A New Formulation for Scattering by Impedant 3D Bodies

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Abstract—A new integral equation formulation is introduced for solving, in the frequency domain, the problem of electromagnetic scattering by an impedant (IBC) or perfect electric/magnetic (PEC/PMC) 3D body of arbitrary shape. It is based firstly, on a special application of the equivalence principle [2] where the 0-field exterior domain is filled with another impedant medium and, secondly, on the widely used PMCHW (Poggio, Miller, Chang, Harrington and Wu) formulation which forces field continuity through the scatterer surface [3]. Unlike other IBC formulations such as [4], this one also applies to PEC/PMC. Furthermore, in this last case, it appears to stabilize the numerical scheme in the vicinity of eigen frequencies. We will provide proofs and conditions of the wellposedness of the problem for impedant as well as for PEC/PMC bodies.

1. Introduction

Since the pioneering work of Leontovitch, Impedance Boundary Condition (IBC) has been widely used to simplify electromagnetic scattering problems. It simulates the material properties of a surface by forcing surface electric and magnetic fields to respect: $\mathbf{E}_{tan} = R\mathbf{n} \times \mathbf{H}_{tan}$, $R \in \mathbb{C}$ [1] where $\mathbf{n}$ is the unit normal to the surface pointing into the outside of the impedant medium. It is absorbing when $Re(R) > 0$. Range of validity of IBC for imperfect conductors has been discussed in [1]. Many specific implementations have been surveyed, but only a few general numerical method are available. The last ones are from Lange [5] and Bendali [4]. Beyond the algebric approach, [5] appears to be very similar to the proposed new formulation. It mimics the widely used PMCHW (Poggio, Miller, Chang, Harrington and Wu) method [3] and introduces a specific parameter which behaves like an impedant "complement medium" whose impedance would be equal to the scatterer's one. But, none of [5] and [4] methods extends to perfect electric (PEC) or magnetic (PMC) conductors. The proposed new formulation follows a more physical approach. It is based on a special application of the equivalence principle [2] where the 0-field exterior domain is filled with another impedant medium and on the use of the PMCHW technic. It does not require scatterer and complement domain impedances to be the same and, most of all, it extends to PEC and PMC bodies ($R_s \longrightarrow 0$ or ∞).

This paper describes a way to generalize [5] formulation. Before posing the concerned integral equation system, we briefly remind how the initial problem is decomposed. The well posedness of the formulation is then demonstrated. Finally we give some numerical illustrations which validate this approach and point out its advantages.

2. Subproblem Decomposition

The equivalent principle [2] conduces to decompose any problem into several subproblems, each one being dedicated to a given portion of the original problem. Given a subproblem, we denote "active domain" the piece of problem extracted from the original one. The space surrounding an active domain is named "complement domain". Fields are expected to be null there. 0 field being a Maxwell Equation solution whatever the medium within a source free domain, this allows to choose any medium for the complement domain. This property is often used to transform a subproblem into a free space problem by replacing a scatterer by free space. It is seldom used in other cases. The proposed formulation uses it twice: once, classically, in the first subproblem, by filling the scatterer volume with free space and, another time, in the second subproblem, by filling the complement domain with an impedant medium.

In order to illustrate this approach, let us consider a scatterer in free space lighted by a plane wave. We refer by D_S to the region of space embodying the scatterer. Its surface is denoted Γ. We refer by D_E ("exterior domain") to the rest of the space interesting the problem. Normal vectors will always be supposed to be unit vectors pointing outside the specified domain: n_S and n_E pointing from D_S, respectively D_E, toward D_E, respectively D_S. The initial problem is decomposed into 2 subproblems as follow (Fig. 1):

PbE: the exterior problem. It includes: an active domain D_E containing free space, a complement domain CD_E filled with free space, a set of surface electric and magnetic fictive currents, respectively $\mathbf{J}_E$ and $\mathbf{M}_E$ on

