Session 1A3a
Scattering and Propagation in Random Media and Rough Surfaces

Designer Emissivities

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The emissivity of an object is defined as the ratio of the brightness temperature $T_{B\nu}$ emitted by the object to its actual physical temperature T_{phys}, under the assumption that the object is at a constant physical temperature [1], $T_{B\nu} = e_\nu\,(\theta,\,\phi)T_{phys}$. In this equation the subscript ν refers to the polarization of the brightness temperature, while $(\theta,\,\phi)$ are the polar and azimuthal angles of observation. Kirchhoffs law [2] relates the emissivity $e_\nu(\theta,\,\phi)$ to the reflectivity of the surface, $e_\nu(\theta,\,\phi) = 1 - r_\nu(\theta,\,\phi)$, where the reflectivity $r_\nu(\theta,\,\phi)$ is the fraction of the power scattered from the surface when $a\,\nu = p$, s wave is incident on it from the direction $(\theta,\,\phi)$. Since the reflectivity of a surface is affected by its roughness [3], it follows that its emissivity is affected by the roughness. The question then arises, can one design a random surface that produces an emissivity with a specified wavelength dependence at a fixed angle of emission?

We investigate this question for a one-dimensional random surface defined by $x_3 = \zeta(x_1)$. The region $x_3 > \zeta(x_1)$ is vacuum, while the region $x_3 < \zeta(x_1)$ is a perfect conductor. This surface is illuminated by an s-polarized plane wave of frequency ω, whose plane of incidence is the x_1x_3 plane. We represent the surface profile function in the form $\zeta(x_1) = nb < x_1 < (n+1)b$, with $n = 0, \pm1, \pm2, \ldots$ Here b is a characteristic length, and the $\{d_n\}$ are independent, identically distributed random deviates. The probability density function (pdf) of d_n, $\langle\delta(\gamma - d_n)\rangle = f(\gamma)$, where the angle brackets denote an average over the ensemble of realizations of $\zeta(x_1)$, is therefore independent of n. In the Kirchhoff approximation, which we use due to its simplicity, the reflectivity is given by $r_s(\theta_0,\omega) = \mid F((2\omega b/c)\cos\theta_0)\mid^2$, where θ_0 is the polar angle of incidence, and $F(\nu) = \int_{-\infty}^{\infty} d\gamma f(\gamma)\exp(-\mathrm{i}\nu\gamma)$. Thus, if we wish to have a particular frequency dependence of the emissivity $e_s(\theta_0,\omega)$ at an angle of emission θ_0, we have to solve the equation $\mid F((2\omega b/c)\cos\theta_0 \mid = [1 - e_s(\theta_0,\,\omega)]^{1/2}$ to obtain $f(\gamma)$. This is done iteratively by the use of a modified Gerchberg-Saxton algorithm [4]. From the result a long sequence of $\{d_n\}$ is generated by the rejection method [5], from which a realization of the surface profile function is generated. To validate the approach described, the emissivity produced by the resulting surface is calculated by solving the scattering problem numerically for a particular example.

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On the Application of the Radiative Transfer Approach to Scattering from a Random Medium Layer with Rough Boundaries

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Abstract—For studying the problem of scattering from a random medium layer with rough boundaries the radiative transfer (RT) approach is widely used. In order to better understand this procedure we compared it with the statistical wave approach. Two such wave approaches are presented in this paper: the surface scattering operator (SSO) approach, and the unified approach. In both wave approaches two conditions are essential for arriving at RT system: the ladder approximation to the intensity operator, and the quasi-stationary approximation of fields. With these approximations one arrives at the integro-differential equations of the RT system. However, to arrive the at the RT boundary conditions, one has to impose further approximations. In the SSO approach weak surface correlation must be imposed. In the unified approach, one has to ignore the terms involving volumetric spectral densities, and consider only single scattering from the rough boundary when deriving the boundary conditions.

1. Introduction

The analysis of scattering from a random medium layer with rough boundaries is a difficult problem. This is the kind of problem one often encounters in remote sensing applications. People have used the phenomenological radiative transfer approach to study this problem (Ulaby et al., 1986; Lam and Ishimaru, 1993; Shin and Kong, 1989). This approach is conceptually simple and yet very effective for studying multiple scattering processes. Here one uses the transport equations corresponding to the random medium of the layer and then one imposes the relevant boundary conditions. Although this procedure appears to be heuristically sound it is not clear what approximations are involved, and under what conditions such a procedure may be used for the problem at hand. One way to better understand this radiative transfer approach is to compare and relate it to the statistical wave approach. For the case of unbounded random media it has been demonstrated how the ladder approximated Bethe-Salpeter equation reduces to the radiative transport equation (Barabanenkov et al., 1971). We found that this procedure can be applied to the problem of random medium layer with planar boundaries and arrive at the radiative transport system as given in Ulaby et al., (1986). However, if the boundaries are statistically rough, the problem is considerably more complicated and we need special procedures to deal with them. We have employed two different statistical wave approaches for such problems. In the first approach we assume that we know the solution of the problem without the volumetric fluctuations. The second approach is based on the solution of the problem where all the fluctuations vanish. We shall compare the results of these two approaches with those of the radiative transfer (RT) approach. This will enable us to understand and meaning and import of the radiative transfer approach as applied to our problem. To keep discussions in a simple setting we will consider the scalar problem and keep the lower boundary alone as rough.

