Session 1A2
Effective Parameters of Metamaterials: Difficulties in Definition, Characterization, and Interpretation of Measurements

On Effective Parameters of Periodical Metamaterials

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Various metamaterials, very actively studied in recent years, usually consist of metal inclusions of complex shapes periodically arranged in space. If the period of the lattice and the dimensions of the inclusions are small compared with the wavelength, the material is usually considered as an effectively homogeneous material characterized by effective material parameters (permittivity and permeability). The shape and dimensions of inclusions define the electromagnetic response, which may be rather exotic (negative material parameters, for example).

It is well known that the effective medium description looses its physical meaning when any of the material dimensions (period, inclusion size) becomes comparable with the wavelength. Retrieved effective parameters cannot be anymore used as effective parameters, as they depend on the sample shape and size, as well as on the wave vector of the incident field. Moreover, in this regime these effective parameters are not response function, so that they do not obey causality, for example. There is another complication in modeling metamaterials — the resonant nature of the inclusions. Due to complex inclusion shape, inclusion resonance takes place at frequencies, where the overall inclusion dimension is still much smaller than the wavelength.

Formal retrieval of material parameters from computed or measured reflection and transmission coefficients of metamaterial slabs often leads to results that clearly violate causality and energy conservation laws, indicating that these parameters do not have the usual physical meaning. This can happen in the frequency regions quite far from the lattice resonance, where usually the effective medium description is meaningful. A typical example can be seen in Fig. 2 (left) of paper [1]. Even at low frequencies, quite far from the resonance, where the losses are small, the retrieved permittivity has a negative derivative with respect to the frequency (violating the reactance theorem), and in the resonance region the energy conservation law does not hold. In the literature, these problems have been attributed to the effect of the periodicity of the medium, and it has been suggested that although the meaning of the effective parameters is quite limited, they may be still useful in understanding the material response (e.g., [2]). This shows that the problem of effective parameter description and parameter retrieval calls for further study.

In this presentation we will discuss this problem using the approach of equivalent periodically loaded transmission lines [3](152–154). Extending this method to resonant inclusions, we will study how the impedance calculated from reflection-transmission data relates with the impedance defined as the ratio of averaged electric and magnetic fields. Limitations for the effective medium description coming from material periodicity and from resonant nature of the inclusions will be discussed.

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Characterization of Metamaterials as General Bianisotropic Effective Media

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Left-handed metamaterials are currently realized as a succession of unit cells within which metallic inclusions are printed. Because of the small size of these unit cells compared to the operating wavelength, the characterization of left-handed metamaterials as effective media has been a major research topic for the last few years. At first, uniaxial parameters were sought [1, 2], independently of the inclusions used within the unit cell. Later, it was was shown that a uniaxial representation is not accurate for some designs of inclusions, and non-zero bianisotropic terms were identified in the constitutive relation tensors [3]. Those terms were first estimated based on equivalent capacitances and inductances generated within the unit cell, thus providing a qualitative representation of their behavior with frequency.

The purpose of this presentation is two-folded. First, we provide a rigorous mathematical scheme for the retrieval of the uniaxial and bianisotropic parameters predicted in [3]. The formulation is based on the measurements of the complex reflection and transmission coefficients of a plane wave impinging onto the metamaterial at three incidences with two polarizations each [4]. In each case, the reflection and transmission coecients are expressed analytically, upon which the index of refraction and impedance are redefined to match a unique relation. This relation is then inverted using the same method as the one used for a uniaxial only retrieval [2] and yields the unknown consitutive tensors as function of frequency.

The second purpose of this presentation is to lift all assumptions on the constitutive tensors of the metamaterial. The starting point is therefore very general, where all nine complex parameters of the four constitutive matrices are unknown. Through the multiple measurements of complex reflection and transmission coefficients and the application of optimization algorithms, we show how to retrieve 72 frequency dependent unknown consitutive parameters.

