Implementation of the PML in the CIP Method

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The constrained interpolation profile (CIP) method, a numerical solver for multiphase problems, can be applied to electromagnetic problems. The method is based on the upwind scheme for the finite difference method, but the variables to be calculated are not only the values of the electromagnetic fields, but also the spatial derivatives. Those variables are used to interpolate the profiles between the grids by means of cubic polynomials, and to calculate them at the next time step with good precision.

Invoking the directional splitting in the Maxwells equations allows us to treat electromagnetic fields as two one-way waves in each direction, and to reduce them into advection equations. For example, in $\pm x$-direction of a 2-dimensional problem with $\frac{\partial}{\partial z} = 0$, the equations in free-space are given by

$$\frac{\partial f^{\pm}(r,t)}{\partial t} \pm c\frac{\partial f^{\pm}(r,t)}{\partial x} = 0, \quad \frac{\partial g^{\pm}(r,t)}{\partial t} \pm c\frac{\partial g^{\pm}(r,t)}{\partial x} = 0, \tag{1}$$

where $f^{\pm}(r,t) = \sqrt{\epsilon}E_z \mp \sqrt{\mu}H_y$, $g^{\pm} = \frac{\partial f^{\pm}}{\partial x}$, and c is the velocity. The reduced equations can be solved by using CIP method.

The CIP method has an absorbing boundary condition (ABC) as good as the 1st Mur's ABC in its nature. But, it is necessary to develop the ABC with better performance if required. In this study, we examine the perfect matched layer (PML) in the CIP scheme.

The application is straightforward, but some considerations are necessary in the computation because the implementation yields non-advective terms:

$$\frac{\partial f^{\pm}(r,t)}{\partial t} \pm c\frac{\partial f^{\pm}(r,t)}{\partial x} = -s(x)f^{\pm}(r,t), \quad \frac{\partial g^{\pm}(r,t)}{\partial t} \pm c\frac{\partial g^{\pm}(r,t)}{\partial x} = -\frac{\partial\{s(x)f^{\pm}(r,t)\}}{\partial x}, \tag{2}$$

where $s(x)$ is the normalized conductivity of the PML. One of the solution is obtained by dividing the equations into advection phase:

$$\frac{\partial f^{\pm}(r,t)}{\partial t} \pm c\frac{\partial f^{\pm}(r,t)}{\partial x} = 0, \quad \frac{\partial g^{\pm}(r,t)}{\partial t} \pm c\frac{\partial g^{\pm}(r,t)}{\partial x} = 0, \tag{3}$$

and then, non-advection phase

$$\frac{\partial f^{\pm}(r,t)}{\partial t} = -s(x)f^{\pm}(r,t), \quad \frac{\partial g^{\pm}(r,t)}{\partial t} = -\frac{\partial\{s(x)f^{\pm}(r,t)\}}{\partial x} = -\frac{ds(x)}{dx}f^{\pm}(r,t) - s(x)g^{\pm}(r,t). \tag{4}$$

Let $f^{\pm,*}$ denote the results of advection phase. The first equation can be evaluated analytically:

$$f^{\pm,n+1} = f^{\pm,*} \cdot e^{-s(x)\Delta t}, \tag{5}$$

where $f^{\pm,n+1}$ stands for the value at the next times step. The evaluation of the second equation in Eq. (4) can be performed numerically:

$$g^{\pm,n+1} = g^{\pm,*} - \Delta t\{-\frac{ds(x)}{dx}f^{\pm,*} - s(x)g^{\pm,*}\}. \tag{6}$$

The successful formulation of the PML in the CIP method enables us to absorb the outgoing waves as much as required by increasing the layers. The numerical experiments show the good performance of the present formulation.