2. Geometry of the Problem

The geometry of the problem consists of a random medium layer with a rough bottom boundary. The permittivity of the layer medium consists of a deterministic part ϵ_2 and randomly fluctuating part $\tilde{\epsilon}\epsilon_2$. $z = 0$ and $z = -d + \zeta(r_\perp)$ describe the upper and lower boundary of the layer. We assume that $\tilde{\epsilon}$ and ζ are small and smooth zero-mean stationary processes independent of each other. The medium above the layer is homogeneous, and we impose the Neumann boundary condition on the lower boundary. This layer is excited by a wave incident from above and we are interested in the scattered waves.

3. Radiative Transfer Approach

The classical equation of radiative transfer is given as

$$\hat{s} \cdot \nabla I(r, \hat{s}) + \eta I(r, \hat{s}) = \int d\Omega' P(\hat{s}, \hat{s}') I(r, \hat{s}') \tag{1}$$

where $P(\hat{s}, \hat{s}')$ is the phase function and η is the extinction coefficient. This equation was originally intended for unbounded scattering medium. However it can be applied to bounded medium with arbitrary geometry by

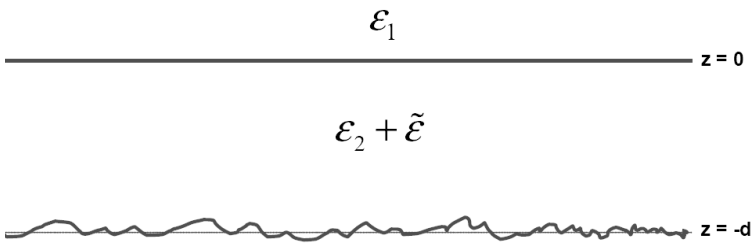

Figure 1: Geometry of the problem.

imposing appropriate boundary conditions. For layer geometry we have the following set of coupled integro-differential equations.

$$\cos\theta \frac{d}{dz} I_u(z,\Omega) + \eta I_u(z,\Omega) = I_u^c + \frac{|k|^4}{4\pi} \int d\Omega' \{\Phi(\theta,\theta';\phi-\phi') I_u(z,\Omega') + \Phi(\theta,-\theta';\phi-\phi') I_d(z,\Omega')\} \quad (2)$$

$$\cos\theta \frac{d}{dz} I_d(z,\Omega) - \eta I_d(z,\Omega) = -I_d^c + \frac{|k|^4}{4\pi} \int d\Omega' \{\Phi(-\theta,\theta';\phi-\phi') I_u(z,\Omega') + \Phi(-\theta,-\theta';\phi-\phi') I_d(z,\Omega')\} \quad (3)$$

Eqs. (2) and (3) follow from (1) noting that the problem is translationally invariant in azimuth. I_u and I_d represent the incoherent part of radiant intensities corresponding to upward and downward travelling waves inside the layer. I_u^c and I_d^c represent the corresponding contributions due to coherent intensities. Φ represents the spectral density of the volumetric fluctuations. Eqs. (2) and (3) are solved using the following boundary conditions.

$$I_d(0,\Omega) = |R_{12}(\Omega)|^2 I_u(0,\Omega) \quad (4)$$

$$I_u(-d,\Omega) = \int d\Omega' \langle |R_{32}(\Omega,\Omega')|^2 \rangle I_d(-d,\Omega) \quad (5)$$

The extinction coefficient η is readily derived from the differential scattering cross section of the random medium. R_{12} is the reflection coefficient at the upper boundary for waves incident from below. R_{32} is the reflection coefficient at the lower boundary for waves incident from above. Thus we see that the formulation in the radiative transfer approach is simple and straight forward, and can be applied to a variety of different geometries. The fundamental quantity in this approach is the radiant intensity and hence is not suitable to represent wave phenomena such as diffraction, interference, etc. A more general approach to this problem is the statistical wave approach. In this paper we will describe two such approaches and compare them with that of radiative transfer.

