Fast Computation of Diffraction by Finite-size Multilayered Arrays of Cylinders

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Scattering by gratings or arrays of cylinders has been extensively studied for many years in the areas of remote sensing and optics. Scattering matrix method (SMM), also called T-matrix method, is the most popular method used for the calculation of multiple scattering among all the cylinders. SMM utilizes the T-matrix to describe the scattering property of each single cylinder and the addition theorems of cylindrical harmonics to take account of mutual couplings. So it is viewed as a kind of semianalytical method. Recently, many authors found new applications of scattering matrix method on the simulation of finite size photonic crystal devices. Scattering matrix method is much faster than those purely numerical methods such as finite difference time domain (FDTD) or finite element method (FEM). However, its computational complexity is still $O(N^2)$. This prohibits its further applications in large size devices, where N stands for the total harmonics numbers used to expand the fields for all cylinders.

In this paper, a novel algorithm, named as fast multipole accelerated scattering matrix method (FMA-SMM), is proposed to speed up the solution of SMM. Fast matrix method (FMM) has already been used to solve the integral equations of 2D scattering problems. The principal formula is the integration expression of zero-order Hankel function over the range of polar angle. It can be named as the fast multipole expression of zero-order Hankel function. Since Higher order, instead of only zero-order, Hankel functions are often involved in SMM, FMM could not be used directly. Fortunately, we derived the general fast multipole expressions for any order Hankel functions by using the lowering and rising operators of cylindrical harmonics. Through numerical investigations, we found that the higher order Hankel functions requires larger group size to reach specific error criterion. This has not been pointed out before since only zero-order Hankel function was investigated in previous publications. This new finding is especially important in successful implementation of the algorithm. The general expressions derived convert the dense coupling matrix in SMM into the combination of sparse matrices, namely aggregation matrix, translation matrix and disaggregation matrix. Thus results in a lower computational complexity of $O(N^{1.5})$ when iterative method is used to solve the final equations. The details of implementation will be presented to guarantee the accuracy of the algorithm. The accuracy and efficiency of FMA-SMM are verified by several numerical examples. A large array with more than 2,000 dielectric rods is analyzed to show the advantage of this novel fast algorithm.