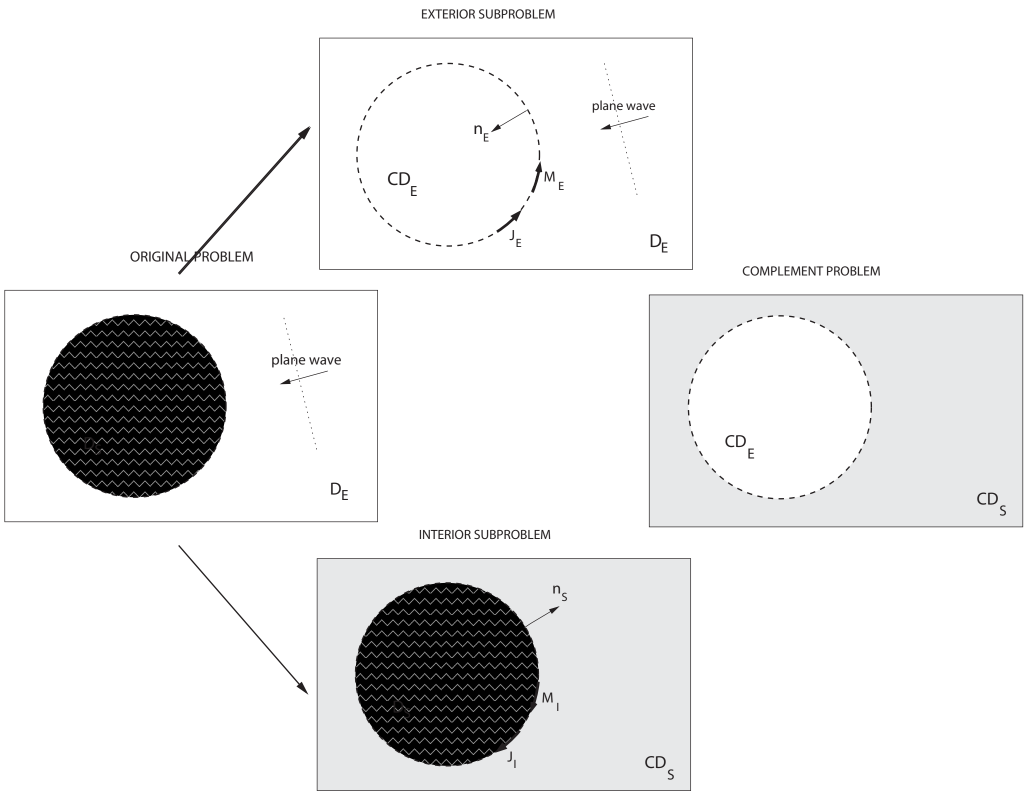

Figure 1: subproblems decomposition.

Γ, impressed sources generating a plane wave incident field. It is well known that this construction leads to a standard problem where fictive sources radiate in a free space environment.

PbI: the interior problem. It includes: an active domain D_S containing the impedant scatterer with local impedance R_s, a complement domain CD_S, filled with an impedant medium caracterised by its local impedance R_c, a set of surface electric and magnetic fictive currents, respectively $\mathbf{J}_I$ and $\mathbf{M}_I$ on Γ, no impressed sources. Due to the complement choice, fields must satisfy an impedance bondary condition on both sides of interface Γ. On the scatterer side of Γ (point denoted x_s):

$$\mathbf{E}_{tan}(x_s) = R_s \mathbf{n}_S \times \mathbf{H}_{tan}(x_s) \tag{1}$$

On the complement side of Γ (point denoted x_{cs}):

$$\mathbf{E}_{tan}(x_{cs}) = -R_c \mathbf{n}_S \times \mathbf{H}_{tan}(x_{cs}) \tag{2}$$

3. Integral Equation Formulation

Once all subproblems posed, we evaluate, for each subroblem independantly, the scattered field on the active side of the interface radiated by fictive currents.

