On the Stability of the Electromagnetic Field in Inhomogeneous Anisotropic Media With Dispersion

N. V. Budko
Delft University of Technology, The Netherlands

A. B. Samokhin
Moscow University of Radioengineering and Electronics, Russia

From the electromagnetic point of view various meta-materials, optical crystals, geophysical formations, ice, magnetized plasma, etc., can be described as inhomogeneous anisotropic media with dispersion. The interaction of the electromagnetic field with such media can be studied by different analytical and numerical methods. When a three-dimensional object is of finite extent and is situated in free space, then the method of choice is the Volume Integral Equation (VIE) method, sometimes referred to as the Domain Integral Equation method (mathematical literature) and the Discrete Dipole Approximation (physics).

In contrast to the one- and two-dimensional cases, where existence of the solution to the scattering problem is a trivial question, in the three-dimensional case solution exists under certain conditions related to the physical properties of the medium in question. Note that the sufficient uniqueness conditions are basically the same for all cases. Previously we have shown that the singular integral operator of the VIE has an essential continuous spectrum, which is given explicitly in terms of the constitutive parameters of an inhomogeneous object. Now we shall extend this result to anisotropic media with dispersion. We shall also prove that in the quasi-static case the discrete eigenvalues are contained within the complex envelope of the continuous spectrum.

For anisotropic media with dispersion, especially for magnetized plasma, the question of consider- able interest is the stability of such a medium. Within the commonly adopted approach, based on the differential form of the Maxwell's equations and Lorentz or Vlasov's equations of motion, very little can be said about the stability of *inhomogeneous* objects of finite extent, whereas the VIE formulation is perfectly suited for this task. Instead of analyzing the stability of the medium itself we propose to analyze the stability of the electromagnetic field in a given medium. Due to self-consistency of the problem the two approaches are in fact identical. Thus, we shall discuss the stability of the field in several practical cases ranging from optical crystals to plasma.