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Negative-definite Media

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The class of negative definite (ND) media is a subclass of lossless bi-anisotropic media with negative definite medium six-dyadic. Thus, it generalizes the class of media which has recently come under great interest, variably labeled as that of double-negative media, negative-index media, backward-wave media, Veselago media or left-handed media. Among examples the class of uniaxially chiral ND media is specially considered. It is shown that the eigenfields are generalizations of TE and TM polarized fields and the Poynting vector of each eigenfield makes an obtuse angle with the propagation vector. When propagating along or transverse to the axis, the eigenwaves are pure backward waves.

Effective Metamaterial Representation by Parameter-fitting of Dispersion Models

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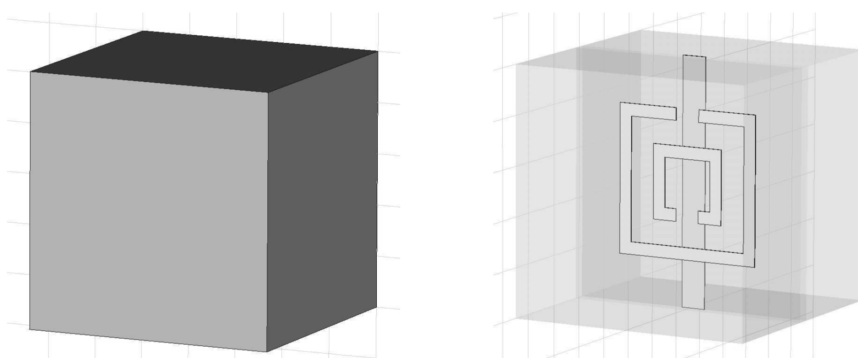
In this paper, a straightforward approach to the extraction of effective electric permittivity and effective magnetic permeability for double negative (DNG) metamaterial structures from 3D field simulation data is presented. Effective parameters are obtained by fitting transmission and reflection of a homogeneous material (Fig. 1(a)) parameterized by frequency dependent Drude model (ϵ_{eff}) and Lorentz model (μ_{eff}) with the transmission and reflection of the simulated metamaterial cell (Fig. 1(b)).

Main difference between the proposed method and the known approaches based on invertion of reflection/transmission results, is that with the presented method one does not need to solve direct equations relating μ_{eff} and ϵ_{eff} with the simulated/measured scattering parameters. In this way one avoids numerical problems connected with the computation of these equations.

Proposed approach is applied to the extraction of effective material parameters for DNG meta- material cells. Optimization results for $\mid s11 \mid$ parameter and extracted effective permittivity obtained with CST Microwave Studio are given in the Fig. 2.



(a) (b)

Figure 1: (a) Effective representation of the SRR/wire structure; (b) SRR/wire reference structure.

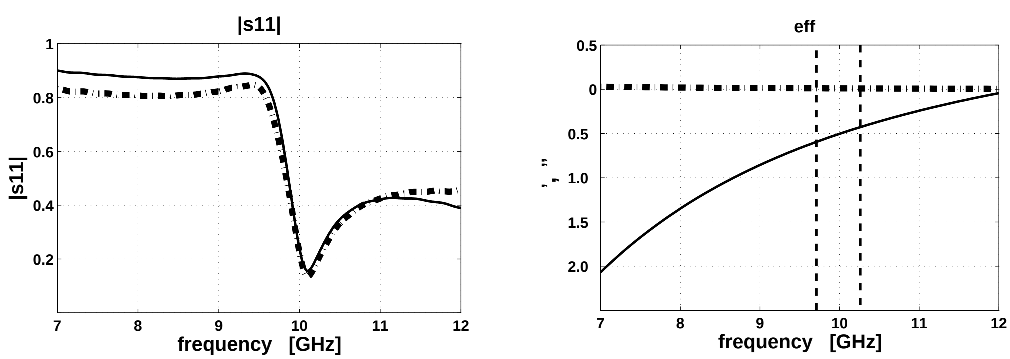


Figure 2: (LHS) Magnitude of the scattering parameter s11 for SRR/wire reference structure from Fig. 1(b) (solid line) and for the optimized effective structure from Fig. 1(a) (dash-dot line); (RHS) Effective electric permittivity $\epsilon_{eff} = \epsilon' - j\epsilon''$ extracted for the optimized structure in Fig. fig:1(a): ϵ' solid line, ϵ'' dash-dot line.