PbE radiating operators In a free space environment, fields radiated by surface currents are controlled by the familiar Stratton-Shu and jump relations on the interface. The field observed at point x_e on the D_E side of surface Γ is given by (Refer to [4] for expressions of Z and operators T and K):

$$\mathbf{E}^{pbE}_{tan}(x_e) = \mathbf{E}^{inc}_{tan}(x) + ikZ(T\mathbf{J_E})_{tan} + (K\mathbf{M_E})_{tan} + \tfrac{1}{2}\mathbf{M_E} \times \mathbf{n}_E$$
$$\mathbf{H}^{pbE}_{tan}(x_e) = \mathbf{H}^{inc}_{tan}(x) - (K\mathbf{J_E})_{tan} + ikZ^{-1}(T\mathbf{M_E})_{tan} - \tfrac{1}{2}\mathbf{J_E} \times \mathbf{n}_E$$

PbI radiating operators A right combination of the usual boundary conditions [2] that links E and H fields on both side of a current sheet running on Γ

$$\left\{ \begin{array}{l} \mathbf{E}^{pbI}_{tan}(x_s) = \mathbf{E}^{pbI}_{tan}(x_{cs}) + \mathbf{M_I} \times \mathbf{n}_S \\ \mathbf{H}^{pbI}_{tan}(x_{cs}) = \mathbf{H}^{pbI}_{tan}(x_s) + \mathbf{J_I} \times \mathbf{n}_S \end{array} \right. \tag{3}$$

and IBC relations (1) and (2) leads to the E and H field expression:

$$\left\{\begin{array}{l}\mathbf{E}^{pbI}_{tan}(x_s) = \frac{R_s}{R_c+R_s}(-R_c\mathbf{J_I} + \mathbf{M_I}\times\mathbf{n}_S)\\ \mathbf{H}^{pb1}_{tan}(x_s) = -\frac{R_c}{R_c+R_s}(\mathbf{J_1}\times\mathbf{n}_S + \frac{\mathbf{M}_I}{R_c})\end{array}\right. \tag{4}$$

When the scatterer medium tends toward PEC ($R_s \to 0$), (4) reduces to:

$$\left\{\begin{array}{l}\mathbf{E}^{pb1}_{tan}(x_s) = 0\\ \mathbf{H}^{pb1}_{tan}(x_s) = -\mathbf{J_1}\times\mathbf{n}_S - \frac{\mathbf{M_1}}{R_c}\end{array}\right.$$

Beyond there simplicity, they appear to be local operators, the numerical implementation of which does not require any long calculation and leads to a sparse matrix.

Connection— According to PMCHW, integral equations are built by forcing equality between surface fields associated to both subproblems:

$$\left\{\begin{array}{l}\mathbf{J_I} = -\mathbf{J_E}\\ \mathbf{M_I} = -\mathbf{M_E}\end{array}\right. \quad and \quad \left\{\begin{array}{l}\frac{R_s}{R_c+R_s}(-R_c\mathbf{J_I} + \mathbf{M_I}\times\mathbf{n}_S) = \mathbf{E}^{inc}_{tan}(x) + ikZ(T\mathbf{J_E})_{tan} + (K\mathbf{M_E})_{tan} + \frac{1}{2}\mathbf{M_E}\times\mathbf{n}_E\\ -\frac{R_c}{R_c+R_s}(\mathbf{J_1}\times\mathbf{n}_S + \frac{\mathbf{M}_I}{R_c}) = \mathbf{H}^{inc}_{tan}(x) - (K\mathbf{J_E})_{tan} + ikZ^{-1}(T\mathbf{M_E})_{tan} - \frac{1}{2}\mathbf{J_E}\times\mathbf{n}_E\end{array}\right. \tag{5}$$

4. Well Posedness

It worth pointing out that the formulation is not a strict application of the equivalence principle. In particular, nowhere it imposes 0 fields outside active domains. This fundamental characteristic must be proven independantly. In this intent, we define a new subproblem called "complement problem" PbC. It is built from the union of the complement domains of both subproblems, CD_E and CD_S plus their interface Γ.

