Effects of the Resonant Scattering of Intensive Fields by Weakly Nonlinear Dielectric Layer

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Abstract—The transverse inhomogeneous, isotropic, nonmagnetic, linearly polarized, weakly nonlinear (a Kerr-like dielectric nonlinearity) dielectric layer is considered. The results of a numerical analysis of the diffraction problem of a plane wave on the weakly nonlinear object with positive and negative value of the susceptibility are shown. The effects: non-uniform shift of resonant frequency of the diffraction characteristics of a weakly nonlinear dielectric layer; itself the channeling of a field; increase of the angle of the transparency of the nonlinear layer when growth of intensity of the field (at positive value of the susceptibility); de-channeling of a field (at negative value of the susceptibility) are found out.

1. The Nonlinear Problem

Let the time dependence be $\exp(-i\omega t)$ and $\vec{E}(\vec{r})$, $\vec{H}(r)$ complex amplitudes of an electromagnetic field. We consider a nonmagnetic, isotropic, transverse inhomogeneous $\varepsilon^{(L)}(z) = 1 + 4\pi\chi_{xx}^{(1)}(z)$, linearly polarized $\vec{E} = (E_x, 0, 0)$, $\vec{H} = (0, H_y, H_z)$ (E-polarized) and Kerr-like weakly nonlinearity $P_x^{(NL)} = \frac{3}{4}\chi_{xxxx}^{(3)} |E_x|^2 E_x$, $\max\limits_{|z|\le 2\pi\delta}\left(|\alpha|\cdot|E_x|^2\right) << \max\limits_{|z|\le 2\pi\delta}\left|\varepsilon^{(L)}(z)\right|$ (where $\vec{P}^{(NL)} = \left(P_x^{(NL)}, 0, 0\right)$ — vector of polarization, $\alpha = 3\pi\chi_{xxxx}^{(3)}$, $\chi_{xx}^{(1)}$ and $\chi_{xxxx}^{(3)}$ is the components of susceptibility tensor) dielectric layer (Fig. 1), [1, 2].

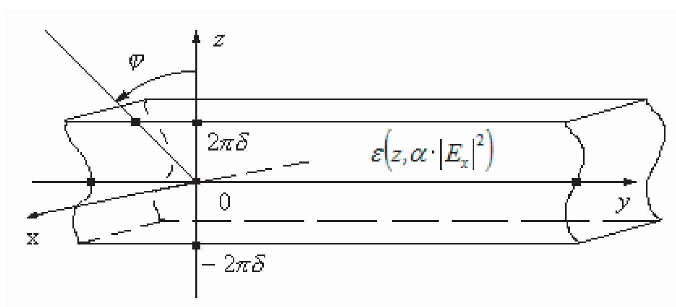

Figure 1: Weakly nonlinear dielectric layer: $\max\limits_{|z|\le 2\pi\delta}\left(|\alpha|\cdot|E_x|^2\right) << \max\limits_{|z|\le 2\pi\delta}\left|\varepsilon^{(L)}(z)\right|$.

The complete diffraction field $E_x(y,z) = E_x^{inc}(y,z) + E_x^{scat}(y,z)$ of a plane wave $E_x^{inc}(y,z) = a^{inc}\exp\left[i(\phi y - \Gamma\cdot(z - 2\pi\delta))\right]$, $z > 2\pi\delta$ on the nonlinear dielectric layer (Fig. 1) satisfies such conditions of the problem:

$$\nabla^2\cdot\vec{E} + \frac{\omega^2}{c^2}\cdot\varepsilon^{(L)}(z)\cdot\vec{E} + \frac{4\pi\omega^2}{c^2}\cdot\vec{P}^{(NL)} \equiv \left(\nabla^2 + \kappa^2\cdot\varepsilon\left(z, \alpha\cdot|E_x|^2\right)\right)\cdot E_x(y,z) = 0, \tag{1}$$

the generalized boundary conditions:

