Session 1P6a
Volume and Rough Surface Scattering: Theory and Photonic Applications

Phase Fluctuations in Scattered Radiation

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Fluctuations in scattered radiation are of considerable practical and theoretical interest [1]. Perhaps due to the prevalence of direct detection systems at optical wavelengths, as well as the relative difficulties encountered in phase sensitive systems, phase statistics and correlations would appear to have received considerably less attention than corresponding results for intensity. Freund and Kessler obtained the phase autocorrelation of a complex Gaussian field from the two-point joint density [2], a calculation that Middleton had performed in earlier investigations [3]. As noted by Sebbah et al., [4], the results from these investigations are applicable to the wrapped phase, or phase modulo 2π, and not the unwrapped phase that can take on arbitrary values. However, there are instances when the variance of the unwrapped or cumulative phase is of most interest, such as characterizing transit times in diffuse waves [5], wavefront sensing and interferometry [6]. In this talk we present unwrapped or cumulative phase results following scattering from one-dimensional phase screens and extended random media. Analytical results are obtained in weak and strong fluctuations regimes, which provide a benchmark for numerical simulations that allow insight under all fluctuation conditions.

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Scattering of an Electromagnetic Wave from 3-dimensional Rough Layers: Small-amplitude Method and Small-slope Approximation

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The scattering of electromagnetic waves from randomly rough surfaces have been extensively studied in different domains such as radio-physics, geophysical remote sensing, ocean acoustics, surface optics and recently plasmonics where metallic surfaces have a dielectric coating. Our purpose is to show how light can interact with several randomly rough surfaces. In this paper, we consider an electromagnetic polarized plane wave incident on a three-dimensional dielectric film with one or two randomly rough surfaces. We assume that the randomly rough surfaces are Gaussian and statistically independent: a Gaussian probability density function is assumed for the random rough surface heights and the autocorrelation function is a Gaussian function. We study two hypothesis, we consider three-dimensional structures bounded by two-dimensional weakly rough surfaces or by two-dimensional randomly rough surfaces with small-slope.

In the case of weakly randomly rough interfaces, we use the small-amplitude perturbation theory, we have generalized the integral equations called reduced Rayleigh equations in the case of a three-dimensional layer with weakly randomly rough interfaces. The electromagnetic polarized plane wave is incident on a dielectric layer whose mean thickness is constant. The dielectric layer is deposited on a metallic film. Illustrative examples are presented for the bistatic diffuse component of the electromagnetic field.

In the second part of the paper, we discuss the extension of the theory using the small-slope approximation method. We study structures with two-dimensional randomly rough surfaces, including scattering from free-standing films or films on a substrate, one or both of whose surfaces are randomly rough. The fourth order term of the perturbative development is required if we want to take into account the interactions between the two randomly rough surface. Some simulations will be given and compared with the small-amplitude perturbation method.

This analysis is relevant to problems of laser cross-section calculation, remote sensing of irregular layered structures and remote detection of chemical coatings.

Fast Modeling of Reflectance Image of Turbid Medium with Full-field Illumination

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Reflectance imaging with oblique full-field illumination is a powerful tool for non-invasive determination of internal structure of turbid materials and diagnosis of lesions in human tissues. Extraction of structural information from the reflectance imaging data, however, requires quantitative modeling of light transportation and distribution in the turbid medium. Radiative transfer theory offers an accurate model and often has to be realized through Monte Carlo simulations, which is time consuming due to its statistical nature. In contrast, the diffusion approximation to the radiative transfer theory can be solved analytically but only applicable to the distribution of multiply scattered photons.

Based on previous studies of radial distribution of reflected light with single-fiber illumination [1–3], we compared different hybrid models that combine the Monte Carlo simulation with calculations based on the diffusion approximation of radiative transfer theory. On the basis of these results, we investigated a hybrid model that is most appropriate for full field illumination in which photon tracking in the Monte Carlo simulation is truncated to significantly increase the calculation speed. The contribution to the reflected light distribution at the surface of the imaged medium by the truncated photons is obtained from the analytical solution of the diffusion equation for these multiply scattered photons. We will present the numerical results on the validity and applicability of this fast hybrid method for modeling reflectance images with fullfield illumination and its potential use in the inverse determination of internal stricture in turbid medium.

