3D and 2.5D AGLID EMS Stirring Modeling in the Cylindrical Coordinate System

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Abstract—We have proposed the new GL and AGILD modeling and inversion in the PIERS 2005 in Hangzhou. In this paper, we propose 2.5D AGILD modeling algorithm for electromagnetic (EM) stirring, motor and generator design. In the cylindrical coordinate system, the EM field is vector function of r, θ, and z. The electrical conductivity is only depended on radial coordinate r and vertical coordinate z. Upon substituting the Fourier serious of the magnetic field into the strip differential integral equation on boundary strip with pole $\rho = 0$ and Galerkin equation in the internal sub domain, we construct 2.5D AGILD EM stirring modeling in cylindrical coordinate system for the steel and metal continuous casting. There are serious difficulties in the EM stirring modeling by using FEM method and FD method. First, there is u/ρ^2 term in the Maxwell magnetic field differential equation in the cylindrical coordinate system, the pole $\rho = 0$ is strong coordinate singularity. The coordinate singularity is difficult in the EM stirring modeling by using FEM and FD method. Our 2.5 AGILDEMS modeling method resolved this difficulty. There is no any coordinate singularity in our 2.5D EM differential integral equation. Second, because the conductivity in air is zero but it is 10^5 in steel, what is a suitable boundary condition on $\rho = 0$ for current, electric field, and magnetic field that is another difficulty when FEM method and FD method to be used. Our AGILDEMS overcome this difficulty. Based on our 2.5D AGILDEMS algorithm, we developed the 2.5D AGILDEMS modeling software. Many applications show that the 2.5D AGILDEMS software is a powerful tool for design of the EM stirring and real time control monitor in the continuous casting. The AGILD K-ε flow modeling and software are developing and joining with our AGILD EMS modeling for continuous casting. GL EMS and AGILD EMS modeling can be used for micro, nano motor, generator and geophysics and materials.

1. Introduction

In the steel and metal continuous caster, the electromagnetic (EM) stirring (EMS) is an established technique and important approach for improving steel quality. Many EMS with variable style have been working in the steel and metal continuous caster industrial in the world. To exactly calculate the EM field and determine the bloom/billet's size and properties in EMS are an important and difficult task. Because the conductivity in the air environmental is zero but 50,000 1/ohm in steel. The sharp high contrast is difficult in inversion. The EM field artificial boundary condition for infinite domain is inaccurate and complicated. The coordinate singularity is another difficulty in FEM for EMS modeling in the cylindrical coordinate system. The existing EM FEM method and software are not accurate to calculate EM field in EMS. The EMS properties inversion for steel material and conductivity is necessary to develop. We have proposed the new GL and AGILD modeling and inversion in the PIERS 2005 in Hangzhou [1, 2]. We propose the GL method and its advantages for resolving the historical difficulties [3] and the stochastic AGILD EM modeling and inversion in Piers 2006 in Cambridge [4]. In this paper, we propose the 2.5D AGILD EMS stirring modeling using our magnetic field differential integral equation and magnetic field Garlekin equation. Our AGILD EMS modeling is an important tool for EMS design and EMS real time processes monitoring in the continuous caster. Also EMS modeling and inversion are useful for variable motor and generator design, environment, geophysics, coaxial antenna, etc. sciences and engineering.

The description order in this paper is as follows. In the section 2, we derive the 3D and 2.5D magnetic field strip differential integral equations in the cylindrical coordinate system. The 3D and 2.5D magnetic field strip Garlekin equations are derived in the section 3. In the section 4, we present the 3D and 2.5D EMS modeling. The applications of the EMS modeling is described in the section 5. In the section 6, we describe conclusions.

2. The 3D and 2.5D Magnetic Field Strip Differential Integral Equations

We derive the 3D and 2.5D magnetic field differential integral equations in the strip domain in the cylindrical coordinate system in this section. We call the equations to be the strip magnetic field differential integral equations.