$$\begin{array}{l} E_{tg} \text{ and } H_{tg} \text{ are continuous at discontinuities } \varepsilon\left(z, \alpha\cdot|E_x|^2\right); \\ E_x(y,z) = U(z)\cdot\exp(i\phi y), \text{ the condition of spatial quasihomogeneity along } y; \end{array} \tag{2}$$

the condition of the radiation for scattered field:

$$E_x^{scat}(y,z) = \left\{\begin{array}{c} a^{scat} \\ b^{scat} \end{array}\right\}\cdot e^{i\,(\phi\, y \pm \Gamma\cdot(z \mp 2\,\pi\delta))},\ z \begin{array}{c} > \\ < \end{array} \pm 2\,\pi\delta \tag{3}$$

Here: $\varepsilon\left(z, \alpha\cdot|E_x|^2\right) = \left\{\begin{array}{cc} 1, & |z| > 2\pi\nabla^2 \\ \varepsilon^{(L)}(z) + \alpha\cdot|E_x|^2, & |z| \le 2\pi\delta \end{array}\right.$; $\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $\alpha = 3\pi\chi_{xxxx}^{(3)}$; $\Gamma = \left(\kappa^2 - \phi^2\right)^{1/2}$; $\phi \equiv \kappa\cdot\sin(\varphi)$; $|\varphi| < \pi/2$ (see Fig. 1); $\kappa = \omega/c \equiv 2\pi/\lambda$; $c = (\varepsilon_0\,\mu_0)^{-1/2}$, ε_0, μ_0 and λ length of the wave are the parameters of environment.

In this case the required solution of the problem (1)–(3) has the form:

$$E_x\left(y,z\right)=U\left(z\right)\cdot e^{i\phi y}=\begin{cases} a^{inc}\cdot\ e^{i(\phi y-\Gamma\cdot(z-2\pi\delta))}\ +\ a^{scat}\cdot e^{i(\phi y+\Gamma\cdot(z-2\pi\delta))}, & z>2\pi\delta,\\ U^{scat}\left(z\right)\cdot\ e^{i\phi y}, & |z|\leq 2\pi\delta,\\ b^{scat}\cdot e^{i(\phi y-\Gamma\cdot(z+2\pi\delta))}, & z<-2\pi\delta.\end{cases}\tag{4}$$

Here $U\left(-2\pi\delta\right)=b^{scat}$, $U\left(2\pi\delta\right)=a^{inc}+a^{scat}$.

The nonlinear problem (1)–(3) is reduced to finding the solutions $U\left(z\right)\in L_2\big([-2\pi\delta,2\pi\delta]\big)$ (see (4)) of the inhomogeneous nonlinear integrated equation of the second kind [3, 4]:

$$U\left(z\right)+\frac{i\,\kappa^2}{2\Gamma}\int\limits_{-2\,\pi\,\delta}^{2\,\pi\,\delta}\exp\left(i\Gamma\cdot|z-z_0|\right)\left[1-\left(\varepsilon^{(L)}\left(z_0\right)+\alpha\left|U\left(z_0\right)\right|^2\right)\right]U\left(z_0\right)\,dz_0=U^{inc}\left(z\right), \qquad |z|\leq 2\pi\delta, \tag{5}$$

where $U^{inc}\left(z\right)=a^{inc}\exp\left[-i\Gamma\cdot\left(z-2\,\pi\delta\right)\right]$.

The integrated equation (5) with application of the quadrature method and use (4) is reduced to system of the nonlinear equations of the second kind [4].