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Optical Tomography of Arbitrarily Shaped Object with Randomly Rough Boundaries

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This paper presents the results of research designed to fulfill two main objectives including development of laser reflectance modeling of complex convex or concave objects with randomly rough boundaries and investigation of tomographic reconstruction of these objects.

This paper addresses the utility of physics based modeling of the laser backscattering of complex rough targets. The physics based model, we present in this paper, is designed to provide accurate results but to also include all of the electromagnetic interaction mechanisms. To model the laser interaction with the randomly rough surface, we use the second order Small-slope Approximation method. Because the problem, we consider in this paper, is three-dimensional, all the scattering coefficients (coherent and incoherent component of the electromagnetic field) are functions of the azimuth angles, and the cross-polarized terms do not vanish. We define, in this case, the Mueller matrix, which gives all the combinations of the polarization states of the scattered electromagnetic waves. The randomly rough surfaces of the complex object are characterized by electromagnetic parameters (permittivity...) and roughness parameters (standard deviation of rough surface height and autocorrelation function). One of the great advantages of this physics based model is its extensibility. Electromagnetic interactions of higher levels of complexity can be added to the model. Illustrative examples are presented for laser scattering from large convex objects. Our model addresses also transparent structures. With this model, we can obtain high temporal resolved laser backscattering from complex objects.

In the second part of the paper, we investigate algorithms for tomographic reconstruction of complex objects. The reconstruction is based on compilations of time-resolved optical backscattering obtained at various angles. The laser backscattered energy at various angles is calculated by our reflectance modeling of complex objects. We use our model to generate sets of data, with which we can compare the different models of reconstruction. We compare direct back-projection method, filtered back-projection method, Fourier-Radon method and stochastic method. We analyze the stability of the different methods when we add noise to the laser backscattering.

Statistical Distribution of Field Scattered by 1-Dimensional Random Slightly Rough Surfaces

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Abstract—We consider a perfectly conducting plane with a local cylindrical perturbation illuminated by a monochromatic plane wave. The perturbation is represented by a random function assuming values with a Gaussian probability density. For each realization of the stochastic process, the spatial average value over the width of the modulated zone is zero. The mean value of the random function is also zero. Without any deformation, the total field is the sum of the incident field and the reflected field. For a locally deformed plane, we consider — in addition to the incident and reflected plane waves — a scattered field. Outside the modulated zone, the scattered field can be represented by a superposition of a continuous spectrum of outgoing plane waves. The method of stationary phase leads to the asymptotic field, the dependence angular of which is given by the scattering amplitudes of the propagating plane waves. Using the first-order small perturbation method, we show that the real part and the imaginary part of scattering amplitudes are uncorrelated Gaussian stochastic variables with zero mean values and unequal variances. Consequently, the probability density for the amplitude is given by the Hoyt distribution and the phase is not uniformly distributed between 0 and 2π.

1. Introduction

The problem of electromagnetic wave scattering from random surfaces continues to attract research interest because of its broad applications. The three classical analytical methods commonly used in random rough-surface scattering are the small-perturbation method, the Kirchhoff method and the small slope approximation [1–5]. The electromagnetic analysis of rough surfaces with parameters close to the incident wavelength requires a rigorous formalism. Numerous method based on Monte Carlo simulations are available for 1D and 2D random rough surfaces [6, 7]. Most of research works focus on the determination of coherent and incoherent intensities. There is not such a voluminous literature on the statistical distribution of scattered field [3]. In this paper, we derive the statistical distribution in the far field zone from the first-order small perturbation method in the particular case of perfectly conducting 1D random rough surface illuminated by an $E_{//}$ polarized monochromatic plane wave.