4. Surface Scattering Operator Approach

We start with the following equations governing the Green's functions of the problem.

$$\begin{aligned} \triangle G_{12} + k_1^2 G_{12} &= 0 \\ \triangle G_{11} + k_1^2 G_{11} &= -I \\ \triangle G_{21} + k_2^2 G_{21} &= -qG_{21} \\ \triangle G_{22} + k_2^2 G_{22} &= -I - qG_{22} \end{aligned} \quad (6)$$

where $q = \omega^2 \mu \tilde{\epsilon} \epsilon_2$ represents the volumetric fluctuations. We write the above system as

$$LG = -I - QG \quad (7)$$

where $G \equiv \{G_{ij}\}$, $L = \text{diag}\{L_1, L_2\}$, $L_j = \triangle + k_j^2$, $Q = q\text{diag}\{0,1\}$. For multiple scattering analysis it is convenient to convert (7) into the following integral equation.

$$G = \breve{G} + \breve{G}QG \quad (8)$$

where $\breve{G}$ is the Green's function of the problem without volumetric fluctuations. In principle, one can construct such Green's functions using surface scattering operators (Voronovich, 1994; Soubret et. al., 2002). First average (8) w. r. t. volumetric fluctuations.

$$\langle G_v\rangle \simeq \breve{G} + \breve{G}\langle Q\langle G\rangle_v Q\rangle\langle G\rangle_v \tag{9}$$

On operating this by L we obtain

$$L\langle G\rangle = -I - \langle Q\langle G\rangle_v Q\rangle\langle G\rangle_v \tag{10}$$

From this we find that

$$L_1\langle G_{11}\rangle_v = -I \tag{11a}$$
$$L_2\langle G_{22}\rangle_v = -I - \langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v \tag{11b}$$

Next average (11) w. r. t. surface fluctuations

$$L_1\langle G_{11}\rangle_{vs} = -I \tag{12a}$$
$$L_2\langle G_{22}\rangle_{vs} = -I - \langle\langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v\rangle_s \tag{12b}$$

We infer from (12a) that the mean propagation constant in Region 1 is unaffected by the fluctuations of the problem. To interpret (12b) we approximate $\langle\langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v\rangle_s$ as $\langle G_{22}\rangle_{vs}\langle qq\rangle\langle G_{22}\rangle_{vs}$. As we shall see, this kind of approximation is essential for arriving at the RT system as given in the previous section. Thus

$$(\triangle + k_2^2)\langle G_{22}\rangle_{vs} = -I - \langle G_{22}\rangle_{vs}\langle qq\rangle\langle G_{22}\rangle_{vs} \tag{13}$$

This implies that

$$k_{2m}^2 = k_2^2 + \langle G_{22}\rangle_{vs}\langle qq\rangle \tag{14}$$

This is the operational definition for the mean propagation constant in the layer region. With this we can proceed to construct the mean Green's functions.

We next turn our attention to the second moments of the fields. Taking the tensor product of (8) with its complex conjugate and performing volumetric averaging leads to

$$\langle G\otimes G^*\rangle_v = \langle G\rangle_v \otimes \langle G\rangle_v^*\{I + K\langle G\otimes G^*\rangle_v\} \tag{15}$$

where K is the intensity operator corresponding to the volumetric fluctuations. Hence the equation for field correlation is

$$\langle\psi\otimes\psi^*\rangle_v = \langle\psi\rangle_v\otimes\langle\psi\rangle_v^* + \langle G\rangle_v\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v \tag{16}$$

Averaging this over surface fluctuations we have

$$\langle\psi\otimes\psi^*\rangle_{vs} = \langle\langle\psi\rangle_v\otimes\langle\psi\rangle_v^*\rangle_s + \langle\langle G\rangle_v\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v\rangle_s \tag{17}$$

Now we employ the following two approximations essential for arriving at the radiative transfer system.

$$\langle\langle G\rangle\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v\rangle_s \simeq \langle\langle G\rangle_v\otimes\langle G\rangle_v^*\rangle_s K\langle\psi\otimes\psi^*\rangle_{vs} \tag{18a}$$
$$K \simeq \langle Q\otimes Q^*\rangle \tag{18b}$$

The first is the weak surface correlation approximation. The second is called the ladder approximation. Thus we arrive at the following equation for the second moment of the fields inside the layer

$$\langle\psi_2\otimes\psi_2^*\rangle_{vs} = \langle\langle\psi_2\rangle_v\otimes\langle\psi_2\rangle_v^*\rangle_s + \langle\langle G_{22}\rangle_v\otimes\langle G_{22}\rangle_v^*\rangle_s K\langle\psi_2\otimes\psi_2^*\rangle_{vs} \tag{19}$$