2. Susceptibility and Effects Resonant Scattering of the Intensive Fields

2.1. Intensity and Resonant Frequency

The effect of non-uniform shift of resonant frequency of the diffraction characteristics of nonlinear dielectric layer is found out at increase of intensity of inciting field [4, 5] (see Fig. 2(a), at positive value of the susceptibility $\alpha=0.01$, and also Fig. 2(b), at negative value of the susceptibility $\alpha=-0.01$). Growth of intensity of

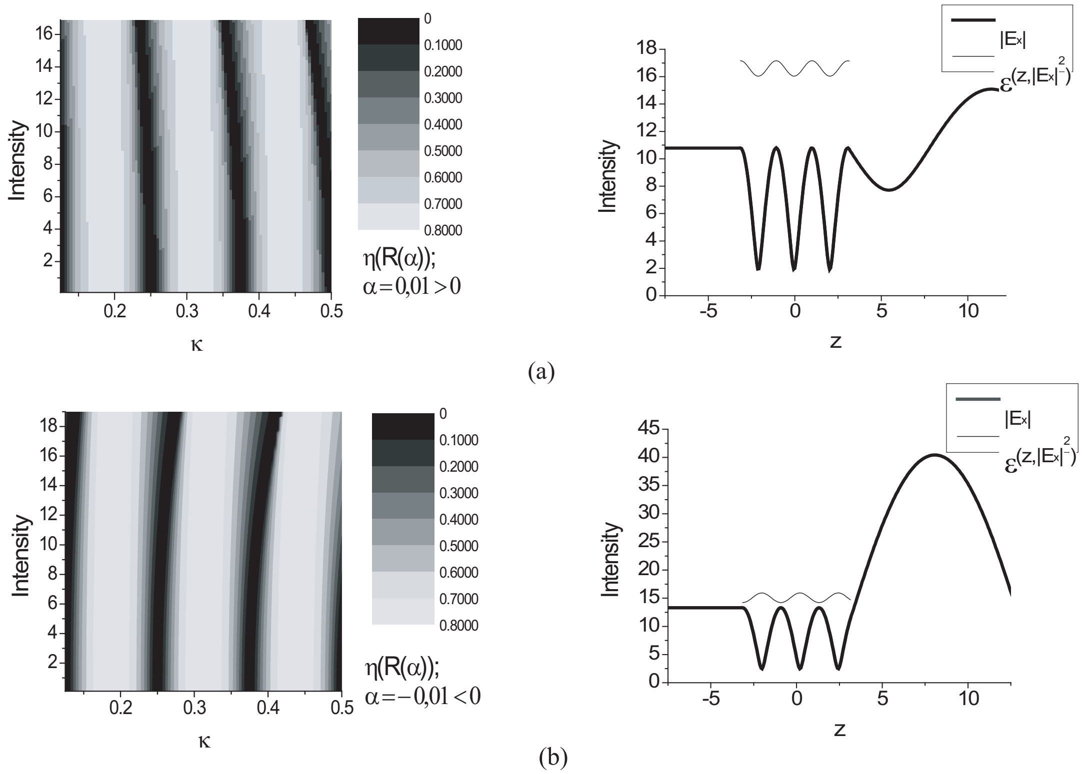

Figure 2: Parameters of structure: $\delta=0.5$; $\varphi=45^0$; $\kappa=0.375$; $\varepsilon^{(L)}=16$. (a)$|I|=\left|{}^{inc}a\right|=11.4$; $\alpha=0.01$, (b)$|I|=\left|{}^{inc}a\right|=22.4$; $\alpha=-0.01$.

the inciting field $|I|=\left|a^{inc}\right|$ results in change of the share of the reflected wave $\eta\left(R\left(\alpha\right)\right)=\left|R\left(\alpha\right)\right|^2\Big/\left|I\right|^2$ (here $\left|R\left(\alpha\right)\right|\equiv\left|a^{scat}\left(\alpha\right)\right|$, $\left|T\left(\alpha\right)\right|\equiv\left|b^{scat}\left(\alpha\right)\right|$, $\left|I\right|^2=\left|T\left(\alpha\right)\right|^2+\left|R\left(\alpha\right)\right|^2$): reduction of value of resonant frequency with increase and reduction of a steepness of the diffraction characteristics before and after resonant frequency

(Fig. 2(a), at $\alpha > 0$); increase of value of resonant frequency with reduction and increase of a steepness of the diffraction characteristics before and after resonant frequency (Fig. 2(b), at $\alpha < 0$).