Homogenisation Theory for Resonant Nonlinear Optical Metamaterials

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Metamaterials for nonlinear optics are constructed from embeddings of resonant particulate or stratified planar materials within a matrix of homogeneous other material, which may itself be electro- magnetically linear or nonlinear. Examples of such materials that have been successfully constructed in this way include semiconductor-doped glass (SDG), rare-earth doped silica fibre, similarly doped glass or silica planar waveguides, quantum-well superlattices and quantum-dot media. In all these ma- terials, the nonlinearities may be passive or even active with suitable coupling to an external energy pump. The resonance in the embedded materials (between the carrier frequency of the electromagnetic radiation and transition energies between quantum electronic states of the embedded atoms) results in saturation effects which are intrinsic sources of electromagnetic nonlinearity.

Here the problem of homogenising such materials into effective medium parameters is examined in detail, using quantum-electronic models to describe the embedded resonances in the background medium. It is very well known that, when the optical carrier frequency Ω is far from all resonant frequencies, a nonlinear inhomogeneous medium can be homogenised to an effective homogeneous medium through the use of nonlinear susceptibility tensors $\chi^{(n)}_{k_1 k_2 \ldots k_n}$ for order-n nonlinearities. The situation when resonant interactions occur is much less clear, because the nonresonant nonlinear sus- ceptibilities have no real meaning due to singularities in their description at the resonant frequencies. In particular, proximity effects and local fields play a significant role in the resonant nonlinear regime, making the homogenisation problem considerably more difficult. It turns out that in the fully resonant case a simplified Maxwell-Bloch model is the appropriate homogenised model, with averaged parameters for dipole moments etc. of the averaged effective medium. The averaged medium parameters are obtained from a full quantum-electronic model of the medium by an averaged Lagrangian method.

This theory permits the definition of, for example, the optical gain g of a pumped electronic medium as if g were an effective single parameter of the medium, along with a homogenised model of snonlinear saturation effects.

The Study of Effective Permittivity and Permeability of Curved Meta-materials

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A new structure of metamaterials which is charactered by curved wires and SRRS, calculate permeability and magnetic permeability of artificial media using Perturbation Theory, simulated effective permittivity and permeability of curved Meta-materials, find which radius of curvature the negative effective permittivity and permeability could appearimple.

Restrictions and Limitations of Parameters in the Description of Complex Media

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The electromagnetic characterization of metamaterials is not very easily incorporated into the traditional machinery to analyze fields and waves in ordinary media. Especially the existence of backward waves and the fact that a wave can be refracted to the "wrong" direction requires that the material parameter values have to be reconsidered. Permittivity and permeability have to be negative. But this is counter-intuitive because we are used to thinking of permittivity (and permeability) as measures of electric (magnetic) energy density. Of course, one can respond to this and defend the possibility of such a behavior by observing that resonating elements can cause it in a limited frequency band through the plasma-type characteristics. This is a high-frequency phenomenon. And then, if the elements to resonate can be made sufficiently complex-shaped so that a resonance can be made to take place within small space, one can bring down the negative band into "low" frequencies.

If we have liberated our minds from the taboo of positivity of the permittivity and permeability, concerning its real part, one may ask are there any restrictions left that it should obey. And another natural question is what kind of limitations can be posed on the imaginary parts. Obviously the sign of the imaginary part of the refractive index is fixed for dissipative materials in order to avoid amplification in the energy. But does this lead to separate restrictions for the imaginary parts of permittivity and permeability? In the presentation, these questions will be discussed concerning both isotropic materials and also more complicated ones that may display anisotropy and/or magnetoelectric coupling.