Observe that $\psi_2 = \langle\psi_2\rangle_{vs} + \tilde{\psi}$ and $\langle\psi_2\rangle_v = \langle\psi_2\rangle_{vs} + \widetilde{\langle\psi_2\rangle_v}$ where tilde is used to denote the fluctuating part. Using these relations in (19) we obtain

$$\langle\tilde{\psi}_2\otimes\tilde{\psi}_2^*\rangle_{vs} = \langle\widetilde{\langle\psi_2\rangle}_v\otimes\widetilde{\langle\psi_2\rangle}_v^*\rangle_s + \langle\langle G_{22}\rangle\otimes\langle G_{22}\rangle_v^*\rangle_s\langle q\otimes q^*\rangle\langle\psi_2\otimes\psi_2^*\rangle_{vs} \tag{20}$$

We next introduce Wigner transforms of the wave functions and the Green's functions in (20) and obtain

$$\tilde{\mathcal{E}}(z,k) = \tilde{\mathcal{E}}^s(z,k) + \frac{|k_2|^4}{(2\pi)^6}\int dz_1\int d\alpha\int d\beta\mathcal{G}(z,k;z_1,\alpha)\Phi_v(\alpha-\beta)\mathcal{E}(z_1,\beta) \tag{21}$$

where $\tilde{\mathcal{E}}$, $\tilde{\mathcal{E}}^s$, $\mathcal{E}$ and $\mathcal{G}$ are the Wigner transforms of $\langle\tilde{\psi}_2\otimes\tilde{\psi}_2{}^*\rangle_{vs}$, $\langle\widetilde{\langle\psi_2\rangle}_v\otimes\widetilde{\langle\psi_2\rangle}_v^*\rangle_s$, $\langle\psi_2\otimes\psi_2^*\rangle_{vs}$ and $\langle\langle G_{22}\rangle_v\otimes\langle G_{22}\rangle_v^*\rangle_s$, respectively. Φ_v is the spectral density of volumetric fluctuations. Boundary conditions relate radiant intensities arriving at and departing the boundary. Therefore, we need to split $\tilde{\mathcal{E}}$ into upward and downward travelling components. Assume that the fields are quasi-stationary and hence only waves travelling over similar paths will be correlated. This leads to the following approximation.

$$\mathcal{G} = \mathcal{G}^o + \mathcal{G}_{uu} + \mathcal{G}_{ud} + \mathcal{G}_{du} + \mathcal{G}_{dd} \tag{22}$$

$\mathcal{G}^o$ is the Wigner transform of $G^o \otimes G^{o*}$ where G^o is the singular part of $\langle G_{22}\rangle$. $\mathcal{G}_{uu}$ is the Wigner transform corresponding to that part of $\langle G_{22}\rangle$ involving the surface scattering operator $\langle S_{uu}\rangle$ and so on. Using this decomposition we split (21) as follows.

$$\begin{aligned}\tilde{\mathcal{E}}^u(z,k) = \tilde{\mathcal{E}}^{su}(z,k) + \frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{z} dz_1 \int d\alpha \int d\beta \mathcal{G}^{>}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\\ + \frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{0} dz_1 \int d\alpha \int d\beta \{\mathcal{G}^{uu}+\mathcal{G}^{ud}\}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\end{aligned} \tag{23a}$$

$$\begin{aligned}\tilde{\mathcal{E}}^d(z,k) = \tilde{\mathcal{E}}^{sd}(z,k) + \frac{|k_2|^4}{(2\pi)^6}\int_{z}^{0} dz_1 \int d\alpha \int d\beta \mathcal{G}^{<}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\\ + \frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{0} dz_1 \int d\alpha \int d\beta \{\mathcal{G}^{du}+\mathcal{G}^{dd}\}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\end{aligned} \tag{23b}$$

On using the expressions for $\mathcal{G}$'s the above pair of equations can be represented as the following integro-differential transport equation system

$$[dz + 2\eta^{''}]\tilde{\mathcal{E}}_u(z,k_\perp) = \mathcal{E}_u^c + \frac{|k_2|^4}{16\pi^2|\eta|^2}\int d\alpha_\perp \{\Phi_v(k_\perp-\alpha_\perp;\eta'-\eta'_\alpha)\tilde{\mathcal{E}}_u(z,\alpha_\perp)+\Phi_v(k_\perp-\alpha_\perp;\eta'+\eta'_\alpha)\tilde{\mathcal{E}}_d(z,\alpha_\perp)\} \tag{24a}$$

$$[dz - 2\eta^{''}]\tilde{\mathcal{E}}_d(z,k_\perp) = -\mathcal{E}_d^c + \frac{|k_2|^4}{16\pi^2|\eta|^2}\int d\alpha_\perp \{\Phi_v(k_\perp-\alpha_\perp;-\eta'-\eta'_\alpha)\tilde{\mathcal{E}}_u(z,\alpha_\perp)+\Phi_v(k_\perp-\alpha_\perp;-\eta'+\eta'_\alpha)\tilde{\mathcal{E}}_d(z,\alpha_\perp)\} \tag{24b}$$