2. The Random Surfaces under Consideration

The geometry of the problem is depicted in Fig. 1. The rough surface is represented in Cartesian coordinates by the equation $y = a_0(x)$ and consists of a small cylindrical random perturbation with length L and zero mean

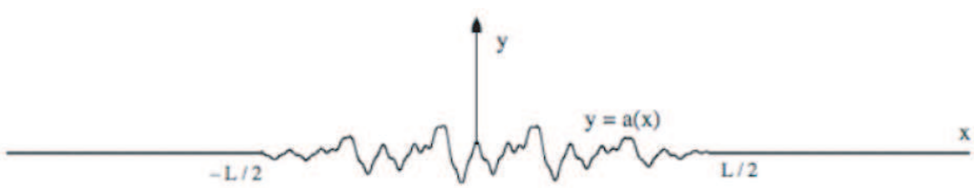

Figure 1: The slightly rough surface.

($< a_0(x) >= 0$) in a perfectly conducting plane $y = 0$. Each realisation can be described by the following equation

$$\begin{aligned} a_0(x) &= a(x) - m \qquad \text{if } |x| \le \frac{L}{2} \\ a_0(x) &= 0 \text{ outside} \end{aligned} \tag{1}$$

where

$$m = \frac{1}{L} \int_{-L/2}^{+L/2} a(x)dx \tag{2}$$

$a(x)$ is a random function assuming values distributed normally with zero mean and variance σ_a^2. Here it's important to distinguish the spatial average m from the statistical mean $< a(x) >$. Insofar $< a(x) >= 0$, we have $< m >= 0$. The random process is assumed stationary with a Gaussian statistical correlation function

$$R_{aa}(x) = \sigma_a^2 \exp\left(- \frac{x^2}{l_c^2} \right) \tag{3}$$

where l_c is the correlation length.

3. The Scattering Amplitudes in the Far Field Zone

The surface is illuminated under incidence θ_i by an z-polarized monochromatic plane wave $E_i \overrightarrow{u}_z$ of wavelength λ. The Oz-electric component of field is

$$E_i(x, y) = \exp(-j\alpha_i x + j\beta_i y) \tag{4}$$

where

$$\alpha_i = k \sin\theta_i\,; \quad \beta_i = k\cos\theta_i\,; \quad k = 2\pi/\lambda \tag{5}$$

The time-dependence factor $\exp(j\omega t)$ where ω is the angular frequency is assumed and suppressed throughout. The total electric field above the rough surface is the sum of the incident field E_i, the field reflected E_r by the plane without deformation (an infinite perfect mirror) and the scattered field E_d.

$$E_t(x, y) = E_i(x, y) + E_r(x, y) + E_d(x, y) \tag{6}$$

where

$$E_r(x, y) = -\exp(-j\alpha_i x - j\beta_i y) \tag{7}$$

Above the highest point on the surface, the scattered field can be represented by a superposition of a continuous spectrum of outgoing plane waves, the so-called Rayleigh integral [5].

$$E_d(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{C}(\alpha) \exp\big(- j\beta(\alpha) y\big) \exp(-j\alpha x) d\alpha \tag{8}$$

with

$$\beta = \sqrt{k^2 - \alpha^2}, \quad \operatorname{Im}\beta < 0 \tag{9}$$

In the far-field zone, the Rayleigh integral is reduced to the only contribution of the propagating waves ($\alpha \leq k$). The method of stationary phase leads to the asymptotic field [8]

$$E_d(r, \theta) \approx \sqrt{\frac{k}{2\pi r}} \hat{C}(k\sin\theta)\cos\theta \exp(-jkr) \exp\left(j\frac{\pi}{4}\right) \tag{10}$$

The angular dependence in the far field zone is given by the function $\hat{C}(\alpha)\cos\theta$ and becomes identified with the propagating wave amplitudes of the continuous spectrum (8) with $\alpha = k\sin\theta$ [9, 10]. Let us recall that the normalized bistatic scattering coefficient $\sigma(\theta)$ is defined by the power scattered per unit angle $d\theta$ normalized with respect to the flux of incident power through the modulated region

$$\sigma(\theta) = \frac{1}{P_i}\frac{dP_d}{d\theta} = \frac{|\hat{C}(k\sin\theta)|^2\cos^2\theta}{\lambda L\cos\theta_i} \tag{11}$$

