Non-lorentzian Electromagnetic Resonances

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In quantum mechanics, scattering amplitude exhibits resonance behavior if the energy of scattered particles is close to the energy of one of the quasi-stationary states, if such quasi-stationary states exist. For potentials which decay fast enough at infinity, the resonance cross sections, as functions of energy, can be accurately approximated by Lorentzians. A similar phenomenon can be found in electromagnetic scattering. Consider scattering of monochromatic waves with the frequency ω by a homogeneous non-magnetic scatterer of arbitrary shape characterized by the dielectric function $\epsilon(\omega)$. We can define an electromagnetic eigenstate as a solution to $\int_V \hat{G}_0(\omega;\mathbf{r},\mathbf{r}')\mathbf{P}_n(\omega;\mathbf{r}')d^3r' = \xi_n(\omega)\mathbf{P}_n(\omega;\mathbf{r})$. Here V is the volume occupied by the scattering material, $\hat{G}_0(\omega;\mathbf{r},\mathbf{r}')$ is the frequency-domain, dyadic free-space Green's function for the Maxwell's equation which gives electric field at the point $\mathbf{r}$ due to a point dipole oscillating at frequency ω at the point $\mathbf{r}'$, $\mathbf{P}_n(\omega;\mathbf{r})$ is the n-th polarization eigenstate and $\xi_n(\omega)$ is the corresponding eigenvalue (generally, complex). The extinction cross section can be written as a sum over the eigenmodes, i.e., $\sigma_e = \sum_n f_n(\omega)/[z(\omega) - \xi_n(\omega)]$, where $f_n(\omega)$ is the generalized oscillator strength for the n-the eigenmode which has no singularities in the complex plane as a function of ω and $z(\omega)$ is the spectral variable defined by $z(\omega) = (4\pi/3)[\epsilon(\omega)+2]/[\epsilon(\omega)-1]$ [1]. Electromagnetic resonances take place when the denominator in the above equation is in some sense small. However, the imaginary part of the denominator can not vanish due to energy conservation considerations. Therefore, we define resonance frequencies ω_n as solutions to $\mathrm{Re}[z(\omega_n) - \xi_n(\omega_n)] = 0$. If ω is close to one of the resonance frequencies ω_n, and if $\xi_n(\omega)$ and $f_n(\omega)$ change slowly in the vicinity of ω_n, one can make the quasi-particle pole approximation and write $\sigma_e \approx [f_n(\omega_n)/z'(\omega_n)]/[\omega - \omega_n + i\gamma_n]$, where $\gamma_n = \mathrm{Im}[z(\omega_n) - \xi(\omega_n)]/z'(\omega_n)$ and prime denotes differentiation. This resonance has the typical Lorentzian structure with the lifetime $\tau_n = 1/\gamma_n$ which is determined by the sum of Ohmic ($\mathrm{Im}[z(\omega_n)]$) and radiative ($-\mathrm{Im}[\xi(\omega_n)]$) losses.

In scatterers which are small compared to the external wavelength, the quasi-particle pole approximation is, typically, quite accurate. This is due to the fact that, within the quasistatics, the real parts of $\xi_n(\omega)$ are ω-independent an satisfy $-8\pi/3 < \mathrm{Re}\xi_n < 4\pi/3$ [2]. In extended systems these statements are, generally, not valid. In particular, in a long periodic chain of nanospheres, real parts of eigenvalues ξ_n diverge logarithmically near certain frequencies which are determined from the synchronism condition [3]. This divergence leads to electromagnetic resonances which are essentially non-Lorentzian. In particular, their width is determined not by relaxation but by the range of frequencies in which the equation $\mathrm{Re}[z(\omega) - \xi_n(\omega)] = 0$ is approximately satisfied. It was shown that these resonances are super-exponentially narrow with the width being proportional to the factor $\exp[-C(h/a)^3]$, where C is a numerical constant of the order of unity, h is the period of the chain and a is the nanosphere radius [4]. The divergence of eigenvalues can also lead to narrow spectral holes which were already reported in [3]. Recent advances in nanofabrication have reinvigorated interest in one-dimensional chains of nanoparticles. A dramatic narrowing of spectral lines and unusual properties electromagnetic resonances were found numerically in chains of large but finite length in [5]. The origin and properties of these resonances in infinite chains were discussed theoretically in [4]. Theoretical treatment of finite chains was recently given in [6].

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