Here $\mathcal{E}_u^c$ and $\mathcal{E}_d^c$ are the contributions due to coherent intensities. The associated boundary conditions are obtained as

$$\tilde{\mathcal{E}}_d(0,k_\perp) = |R_{12}(k_\perp)|^2\tilde{\mathcal{E}}_u(0,k_\perp) \tag{25a}$$

$$\tilde{\mathcal{E}}_u(-d,k_\perp) = \langle |R_{32}(k_\perp,k'_\perp)|^2\rangle\tilde{\mathcal{E}}_d(-d,k_\perp) \tag{25b}$$

where R_{12} and R_{32} are the reflection coefficients at the lower and upper boundaries for waves in the layer. In the process of obtaining (25) we had to impose the following approximation

$$\langle [R_{32}\otimes R_{32}^*][(I+S_{dd})\otimes(I+S_{dd})^*]\rangle \simeq \langle R_{32}\otimes R_{32}^*\rangle\langle (I+S_{dd})\otimes(I+S_{dd})^*\rangle \tag{26}$$

This is similar to the weak surface correlation approximation in the sense that we assume that the influence of the boundary fluctuations result in local relations. On observing that $I(z,\Omega) = \frac{\epsilon c k_2^2}{(2\pi)^2}\mathcal{E}(z,k_\perp)\cos\theta$ we find that the system of integro-differential Eqs. (24) and (25) is identical to the radiative transfer equation system (2)–(5). The conditions under which this has been possible are:

1. ladder approximation to the intensity operator
2. quasi-stationary approximation for fields
3. weak surface correlation

For unbounded random media and random medium layer with planar boundaries we find that the first two conditions are sufficient. But for random media with rough boundaries we need in addition the third approximation.

5. Unified Approach

The system of equations that we start here is the same as that in the surface scattering operator (SSO) approach, viz., (6) and (7). However, the integral equation representation is different. In the SSO approach we did not directly deal with the boundary conditions. The role of the boundaries are represented entirely by the SSO. Indeed the boundary conditions are essential to determine the SSO. However, in the unified approach we will directly make use of the boundary conditions. At the top surface the boundary conditions are given as

$$\begin{aligned} G_{12}(r_\perp,0;r') &= G_{22}(r_\perp,0;r')\\ \epsilon_2\partial_z G_{12}(r_\perp,0;r') &= \epsilon_1\partial_z G_{22}(r_\perp,0;r')\end{aligned} \tag{27}$$

There is a similar pair of relations at the top surface involving G_{11} and G_{21}. At the bottom surface we have

$$\partial_n G_{21}(r_\perp,\zeta;r') = \partial_n G_{22}(r_\perp,\zeta;r') = 0 \tag{28}$$

These boundary conditions are translated on the plane $z = -d$ by using the following approximation which applies when the surface fluctuations are small and smooth.

$$\begin{aligned}\partial_z G_{21}(r_\perp, -d; r') &= \mathcal{H} G_{21}(r_\perp, -d; r') \\ \partial_z G_{22}(r_\perp, -d; r') &= \mathcal{H} G_{22}(r_\perp, -d; r')\end{aligned} \tag{29}$$

where $\mathcal{H} = \nabla_\perp \zeta \cdot \nabla_\perp - \zeta \partial_z^2$. Using (29) we can convert the differential equation system of our problem into the following integral equation system.

$$G = G^o + G^o Q G \tag{30}$$

where

$$Q = Q_v + Q_s \tag{31a}$$

$$Q_v = qN \qquad Q_s = -\mathcal{H}\delta(z+d)N \tag{31b}$$

Q_v and Q_s represent the volumetric fluctuation and the surface fluctuation, respectively. G^o is the Green's function for the unperturbed problem, viz., the problem when all the fluctuations vanish. Notice that, in this approach, volumetric and surface fluctuations are treated on equal footing. Thus statistical averaging over volumetric and surface fluctuations are carried out at the same step. Therefore we do not have any subscripts for the averaging operations. This is in contrast with the SSO approach where these operations are carried out separately at different stages of the analysis and we had to use subscripts to indicate whether it is w. r. t. volumetric fluctuations or surface fluctuations.