For a random process, the scattered field is a random function of position (r, θ) but the scattering amplitude $\hat{C}(\alpha)$ is a random function of the observation angle θ only [10]. The scattering amplitude can be written as the sum of an average amplitude $< \hat{C}(\alpha) >$ which gives the coherent far-field from (11) and a fluctuating amplitude which leads to the incoherent far-field. The first order small perturbation method applied to the Rayleigh integral (8) and the Dirichlet boundary condition gives an approximation of the scattering amplitudes [1, 2]

$$\hat{C}(\alpha) = -2j\beta_i \int_{-L/2}^{+L/2} a_0(x) \exp\big(+ j(\alpha - \alpha_i)x\big) dx \tag{12}$$

Making a change of variable $\gamma = \alpha - \alpha_i$, real part $\hat{C}_r(\alpha)$ and imaginary part $\hat{C}_i(\alpha)$ of scattering amplitudes can be expressed as

$$\hat{C}_r(\gamma) = +2\beta_i \int_{-L/2}^{+L/2} a(x)\sin(\gamma x)dx \tag{13}$$

$$\hat{C}_i(\gamma) = -2\beta_i \left(\int_{-L/2}^{+L/2} a(x)\cos(\gamma x)dx - mL\sin c(\gamma L/2) \right) \tag{14}$$

where $\sin c(x) = \sin x/x$. It can be noticed that the scattering amplitude is zero in the specular direction $\gamma = 0$. $\hat{C}_r$ and $\hat{C}_i$ are obtained from mathematical linear operations applied to the Gaussian random function $a(x)$. Consequently, $\hat{C}_r$ and $\hat{C}_i$ are also quantities distributed with Gaussian probability densities.

4. The Statistical Distribution of Scattering Amplitudes

4.1. The Incoherent Intensity

From (13) and (14), we derive $< \hat{C}(\gamma) >= 0$. Consequently, the coherent density is zero. Moreover, after some extensive mathematical manipulations, we deduce the variances

$$r =< \hat{C}_r^2 > = 4\beta_i^2 \int_0^{+L} (L-x)\Big[\cos\gamma x - \sin c\big(\gamma(L-x)\big)\Big] R_{aa}(x)dx \tag{15}$$

$$\begin{aligned} s =< \hat{C}_i^2 > &= 4\beta_i^2 \int_0^{+L} (L-x)\Big[\cos\gamma x + \sin c\big(\gamma(L-x)\big)\Big] R_{aa}(x)dx - 4\beta_i^2 \sin c(\gamma L/2) \\ &\Bigg[\sin c(\gamma L/2)\int_0^{+L} xR_{aa}(x)dx + 2\int_0^{+L}(L/2-x)\sin c\big(\gamma(L/2-x)\big)R_{aa}(x)dx\Bigg] \end{aligned} \tag{16}$$

where the statistical correlation function $R_{aa}(x)$ is given by (3).

The variances depend on the width L of the modulated zone. But, outside the specular reflection zone, if L goes to infinity, the variances of the real and imaginary parts become identified. Using (11), (15) and (16), we obtain the incoherent intensity $I_f(\theta) =< \sigma(\theta) >$

$$I_f(\theta) = \frac{< \big|\hat{C}(k\sin\theta - k\sin\theta_i)\big|^2 > \cos^2\theta}{\lambda L\cos\theta_i} \quad \text{with} < \big|\hat{C}(\gamma)\big|^2 >=< \hat{C}_r^2 > + < \hat{C}_i^2 > \tag{17}$$

We note that the incoherent intensity is not proportional to the surface power spectrum.