2.1. The 3D Magnetic Field Strip Differential Integral Equation

Upon substituting the field and coordinate transformation between the rectangle and cylindrical coordinate system, we derive the 3D magnetic field strip differential integral equation in the cylindrical coordinate system as follows

$$\begin{aligned}
&FH3(H,H_{b\rho},H_b^{M_\rho},E_b^{M_\rho})\\
&=H_{b\rho}+\int_\Omega \frac{((\sigma+i\omega\varepsilon)-(\sigma_b+i\omega\varepsilon_b))}{(\sigma+i\omega\varepsilon)}\left(E_{b\rho}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial H_z}{\partial\theta}-\frac{\partial H_\theta}{\partial z}\right)+E_{b\theta}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)+E_{bz}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}(\rho' H)_\theta-\frac{1}{\rho'}\frac{\partial H_\rho}{\partial\theta}\right)\right)(r')d\rho' d\rho' d\theta dz\\
&+\int_{\partial\Omega-}\frac{1}{(\sigma_b+i\omega\varepsilon_b)}E_b^{M_\rho}\times H\cdot d\vec{S}\\
&-\int_{\partial\Omega-}\frac{1}{(\sigma+i\omega\varepsilon)}\left(\left(H_{b\theta}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho' H_\theta-\frac{1}{\rho'}\frac{\partial H_\rho}{\partial\theta}\right)-H_{bz}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)\right)\rho dz+\left(H_{b\rho}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)-H_{b\theta}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial H_z}{\partial\theta}-\frac{\partial H_\theta}{\partial z}\right)\right)\right)\rho' d\rho',
\end{aligned}\tag{1}$$

$$\begin{aligned}
H_\rho(r)&=FH3(H,H_{b\rho},H_b^{M_\rho},E_b^{M_\rho}),\\
H_\theta(r)&=FH3(H,H_{b\theta},H_b^{M_\theta},E_b^{M_\theta}),\\
H_z(r)&=FH3(H,H_{bz},H_b^{M_z},E_b^{M_z}),
\end{aligned}\tag{2}$$

where E is the electric field, H is the magnetic field, E_b^m and H_b^m is Green function exciting by the magnetic dipole source, $E_b^m(r',r)$ has weak and integrative singular at $r=r'$, the r locates in the outside boundary of the strip or in the subsurface with $\rho'=0$, the r' locates in $\partial\Omega-$, the internal boundary of the strip, therefore, the 3D strip magnetic field differential integral equation has no coordinate singular at pole $\rho'=0$. It has integrative weak singular kernel.

2.2. The 2.5D Magnetic Field Differential Integral Equation

Substituting the EM field Fourier series, $H(\rho,\theta,z)=\sum_{m=-\omega}^{\infty}H_m(\rho,z)e^{im\theta}$, into the 3D strip magnetic field differential integral equation (2), we derive the 2.5D equations in the cylindrical coordinate system

$$\begin{aligned}
&FH25(H,H_{b\rho},H_b^{M_\rho},E_b^{M_\rho})\\
&=H_{b\rho}+\int_\Omega\frac{((\sigma+i\omega\varepsilon)-(\sigma_b+i\omega\varepsilon_b))}{(\sigma+i\omega\varepsilon)}\left(E_{b\rho}^{M_\rho}\left(\frac{1}{\rho'}imH_z-\frac{\partial H_\theta}{\partial z}\right)+E_{b\theta}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)+E_{bz}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho H_\theta-\frac{1}{\rho'}imH\rho\right)\right)(r')\rho' d\rho' d\theta' dz'\\
&+\int_{\partial\Omega}\frac{1}{(\sigma_b+i\omega\varepsilon_b)}E_b^{M_\rho}\times H\cdot d\vec{S}\\
&-\int_{\partial\Omega}\frac{1}{(\sigma+i\omega\varepsilon)}\left(\left(H_{b\theta}^{M_\rho}\left(\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\rho' H_\theta-\frac{1}{\rho'}imH_\rho\right)-H_{bz}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)\right)\rho dz+\left(H_{b\rho}^{M_\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho'}\right)-H_{b\theta}^{M_\rho}\left(\frac{1}{\rho'}imH_z-\frac{\partial H_\theta}{\partial z}\right)\right)\right)\rho' d\rho',
\end{aligned}\tag{3}$$

$$\begin{aligned}
H_\rho&=FH25(H,\,H_{b\rho},\,H_b^{M_\rho}E_b^{M_\rho}),\\
H_\theta&=FH25(H,\,H_{b\theta},\,H_b^{M_\theta}E_b^{M_\theta}),\\
H_z&=FH25(H,H_{bz},\,H_b^{M_z}E_b^{M_z}),
\end{aligned}\tag{4}$$

3. The 3D and 2.5D Magnetic Filed Garlekin Equation

We derive the 3D and 2.5D magnetic field Garlekin equation in the cylindrical coordinate system.

