Source Representations of the Debye Potentials in Spherical Coordinates

M. J. Lahart
U.S. Army Research Laboratory, USA

In a widely cited paper [1], Bouwkamp and Casimir derived the relationship between the Debye scalar potentials and their charge and current sources. They did this by computing the electric and magnetic fields associated with each of the potentials and using Maxwell's equations to compute the charge and current sources. Their derivation specified that the scalar potentials could only by defined outside a sphere that contained all of the charge and current sources. Nisbet [2] challenged the need for this restriction and claimed that the Debye potentials could be defined everywhere, including regions that contained charge and current sources.

This presentation examines the need to define the Debye potentials only in regions where the charge and current sources are zero. Some possible definitions of scalar potentials in terms of magnetic and electric vector potentials will be examined. It will be shown that, to be consistent with Maxwell's equations, some definitions require that the scalar potentials obey the wave equation, while others require that only the components of the gradients of the potentials in two orthogonal directions obey the wave equation. In spherical coordinate systems, only the latter type of definition is possible; potentials that obey the wave equation cannot be defined, but potentials whose gradient components in the θ and ϕ directions can be. By expressing the Debye potentials in terms of the magnetic and electric vector potentials and examining the consistency of the expressions with Maxwell's equations, it will be shown that one of the potentials can be defined in regions that contain charges and currents and the other cannot.

For comparison, it will be shown that scalar potentials that obey the wave equation can be defined in rectangular coordinates. Because of this, a pair of potentials can be defined in regions where charge and current sources are present. An example will be given by expressing fields in a waveguide in terms of scalar potentials and their charge and current sources.

REFERENCES

1. Bouwkamp, C. J. and H. B. G. Casimir, "On multipole expansions in the theory of electromagnetic radiation," *Physica (Utrecht)*, Vol. 20, 539–554, 1954.
2. Nisbet, A., "Source representations for Debye's electromagnetic potentials," *Physica(Utrecht)*, Vol. 21, 799–802, 1955.