4.2. Probability Densities of the Amplitude and Phase

Random quantities $A = \hat{C}_r(\alpha)$ and $B = \hat{C}_i(\alpha)$ are distributed normally with zero mean values and unequal variances r and s. Moreover, we show that they are uncorrelated. Consequently, they are independent and we can write:

$$p_{AB}(a,b) = p_A(a)p_B(b) = \frac{1}{2\pi\sqrt{rs}} \exp\left(-\frac{a^2}{2r} - \frac{b^2}{2s} \right) \tag{18}$$

where $p_{AB}(a,b)$ is the two-dimensional normal distribution of $\hat{C}_r(\alpha)$ and $\hat{C}_i(\alpha)$. Transforming to polar coordinates,

$$A = M\cos\psi\,; \qquad B = M\sin\psi \tag{19}$$

we obtain the required distributions for the modulus M and the phase ψ:

$$p_M(m) = \int_0^{2\pi} p_{M\psi}(m,\varphi)m\,d\varphi = \frac{m}{\sqrt{rs}}\exp\left(-\frac{m^2}{4r} - \frac{m^2}{4s} \right) \tag{20}$$

$$p_\psi(\varphi) = \int_0^{+\infty} p_{M\psi}(m,\varphi)m\,dm = \frac{1}{2\pi}\frac{\sqrt{rs}}{s\cos^2\varphi + r\sin^2\varphi} \tag{21}$$

These formulas show that $p_M(m)$ is the Hoyt distribution [3] and that the phase is not uniformly distributed between 0 and 2π. Nevertheless, outside the specular reflection zone and if L goes to infinity, $p_M(m)$ is reduced to the Rayleigh distribution and the phase is uniformly distributed.

5. Results

Figure 2 gives the incoherent intensity for a Gaussian random profile having a modulation length $L = 24\lambda$, a rms height $h = 3\lambda/100$ and a correlation length $l_c = 2\lambda$. We can note the zero value of $I_f(\theta)$ in the specular direction ($\theta = \theta_i = 30°$). Outside the specular zone, the comparison with results obtained by the C method [10] is good. The dashed curve and the solid curve show the values obtained by (15) and by the C method.

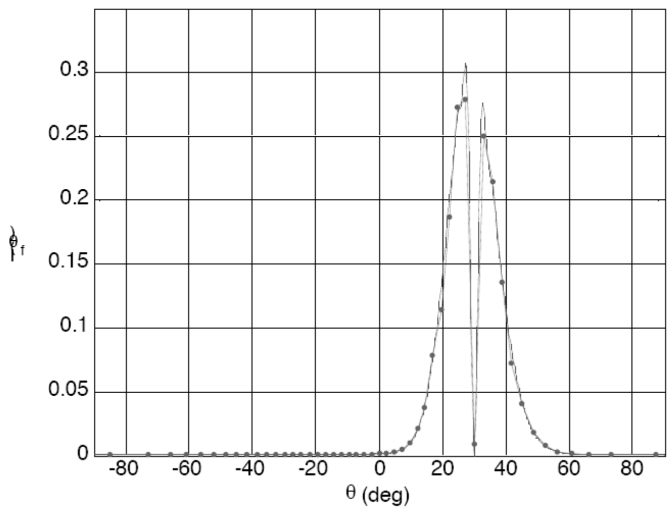

Figure 2: Incoherent intensity for a Gaussian random profile.