3.1. The 3D Magnetic Field Garlekin Equation

Substituting field and coordinate transformation from rectangle to cylinder into the magnetic field Galerkin equation [2], we derive the 3D magnetic field Garlekin equation in the cylindrical coordinate system as follows

$$\begin{aligned}
&\int_\Omega\frac{1}{\sigma+i\omega\varepsilon}\left(\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho}\right)\frac{\partial\phi}{\partial z}-\left(\frac{\partial}{\partial\rho}\rho H_\theta-\frac{\partial H_\rho}{\partial\theta}\right)\frac{1}{\rho^2}\frac{\partial\phi}{\partial\theta}\right)\rho d\rho d\theta dz+i\omega\int_\Omega\mu H_\rho\phi\rho d\rho d\theta dz=-i\omega\int_\Omega\mu M_\rho\phi\rho d\rho d\theta dz,\\
&\int_\Omega\frac{1}{\sigma+i\omega\varepsilon}\left(-\left(\frac{1}{\rho}\frac{\partial H_z}{\partial\theta}-\frac{\partial H_\theta}{\partial z}\right)\frac{\partial\phi}{\partial z}+\left(\frac{\partial}{\partial\rho}\rho H_\theta-\frac{\partial H_\rho}{\partial\theta}\right)\frac{1}{\rho^2}\frac{\partial\rho\phi}{\partial\rho}\right)\rho d\rho d\theta d+i\omega\int_\Omega\mu H_\theta\phi\rho d\rho d\theta dz=-i\omega\int_\Omega\mu M_\theta\phi\rho d\rho d\theta dz,\\
&\int_\Omega\frac{1}{\sigma+i\omega\varepsilon}\left(\left(\frac{1}{\rho}\frac{\partial H_z}{\partial\theta}-\frac{\partial H_\theta}{\partial z}\right)\frac{1}{\rho}\frac{\partial\phi}{\partial\theta}-\frac{\partial\phi}{\partial\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho}\right)\right)\rho d\rho d\theta dz+i\omega\int_\Omega\mu H_z\phi\rho d\rho d\theta dz=-i\omega\int_\Omega\mu M_z\phi\rho d\rho d\theta dz,
\end{aligned}\tag{5}$$

3.2. The 2.5D Magnetic Field Garlekin Equation

Upon substituting the Fourier series, $H(\rho,\,\theta,\,z)=\sum_{m=-\omega}^{\infty}H_m(\rho,\,z)e^{im\theta}$ into the 3D Garlekin equation (5), we derive the 2.5D magnetic field Garlekin equation in the cylindrical coordinate system as follows

$$\begin{aligned}
&\int\limits_{\Omega}\frac{1}{\sigma+i\omega\varepsilon}\left(\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial \rho}\right)\frac{\partial\phi}{\partial z}-\left(\frac{\partial}{\partial\rho}\rho H_\theta-imH_\rho\right)\frac{1}{\rho^2}im\phi\right)\rho d\rho dz+i\omega\int\limits_{\Omega}\mu H_\rho\phi\rho d\rho dz=-i\omega\int\limits_{\Omega}\mu M_\rho\phi\rho d\rho dz,\\
&\int\limits_{\Omega}\frac{1}{\sigma+i\omega\varepsilon}\left(-\left(\frac{1}{\rho}imH_z-\frac{\partial H_\theta}{\partial z}\right)\frac{\partial\phi}{\partial z}+\left(\frac{\partial}{\partial\rho}\rho H_\theta-imH_\rho\right)\frac{1}{\rho^2}\frac{\partial\rho\phi}{\partial\rho}\right)\rho d\rho dz+i\omega\int\limits_{\Omega}\mu H_\theta\phi\rho d\rho dz=-i\omega\int\limits_{\Omega}\mu M_\theta\phi\rho d\rho dz,\\
&\int\limits_{\Omega}\frac{1}{\sigma+i\omega\varepsilon}\left(\left(\frac{1}{\rho}imH_z-\frac{\partial H_\theta}{\partial z}\right)\frac{1}{\rho}im\phi-\frac{\partial\phi}{\partial\rho}\left(\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial\rho}\right)\right)\rho d\rho dz+i\omega\int\limits_{\Omega}\mu H_z\phi\rho d\rho dz=-i\omega\int\limits_{\Omega}\mu M_z\phi\rho d\rho dz,
\end{aligned}\tag{6}$$