Experimental Extraction of the Effective Properties of Metamaterials from Measured S-parameters

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This talk overviews experimental work related to metamaterials and chiral materials at microwave frequencies and address some of the controversies surrounding effective property extraction for metamaterials. A focused plane wave beam is used to illuminate planar samples in a free space set-up. TRL calibration in conjunction with a vector network analyzer leads to the establishment of two phase and amplitude reference planes for homogenization and effective medium modeling and subsequent extraction of complex permittivity, permeability and magneto-electric coupling if any from S-parameter measurements [1, 2]. Experimental results showing negative permittivity, permeability and refractive index are presented for ordered and random metamaterials that exhibit plasmonic resonances at the frequency range of interest. Several different types of scattering elements were used to make the samples, such as C left, right and racemic mixtures of small metallic springs, metallic Omega shaped elements, combinations of split ring resonators and wire elements. It is shown that it is possible for all such scatterers to exhibit negative refraction. Good agreement is obtained with numerically simulated data for effective properties. It is shown that disordered structures can also lead to NIM behavior thus periodicity is not required. There is current controversy regarding the allowed sign of the imaginary parts of the permittivity and permeability. Experimental data confirming loss in such materials from plots of power absorption, resistive part of the complex impedance and attenuation constant is shown for those cases where one of the material properties exhibits a negative imaginary part. Homogenization methods are used by experimentalists and theorists alike to derive effective medium properties and this is discussed with reference to the experimental study. A new method for extracting the effective properties of Omega media is presented in order to obtain a third material property, the magneto-electric coupling coefficient. Omega media are non-symmetric ($S11 \neq S22$). This however does not resolve the question of negative imaginary parts of the complex properties as had been suggested by some researchers.

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Metamaterials: Mechanisms of Subwavelength Imaging

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Metamaterials are artificial structures comprising of arrays of small resonant elements in which both the size and the distance between the elements are much smaller than the wavelength. A number of metamaterial devices have been designed to manipulate and control the near field following the idea of the perfect lens proposed by Pendy [1]. His idea was to image an object of subwavelength dimensions a distance away from the source plane by a plane parallel slab having a refractive index ?. A silver slab [1] or a multilayered silver metamaterial [2] can serve as a near-perfect lens, with the limit of resolution determined by surface plasmon resonances and by a high-frequency cut-off of the transfer function. There have been imaging experiments with silver slabs (see e. g., [3, 4]), with a single layer of a 2D array of Swiss Rolls [5, 6], and with two or more layers of 2D arrays of split ring resonators [7–10].

The purpose of the present work is to review the mechanisms of subwavelength imaging and discuss the relationship between imaging and focussing. The mechanisms discussed will be the excitation of (i) surface plasmons, (ii) magnetoinductive surface waves and (iii) phase conjugate waves at the outer boundary of the lens. In addition we shall consider microchannelling [11] based on the eigenmodes of the periodic medium and also on a set of magnetoinductive waves evanescent in the transverse direction and propagating in the longitudinal direction. Imaging and focussing will also be described in terms of coupled resonators [7] and finally the general topic of Poynting vector optics [12] will be discussed with indefinite media [8] as one of the examples.

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FDTD Simulation of Perfect Lens Imaging System

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The ability of the Finite-Difference Time-Domain (FDTD) method to model a perfect lens made of a slab of homogeneous left-handed material (LHM) is investigated. It is shown that because of the frequency dispersive nature of the medium and the time discretization, an inherent mismatch in the constitutive parameters exists between the slab and its surrounding medium. This mismatch in the real part of the permittivity and permeability is found to have the same order of magnitude as the losses typically used in numerical simulations. Hence, when the LHM slab is lossless, this mismatch is shown to be the main factor contributing to the image resolution loss of the slab.