First average (30) to get

$$\langle G \rangle = G^o + G^o \langle Q \langle G \rangle Q \rangle \langle G \rangle \tag{32}$$

This is the mean Green's function that we will use in our analysis of the second moments of the fields. Details of the analysis of (32) are given in Mudaliar (2005). We proceed to the calculation of the field correlation described by the following equation

$$\langle \psi \otimes \psi^* \rangle = \langle \psi \rangle \otimes \langle \psi \rangle^* + \langle G \rangle \otimes \langle G \rangle^* K \langle \psi \otimes \psi^* \rangle \tag{33}$$

where

$$K \simeq \langle Q \otimes Q^* \rangle = \langle Q_v \otimes Q_v^* \rangle + \langle Q_s \otimes Q_s^* \rangle \tag{34}$$

We employ the Wigner transforms in (33) as before and obtain

$$\mathcal{E}(r,k) = \tilde{\mathcal{E}}^m(r,k) + \frac{1}{(2\pi)^6} \int dr_1 \int d\alpha \int d\beta \mathcal{G}(r,k;r_1,\alpha)\{T_v + T_s\}\mathcal{E}(r_1,\beta) \tag{35}$$

where T_v and T_s are spectral representations of $\langle Q_v \otimes Q_v^* \rangle$ and $\langle Q_s \otimes Q_s^* \rangle$, respectively. As before we employ the quasi-stationary field approximation, use (22), and hence arrive at a system of integro-differential equations. The system thus obtained in identical to that in SSO approach. However, the boundary conditions are quite complicated and we have

$$\tilde{\mathcal{E}}(0,k_\perp) = \tilde{\mathcal{E}}^o(0,k_\perp) + \int_{-d}^{0} dz_1 \int d\alpha_\perp W(0,k_\perp;z_1,\alpha_\perp)\tilde{\mathcal{E}}(z_1,\alpha_\perp) \tag{36a}$$

$$\tilde{\mathcal{E}}(-d,k_\perp) = \tilde{\mathcal{E}}^o(-d,k_\perp) + \int_{-d}^{0} dz_1 \int d\alpha_\perp W(-d,k_\perp;z_1,\alpha_\perp)\tilde{\mathcal{E}}(z_1,\alpha_\perp) \tag{36b}$$

where $\tilde{\mathcal{E}}^o$ is the single scattering solution, and W is a 2×2 matrix given in the appendix. Observe that the boundary conditions are not localized. Furthermore, W involves both surface scattering and volumetric scattering. Thus our system incorporates volumetric and surface scattering interactions. However, if we let $\Phi_v \to 0$ and consider only single scattering from the rough boundary, then we obtain the boundary conditions used in the radiative transfer approach, viz., (25).

6. Conclusion

Radiative transfer approach is very efficient and at the same time simple for describing multiple scattering phenomena. Quite rightly this approach is very popular and is used in a wide variety of applications. Consequently, there are several different interpretations of the meaning and domain of applicability of this approach. One good way to understand this approach is to compare and relate it to the statistical wave approach.

Its relation to the wave approach has been well established for the case of unbounded random media. The primary conditions for establishing this equivalence are: ladder approximation to the intensity operator and quasi-stationary approximation of the fields. We find these two conditions are the only requirements even for random media with plane parallel boundaries. However, if the boundaries are statistically rough we need to impose additional restrictions. To illustrate this point we considered two statistical wave approaches: the surface scattering operator approach and the unified approach. In both approaches the integro-differential equations for intensities are the identical to those used in the RT approach. However the boundary conditions are different from those in the RT approach. In the case of SSO approach we need to impose the weak surface correlation approximation to arrive at the boundary conditions of the RT approach. In the case of unified approach we had to let $\Phi_v \to 0$ and consider only single scattering from the rough surface while deriving the boundary conditions. With these additional conditions all the three approaches result in the same system of equations. This study has thus helped us to better understand the three approaches and in particular the relation between the radiative transfer approach and the statistical wave approach when applied to the problem of scattering from a random medium layer with rough boundaries.

Acknowledgment

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Appendix

$$
\begin{aligned}
W_{uu} &= \frac{1}{(2\pi)^2} e^{-2q'' z} \{|k_2|^4 T^v_{uu} + T^s_{uu}\} \\
&\quad T^v_{uu} = \mathrm{rect}\{z, -d\} |S^{>}|^2 e^{2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; \eta' - \eta'_\alpha) + |S^{uu}|^2 e^{2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; \eta' - \eta'_\alpha) \\
&\qquad + |S^{ud}|^2 e^{-2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; -\eta' - \eta'_\alpha) \\
&\quad T^s_{uu} = \{|S^{uu}|^2 e^{-2\eta'' d} + |S^{ud}|^2 e^{2\eta'' d}\} \Phi_s(k_\perp - \alpha_\perp) \{(k_\perp - \alpha_\perp) \cdot \alpha_\perp - \eta'^2_\alpha\}^2 \\
W_{ud} &= W_{uu}\{\eta'_\alpha \to -\eta'_\alpha\} \\
W_{du} &= \frac{1}{(2\pi)^2} e^{2q'' z} \{|k_2|^4 T^v_{du} + T^s_{du}\} \\
&\quad T^v_{du} = \mathrm{rect}\{0, z\} |S^{<}|^2 e^{-2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; -\eta' - \eta'_\alpha) + |S^{du}|^2 e^{2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; \eta' - \eta'_\alpha) \\
&\qquad + |S^{dd}|^2 e^{-2\eta'' z_1} \Phi_v(k_\perp - \alpha_\perp; -\eta' - \eta'_\alpha) \\
&\quad T^s_{du} = \{|S^{du}|^2 e^{-2\eta'' d} + |S^{dd}|^2 e^{2\eta'' d}\} \Phi_s(k_\perp - \alpha_\perp) \{(k_\perp - \alpha_\perp) \cdot \alpha_\perp - \eta'^2_\alpha\}^2 \\
W_{dd} &= W_{du}\{\eta'_\alpha \to -\eta'_\alpha\}
\end{aligned}
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Polarization Dependent Backscatter Cross Sections of Composite Random Rough Surfaces for Normal to Near Grazing Incidence