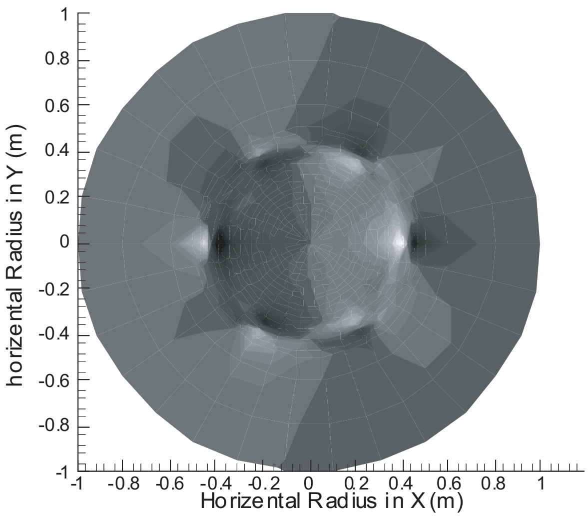

Figure 1: Rotation magnetic field H_θ in time =0 s.

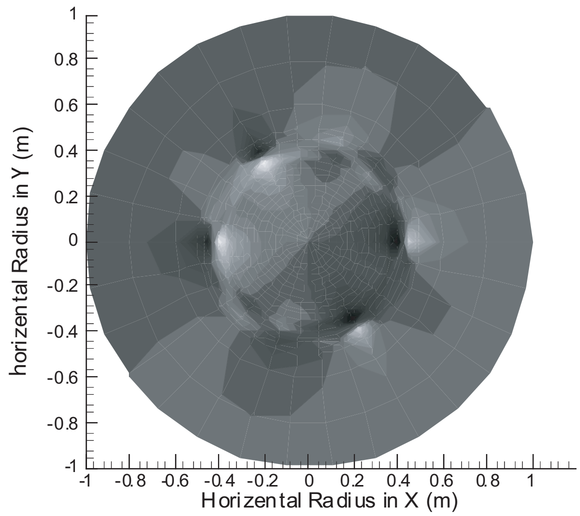

Figure 2: Rotation magnetic field H_θ in time =0.1 s.

4. 3D and 2.5 D EMS Modeling

4.1. 3D EMS Modeling

We use collocation FEM the 3D strip magnetic field differential integral equation (2) in the boundary strip domain including pole point $\rho=0$, and the 3D magnetic field Galerkin equation (5) in the reminder internal domain without pole $\rho=0$ to construct 3D AGILD EMS magnetic field modeling for EM field in the Stirring and motor etc industrial engineering and sciences.

4.2. 2.5D EMS Modeling

We use collocation FEM the 2.5D strip magnetic field differential integral equation (4) in the boundary strip domain including pole point $\rho=0$, and use the 2.5D magnetic field Galerkin equation (6) in the reminder internal domain without pole $\rho=0$ to construct 2.5D AGILD EMS magnetic field modeling for EM field in the stirring and motor etc industrial engineering and sciences.

5. The Applications of the AGILD EMS Modeling

Our 3D and 2.5D AGILD and GL EMS modeling has been used to calculate the EM field for several EM stirring with variable style. Some asynchronous EMS stirring is designed as follows: its outer radius is 500 mm, the internal radius is 350 mm, and it is divided 6 sectors. The electric current has inverse direction for any adjoining two sectors. The input electric current density intensity is 1 A/mm^2. The frequency is 4 Hz. Before installation of the stirring without steel flow, the factor did measure the magnetic field intensity. By using digit magnetic GAUSS meter, the measurement value of the magnetic field intensity at center of the stirring is 1500 Gauss. By using our 2.5D AGILD EMS modeling simulation, the evaluated magnetic field intensity is 1513.28 Gauss at center of the stirring. The rotational EM field is very accurate and very stable. The AGILD EMS rotation magnetic field in caster $H\rho(\rho, \theta, \text{zc}, \text{t})$ at the 0.0∼0.25 second are plotted in the Figures 1 and 4. They show that by using the GL EMS and AGILD EMS modeling, the rotational magnetic field's frequency is exactly 4 Hz. The GL EMS [3, 5] and 2.5D AGILD EMS magnetic field H_ρ, H_θ intensity are plotted in Figures 5 and 6, the red curve is the GL magnetic field and blue curve is AGILD magnetic field, the two curves are close matched. GL EMS and AGILD EMS modeling can be used for micro, nano motor, generator and group holes geophysics and materials etc. We are developing GL and AGILD K-ε model steel flow driving by the EMS Lorentz force and join it with AGILD EMS modeling to work for the steel and metal continuous casters.