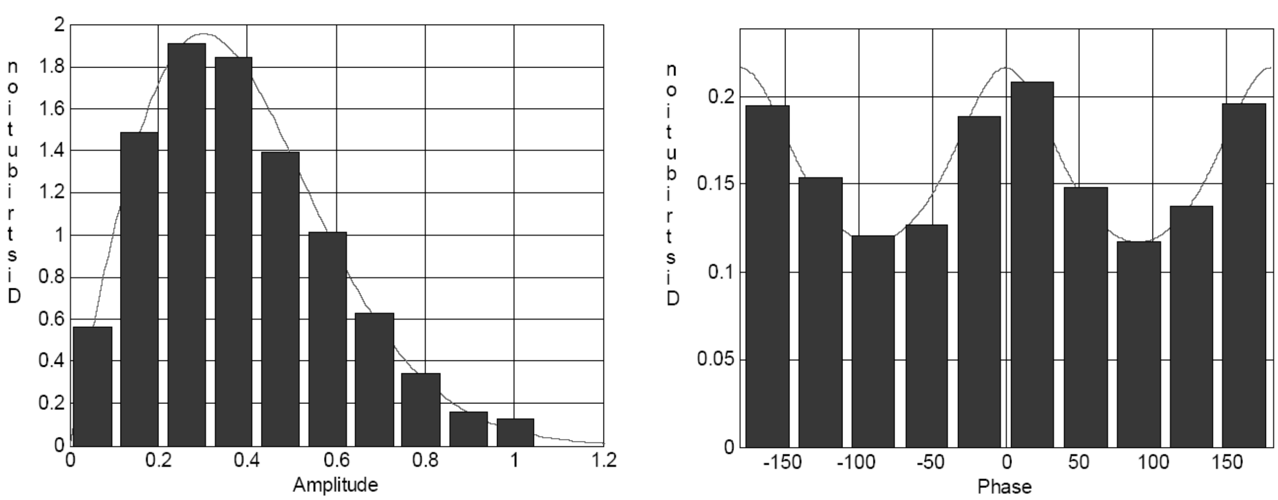

Figure 3: Amplitude and phase distributions.

Figure 3 show the values of the Hoyst distribution and the phase distribution (given by (20) and (21), respectively) for an observation angle $\theta = 10°$. The comparison with the normalized histogram obtained by a Monte-Carlo simulation with 10000 surface realizations is good.

6. Conclusion

We have derived the statistical distribution in the far field zone from the first-order small perturbation method in the particular case of perfectly conducting 1D random rough surfaces illuminated by an $E_{//}$ polarized monochromatic plane wave. We have shown that the real part and the imaginary part of scattering amplitudes are uncorrelated Gaussian stochastic variables with zero mean values and unequal variances. The probability density for the amplitude is given by the Hoyt distribution and the phase is not uniformly distributed between 0 and 2π. Comparisons with statistical observation over 10000 surfaces confirm the result. This approach can be extended to dielectric random rough surfaces illuminated by a polarized plane wave $E_{//}$ or $H_{//}$. The generalization of these results to slightly rough surface with an arbitrary statistical height distribution with an arbitrary correlation function is in progress.

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Experimental and Theoretical Studies of Specular and Diffuse Scattering of Light from Randomly Rough Metal Surfaces

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We present experimental results for the reflectivity of two-dimensional randomly rough metal surfaces, as well as the contribution to the mean differential reflection coefficient from the light scattered incoherently by such surfaces. The measurements were done with s- and p-polarized light. The samples were fabricated on photoresist and coated with gold. Their surface profiles constitute good approximations to Gaussian random processes with a Gaussian surface height autocorrelation function. The measurements were done in the infrared at a wavelength of $10.6\,\mu$m. The experimental results for the reflectivity are compared with the results of small-amplitude perturbation theory, phase perturbation theory, and self-energy perturbation theory, and with results obtained on the basis of the Kirchhoff approximation. Rough surfaces with rms heights a small fraction of the wavelength of the incident light were employed, so that meaningful comparisons with the results of the perturbation theories could be made. In the case of the light scattered incoherently, the experimental results are compared with results obtained by means of the Kirchhoff approximation and with the results of small-amplitude perturbation theory and phase perturbation theory. The theoretical results for the reflectivity obtained on the basis of phase perturbation theory are closest to all the experimental results in both s and p polarization. Phase perturbation theory also produces the best overall agreement with the experimental results for the contribution to the mean differential reflection coefficient from the incoherent component of the scattered light for in-plane, co-polarized scattering, although small-amplitude perturbation theory produces better results in p polarization for samples with very small transverse correlation lengths of the surface roughness.