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The small height/slope perturbation solutions, for the vertically and horizontally polarized backscatter cross sections of random rough surfaces have significantly different dependence upon the incident angle. Thus, for highly conducting surfaces, the ratio of vertically to horizontally polarized backscatter cross sections based on these solutions becomes very large for grazing angles of incidence and the ratio is independent of the roughness characteristics. The corresponding backscatter cross sections based on the physical optics solutions are polarization independent and the ratio is also independent of the roughness characteristics.

Ample experimental data, however, indicate that at near grazing angles of incidence the vertically and horizontally polarized backscatter cross sections of the earth surface, can be of the same order of magnitude and they are also significantly large than the values predicted by both physical optics and small perturbation theory. A unified full wave approach is used to express the backscatter cross sections as weighted sums of two cross sections. The first, associated with the larger scale rough surface height is given by the physical optics cross section, multiplied by the magnitude squared of the smaller scale rough surface height characteristic function. The second is the cross section associated with the smaller scale surface height, modulated by the slopes of the larger scale surface. When the composite rough surface is characterized by a continuous surface height spectral density function, it becomes necessary to judiciously separate the smaller scale surface from the lager scale surface. It is shown that this can be done using the unified full wave approach by seeking stationary solutions to the cross sections over a wide range of the variational parameters $r = \langle h_s^2 \rangle / \langle h^2 \rangle$ where $\langle h_s^2 \rangle$ and $\langle h^2 \rangle$ are the mean square heights of the smaller scale surface and the total surface respectively. These stationary values for the polarization dependent backscatter cross sections at grazing incidence can be practically equal and they are also significantly larger than the corresponding physical optics and perturbation results. These investigations also impact on the feasibility of relating the backscatter cross section to the remote sensing of moisture content of soil surfaces over gently undulating fields.

Time-reversal Strategies for Extended Target Imaging and Focusing, and Clutter Nulling

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Time-reversal (TR) imaging has been shown (see [1], and the references therein) to be a useful strategy for super-resolved imaging of M point targets from the multistatic response matrix (MRM) K measured by an array of $N > M$ co-located transmitters and receivers. This method works even if there is significant multiple scattering among the targets. For the particular non-multiply scattering case the MRM $K = \sum_{m=1}^{M} \tau_m g(\mathbf{x}_m) g^{*\dagger}(\mathbf{x}_m)$ where $\dagger$ denotes the adjoint, the τ_m's are the target scattering strengths, the $g(\mathbf{x}_m)$'s are linearly independent N-long Green function vectors or "propagators" corresponding to the target locations $\mathbf{x}_m$, and the $g^*(\mathbf{x}_m)$'s are the associated "backpropagators". If (u_i, v_i, λ_i) is the singular system of K, where $K_{u_i} = \lambda_i v_i$, then if $\lambda_i = 0$ the inner product $u_i^\dagger g^*(\mathbf{x}_m) = 0$ for all $m = 1, 2, \ldots, M$ so that in the absence of additive noise the TR MUSIC pseudospectrum $P(\mathbf{x}_m) = (\sum_{\lambda_i=0} |u_i^\dagger g^*(\mathbf{x}_m)|^2)^{-1}$ peaks at the correct target locations while in the presence of noise it yields an image of the targets. In this theory there is a single propagator $g(\mathbf{x}_m)$ and its companion backpropagator $g^*(\mathbf{x}_m)$ per point target. But, as is well known (see [2–4]), for extended targets whose size is not small relative to the smallest probing wavelength one must associate a set of propagators and backpropagators per each target (they depend on both the target's shape and position). The question then arises how to optimally generalize TR imaging when the targets are extended. This generalization is desirable for realistic applications in ground-penetrating radar, SAR imaging, and inverse scattering of large objects. A treatment based on the Born approximation is given in ([5]) which focuses on extended targets that have a uniform scattering potential. The present work considers the more general exact scattering regime and non-uniform scatterers, and also provides a new framework to treat space-time information in TR. The general theory is based on the fact that for a broad class of problems the MRM K essentially takes the more general form $K \simeq \sum_{m=1}^{M} \sum_{q=1}^{\alpha_m(\mathbf{x}_m)} \tau_{m,q} \pi_m^{(q)}(\mathbf{x}_m) [\Pi_m^{(q)}]^\dagger(\mathbf{x}_m)$ where $\alpha_m(\mathbf{x}_m)$ is a finite number of degrees of freedom corresponding to the relevant signal-to-noise ratio (SNR) and where the $\pi_m^{(q)}(\mathbf{x}_m)$'s and $\Pi_m^{(q)}(\mathbf{x}_m)$'s form respective sets of propagators and backpropagators *per target*. The generalized form of the pseudospectrum valid for extended targets can then be shown to be $P(\mathbf{x}_m) = (\sum_{\lambda_i=0} \sum_{q=1}^{\alpha_m(\mathbf{x}_m)} |u_i^\dagger \Pi_m^{(q)}(\mathbf{x}_m)|^2)^{-1}$ which theoretically peaks at the correct target locations. This method and yet more generalized variants of it are validated in this talk with the aid of computer simulations.