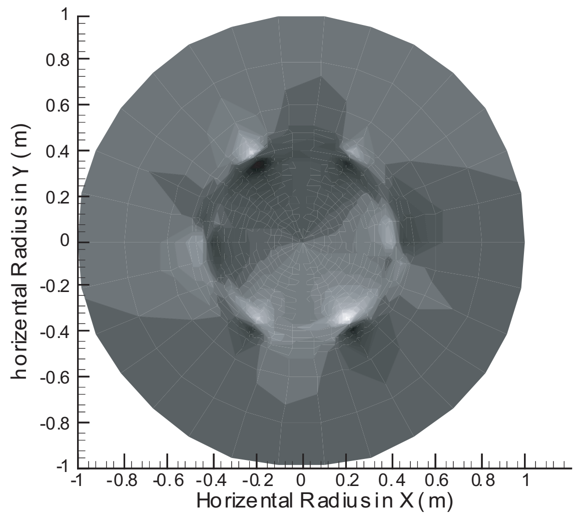

Figure 3: Rotation magnetic field H_θ in time =0.2 s.

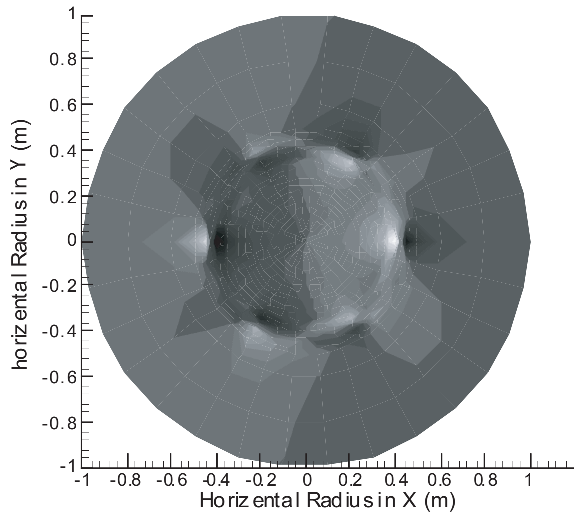

Figure 4: Rotation magnetic field H_θ in time =0.25 s.

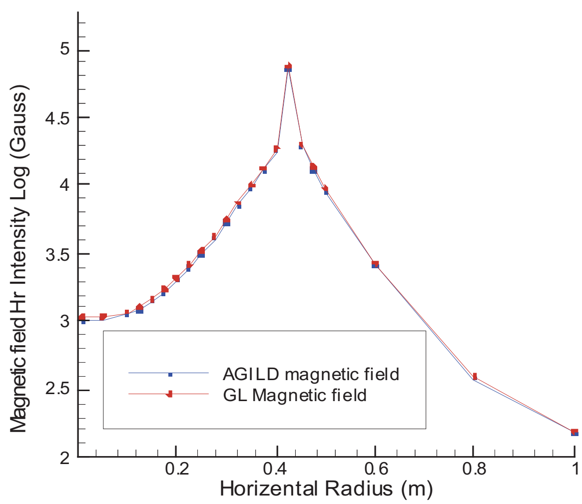

Figure 5: The magnetic field H_ρ intensity ,The red line is GL magnetic field, The blue line is AGILD magnetic field.

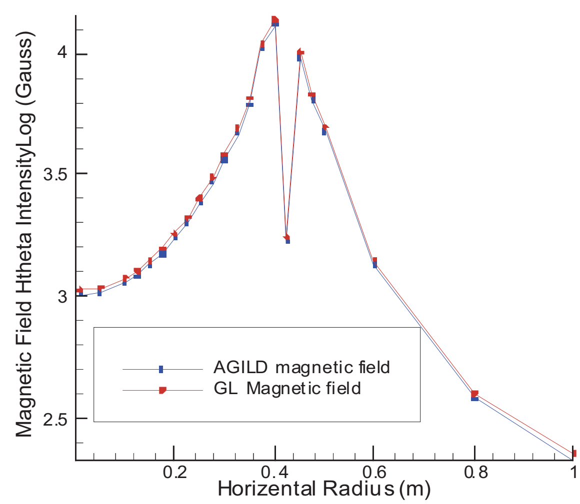

Figure 6: The magnetic field H_θ intensity, The red line is GL magnetic field, The blue line is AGILD magnetic field.

6. Conclusions

Many EM field in the stirring and motor simulations show that the 3D and 2.5D AGILD, and GL EMS modeling are accurate and fast and stable. The AGILD EMS has merits over existing FEM, FD, and Born approximation. The 3D and 2.5D AGILD and GL EMS modeling will be new tools for widely applications in the sciences and engineering.

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