Generation of Diverse Time-series Data though Monitoring a Death-multiple Immigration Population Model

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Discrete population models have been employed in combination with random walk techniques to model successfully non-Gaussian clutter occurring in coherent imaging systems [e.g., 1, 2]. The method models the coherent returns from an ensemble of scatterers as a random walk comprising a fluctuating number of steps. Non-Gaussian limiting distributions obtain when the stochastic process describing the discrete distribution is subject to clustering. In particular, a simple mathematical paradigm for turbulence is the birth-death-immigration process, where turbulent eddies nucleate (immigration), are shed (birth) and dissipate (death). The equilibrium distribution is then of the negative binomial class, this being the discrete analogue of continuous gamma-distributed fluctuations, and the clutter is then K-distributed [2]. Here we discuss the properties of a death-multiple immigration model [3], which allows for pairs, triplets, ... n-tuplets to enter the population, and which has the useful property of enabling a very wide class of equilibrium distributions to be constructed, including the negative-binomial class and distributions with scale free-characteristics. Allowing "individuals" to leave the population creates a series of events in time [4], whose characteristics can be tailored to exhibit a wide range of behaviours, together with correlation properties including non-Poissonian processes and fractals. The utility to model non-Gaussian fractal processes using the technique will be discussed [5], together with the wider implications for the generation of time series.

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