The proposed approach can be implemented for cases of increasing complexity ranging from known targets, to targets of partially known support, to completely unknown targets whose support one attempts to deduce from the data. An application emphasized in this talk is imaging of an extended target in the presence of certains kinds of clutter which do not limit the information space versus data space dimensionality restrictions of all TR approaches. This includes ways of suppressing the clutter by reducing the data subspaces that are associated mainly to clutter (interference) which enhances signal-to-interference ratio. Another application, within the medical context, is the minimally invasive focusing of wave energy in a target surrounded by clutter (e.g., surrounding organs).

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A New Fractal-based Approach to Model Scattering from Natural Surfaces with Hurst Exponent H(R)

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Natural surfaces have properties, such as the root-mean square (RMS) slope, which depend on the horizontal scale, R, over which they are estimated. Fractal surface models mimic these observed scale-dependent characteristics and, thus, have come to play an important role in the study of wave scattering from different types of natural rough surfaces. Franceschetti et al. [1], for instance, used the Kirchhoff approximation and a fractional Brownian surface model to develop a practical fractal-based scattering law. Their law subsumes as special cases the Hagfors and Gaussian scattering laws, widely used to model scattering from planetary surfaces, when the Hurst exponent parameter, H, of the fractal model equals 1/2 and 1, respectively. The exponential scattering law, which is a better modeler of the scattering behavior from certain surfaces, is not covered by the fractal-based scattering law of Franceschetti et al., however. In order to find a fractal counterpart to the exponential law, we allow the Hurst exponent to be a function of horizontal scale, $H(R)$. Given an exponential scattering law, whose parameters are obtained through fitting backscatter radar cross section observations, an integral equation is solved, via the Hankel transform, to obtain the Hurst exponent and, consequently, the RMS slope of the surface as a function of horizontal scale [2]. This approach can be extended to provide a scale-explicit parameterization of surfaces for which near-nadir quasi-specular observations are available, and whose scattering behavior is modeled by any linear combination of the aforementioned scattering laws or any others that have a Hankel transform.

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A Semi-rigorous Method for Scattering from 2D Rough Heterogeneous Surfaces

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Scattering from three-dimensional rough heterogeneous media is a challenging problem that can hardly be addressed by the classical numerical methods (FDTD, Coupled-Dipole Approximation, ...) due to the cost of computing over a large three-dimensional domain.

We propose an efficient numerical method to calculate scattering from two-dimensional rough surfaces on top of an assembly of random inclusions. The roughness has large scales compared to radiation wavelength and the scatterers are small particles that radiate like dipoles.

Our approach is semi-rigorous as multiple scattering between particles and between surface and scatterers are accounted for. The Kirchhoff Approximation (KA), however, is used to compute the electric and magnetic currents produced at the interface.

In the KA, the surface currents depend only on the local incident field on the surface, which is the sum of the incident beam and fields scattered by all the particles. The latter are the dipolar response of each particle excited by both the field radiated by the surface currents and the field steeming from the other particles. All these interactions are expressed trough operations on the (unknown) dipolar moments of the scatterers. Finding these dipolar moments then amounts to solve a linear system of size 3N, N being the number of scatterers.

Once this is done, surface currents can be made explicit, as well as the field scattered above by the heterogeneous surface.

Some numerical test cases will be presented. A cross-comparison with other methods will be made on simple geometries to validate the results.