Using the Auxiliary Differential Equation (ADE) method to implement the frequency dispersive permittivity and permeability for Drude model, we show that after the FDTD discretization, the numerical permittivity (so as permeability) can be described as

$$\epsilon_r = 1 - \frac{\omega_{pe}^2}{4\sin(\omega_o \Delta t/2)/(\Delta t)^2} \tag{1}$$

It is clear that Eq. 1 approaches the Drude model $\epsilon_r = 1 - \omega_{pe}^2/\omega_o^2$ in the limit of $\Delta t \to 0$, which gives a value of -1 when $\omega_{pe} = \sqrt{2}\omega_o$. However, for a finite Δt used in an actual simulation, ϵ_r presents a slight deviation from exactly -1 at the same ω_{pe}. As an example, the value of ϵ_r from Eq. 1 is about -1.000297 for a typical grid size of $\lambda/100$, which is also the value of the refractive index since we choose here a magnetic plasma frequency identical to the electric one. This small perturbation does not affect the propagating waves significantly. However, the resolution of a subwavelength imaging system is critically dependent on the reconstruction of the evanescent waves, or part of the evanescent wave spectrum, by the LHM slab. This reconstruction is in turn critically dependent on the slab's constitutive parameters, and the slight mismatch of 0.03% in the real part has an important impact on the resolution of the image as we shall see hereafter. It is important to note that this small perturbation in the real part of the constitutive parameters has often been overlooked and the imaginary parts with a value in the same order are typically considered to be the main contributor for limiting the image resolution. By comparing the simulation results and analytical calculations, we demonstrate that the simulated image resolution of an LHM perfect lens is mainly limited by this mismatch. In other applications such as the simulation study of surface plaritons at LHM/RHM interfaces where the matching condition is required, the understanding of this limitation in FDTD can also be very important.

Permeability and Permittivity of Metamaterials Determined by the Field Summation Method

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Negative index materials have been recently demonstrated using adequate topologies of metallic inclusions exhibiting both engineered permeability and permittivity [1]. The opportunities associated to these so-called metamaterials, first envisioned 40 years ago, are continuously expanding to new field of applications and extended wavelengths.

The effective parameters ε and μ are fundamental quantities in the conception of a metamaterial. As a consequence, it is of foremost importance to be able to enhance our ability to predict these quantities for different inclusion topologies, and retrieve them from experimental results. A method has been proposed [2] to determine the effective parameters of heterogeneous materials, from the knowledge of the fields inside the material. This paper shows that this method can be fruitfully applied to the determination of permeability and permittivity of a metamaterial.

As an example, a composite consisting of resonant permeability inclusions has been considered. The fields in the composite have been determined using HFSS commercial software. The permeability and permittivity have been deduced using this field summation method. Reflexion and transmission coefficients have been computed from these values, and are compared with raw numerical prediction by HFSS. It is evidenced on Fig. 1 that a satisfactory agreement is observed.

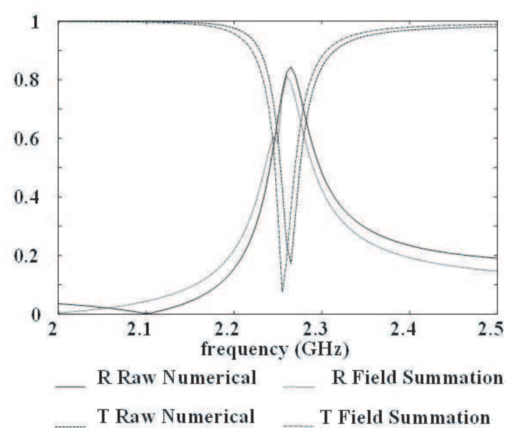

Figure 1: Reflection and transmission coefficient derived from effective parameters obtained by field summation, and compared to raw numerical results, for a composite made of split ring resonators.

Different types of composite materials are considered. In some cases, the field summation method indicates that it is not adequate to describe a metamaterial through an homogenization approach at frequencies close to the resonance. This observation may account for some difficulties in retrieving effective parameters from experimental results, and the scarcity of direct experimental determinations of ε and μ on metamaterials.

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