2.2. Intensity and Angle

The effects: itself the channeling of a field — increase of the angle of the transparency of the nonlinear layer ($\alpha \neq 0$) when growth of intensity of the field (Fig. 3(a), at positive value of the susceptibility, $\alpha > 0$); de-channeling of a field (Fig. 3(b), at negative value of the susceptibility, $\alpha < 0$) are found out, [4, 5].

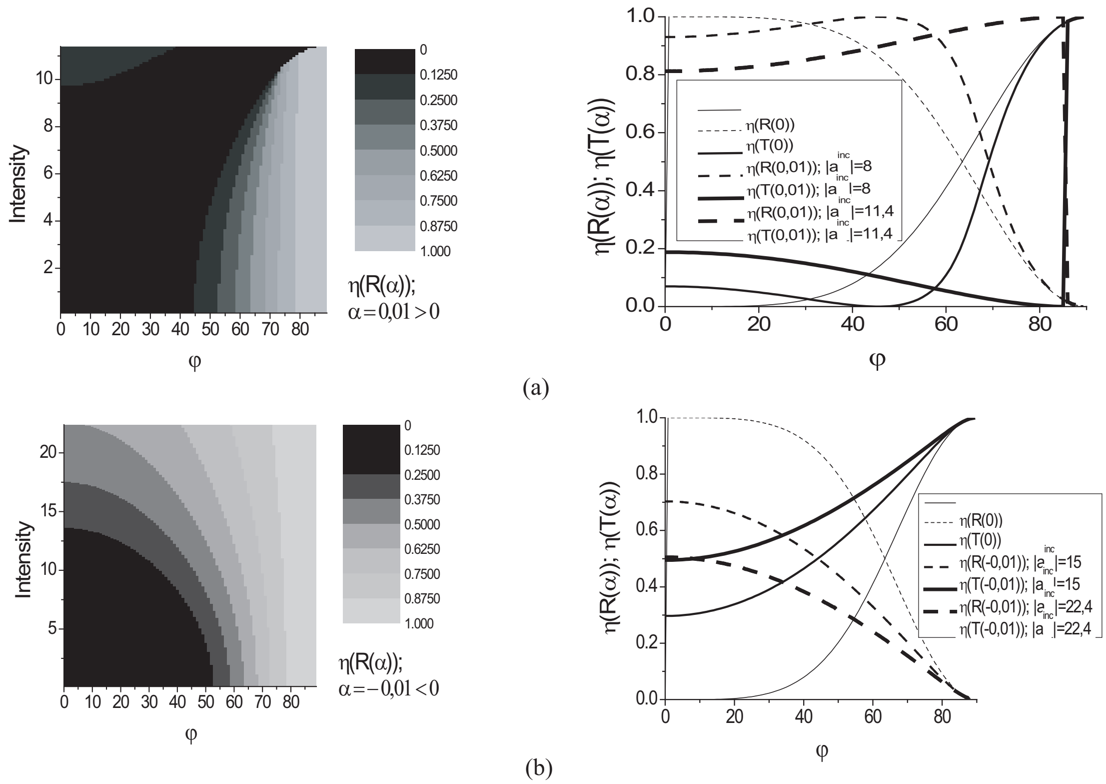

Figure 3: Parameters of structure: $\delta = 0.5$; $\kappa = 0.375$; $\varepsilon^{(L)} = 16$; for linear layer with $\alpha \equiv 0$ and for nonlinear layer: a with $\alpha = 0.01$; b with $\alpha = -0.01$.

The increase of the angle of a transparency with growth of intensity at positive value of the susceptibility $\alpha = 0.01$ is easy for tracking on Fig. 3(a): $\left|a^{inc}\right| = 8$, $\varphi \approx 46^\circ$ and $\left|a^{inc}\right| = 11.4$, $\varphi \approx 85^\circ$.

