

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Combining an FEM Domain Decomposition Method with BEM for Accurate Antenna Array Analysis

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Finite Element Method-Domain Decomposition (FEM-DD) methods have been proven very efficient and effective numerical techniques for the analysis of Maxwell's equations. Among other advantages, it suffice to stress their parallelization ability, ability to systematically couple different numerical methods into hybrid schemes, efficient exploitation of geometrical redundancies and symmetries, and relaxing meshing and adaptive meshing strategies. On the other hand, in light of the resent fast Integral Equation developments, see for example MLFMA, AIM, P-FFT, etc. Boundary Element Methods (BEM) are best suited for the fast analysis of unbounded problems.

This paper attempts to modularly couple the two approaches. The result is a very robust, accurate and efficient method for unbounded electromagnetic problem analysis. The method is extremely efficient when repeating structures are involved in the computational domain. The FEM-DD is coupled with BEM using DD concepts. In other words, the FEM-DD and BEM are viewed as another domain level in the domain decomposition. In overall this is a 2-level DD, where the inner level of the DD is the FEM-DD whereas the outer level DD is the coupled FEM-DD and BEM problem. The method is non-conforming thus it allows for maximum exploitation of geometry repetitions, local adaptation schemes and efficient structured BEM solvers. The overall method is variational and free of internal resonances. Various solutions strategies will be proposed.

New results of the coupled FEM-DD and BEM will be given. Comparisons with other methods, convergence curves and computational statistics will be presented in order to demonstrate the accuracy, efficiency and versatility of the method. Some results on real-world challenging radiation and scattering problems such as very large antenna arrays, hybrid radomes and EBGs will be presented.