Weak nonlinearity of a dielectric layer $\varepsilon\left(z, \alpha \cdot |E_x|^2\right) \equiv \varepsilon\left(z, \alpha \cdot |U|^2\right)$,

$$\max_{|z| \leq 2\pi\delta} \left(|\alpha| \cdot |E_x|^2\right) << \max_{|z| \leq 2\pi\delta} \left|\varepsilon^{(L)}(z)\right|, \tag{6}$$

i. e., the small nonlinear additive $\alpha\left|U(z)\right|^2$ to a linear part $\varepsilon^{(L)}(z)$ of the dielectric permeability, caused by intensity $\left|U^{inc}\right|$ of a field of excitation of nonlinear object, results in essential changes diffraction characteristics. Exceeding some critical threshold of intensity the statement (6) loses force, computing process is broken. For example, diffraction characteristics reach critical values with growth of intensity of field, see lines for $\alpha > 0$ on Fig. 3(a): point of a transparency $\varphi = \varphi^*(|a^{inc}|)$, where $\eta(R)|_{\varphi=\varphi^*(|a^{inc}|)} = 0$ and $\eta(T)|_{\varphi=\varphi^*(|a^{inc}|)} = 1$, here $\varphi^*(|a^{inc}|)$ defined from: $\frac{d\eta(R)}{d\varphi}|_{\varphi=\varphi^*(|a^{inc}|)} = \frac{d\eta(T)}{d\varphi}|_{\varphi=\varphi^*(|a^{inc}|)} = 0$, weakly nonlinear layer aspires to limiting value $\varphi^*(|a^{inc}|) \to 90^\circ$ at $|a^{inc}| \to \max\{|a^{inc}|\} = 11.5$. The analysis of results for $\alpha < 0$ on Fig. 3(b) shows, that limiting critical values $\eta(R)|_{\varphi=\varphi^*(|a^{inc}|)\equiv 0^\circ} \to 0.5$ and $\eta(T)|_{\varphi=\varphi^*(|a^{inc}|)\equiv 0^\circ} \to 0.5$ at $|a^{inc}| \to \max\{|a^{inc}|\} = 22.4$ lay on curves of translucent $\eta(R) = \eta(T) = 0.5$ weakly nonlinear structure. It allows to estimate numerically size of required intensity of a field of excitation

$$\max_{|z| \leq 2\pi\delta} \left(|\alpha| \cdot \left|U(z)\right|^2\right) \leq \max_{|z| \leq 2\pi\delta} \left(|\alpha| \cdot \left|U^{inc}(z)\right|^2\right) < C \cdot \max_{|z| \leq 2\pi\delta} \left|\varepsilon^{(L)}(z)\right| \tag{7}$$

to make an estimation weakly sizes C, at which (6) does not lose force with growth of intensity of a field of excitation of a nonlinear layer.

For example, see Fig. 3(a), (where: $\varepsilon^{(L)}(z) = 16$, $\alpha = 0.01$), convergence of iterative process is broken when $\left|U^{inc}\right| > 11.5$. From (7) it is received: $C = 0.083$. Hence, weak nonlinearity proves at intensity not surpassing $\left|U^{inc}\right| = 11.5$ and variations of small nonlinearity layer: $\max\limits_{|z|\leq 2\pi\delta}\left(|\alpha| \cdot |U(z)|^2\right) < 1.328$.

These effects (see sections 2.1 and 2.2) are connected to resonant properties of a nonlinear dielectric layer and caused by increase at positive value of the susceptibility or reduction at negative value of the susceptibility of a variation of dielectric permeability of a layer (its nonlinear components) when increase of intensity of a field of excitation of researched nonlinear object.

3. Conclusion

The principal fields where the results of our numerical analysis are applicable are as follows: the investigation of wave self-influence processes; the analysis of amplitude-phase dispersion of eigen oscillation-wave fields in the nonlinear objects, see [6]; extending the description of evolutionary processes near to critical points of the amplitude-phase dispersion of nonlinear structure; new tools for energy selecting, transmitting, and remembering devices; etc.

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