According to the way subproblems are built, PbE and PbI solutions restricted to their respective complement domains CD_E and CD_S are solutions of the complement problem PbC:

on CD_S side $\left\{\begin{array}{l}\mathbf{E}^{pbC}_{tan}(x_{cs}) = \mathbf{E}^{pbI}_{tan}(x_{cs})\\ \mathbf{H}^{pbC}_{tan}(x_{cs}) = \mathbf{H}^{pbI}_{tan}(x_{cs})\end{array}\right.$ and on CD_E side $\left\{\begin{array}{l}\mathbf{E}^{pbC}_{tan}(x_{ce}) = \mathbf{E}^{pbE}_{tan}(x_{ce})\\ \mathbf{H}^{pbC}_{tan}(x_{ce}) = \mathbf{H}^{pbE}_{tan}(x_{ce})\end{array}\right.$.

Furthermore, PMCHW formulation forces equality between surface fields located into PbE and PbI active sides. By applying (3), one can easily prove that PMCHW formulation works as well with fields observed into the complement sides: $\left\{\begin{array}{l}\mathbf{E}^{pbI}_{tan}(x_{cs}) = \mathbf{E}^{pbE}_{tan}(x_{ce})\\ \mathbf{H}^{pbI}_{tan}(x_{cs}) = \mathbf{H}^{pbE}_{tan}(x_{ce})\end{array}\right.$. Consequently, in the Complement problem $\left\{\begin{array}{l}\mathbf{E}^{pbC}_{tan}(x_{cs}) = \mathbf{E}^{pbC}_{tan}(x_{ce})\\ \mathbf{H}^{pbC}_{tan}(x_{cs}) = \mathbf{H}^{pbC}_{tan}(x_{ce})\end{array}\right.$, tangential components of field are continuous through Γ and, finally, PbC appears to be a source free problem. AS FAR AS IT IS NOT A SINGULAR PROBLEM subject to eigen modes, its unique solution is ZERO. This proves that field solutions are equal to 0 in all complement domains whatever the subproblem.

Consequently, PbI and PbE solutions are the same as the ones provided by the equivalence principle, combination of which is known to be the unique solution of the original problem.

Finally, we can conclude that the well posedness condition requires that *the problem built on the complement domains union is a non singular problem.*

5. Numerical illustrations

Numerical results obtained with a unit sphere meshed with planar triangles (750 edges) confirm the formulation validity and advantages. Equivalent currents and test functions are expanded using RWG elements [6].

- Accuracy: in the case of an IBC sphere ($R_s = 100$), we have compared numerical results obtained from three formulations: the new formulation, CERFACS implementation of Leontovitch problem [4] and Mie series with boundary condition (1) imposed at the sphere surface [2]. The sphere is lighted from the bottom ($+z$ direction) by a x-polarised plane wave which wave number is set to $k = 2$. Complement medium impedance is $R_c = 2$. Fig. 2 reports the radar cross section (RCS) observed in different direction using the 3 methods. Angle 0 corresponds to the direction of incidence. The 3 resulting curves are in perfect agreement. New formulation and CERFACS RCS results are strictly superimposed. This visual feeling is confirmed by the relative errors values on equivalent currents computed via the 3 methods (see Tab. 1).
- Numerical stabilization: the behaviour of one selected RWG current element of a PEC sphere ($R_s = 0$) has been followed when wave number k varies in the vicinity of the first eigenfrequency of the spherical

cavity: $k_e = 2,76$. In this case, we use edge excitation by turning on edge 1 (excitation vector set to $\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}$). Fig. 3 reports real and imaginary parts of the observed flux as a function of k when EFIE or proposed formulation is used. One can easily notice that the resonance peak, that clearly appears with EFIE, is suppressed by the new method.The proposed formulation is thus no subject to spurious solutions when $R_s \longrightarrow 0$ or ∞.

Table 1: Equivalent current relative error.

degree of freedom	formulations	error
electric	Cerfacs / New formulation	1,6%
electric	New formulation/ Mie	1,7%
magnetic	New formulation / Mie	1,6%

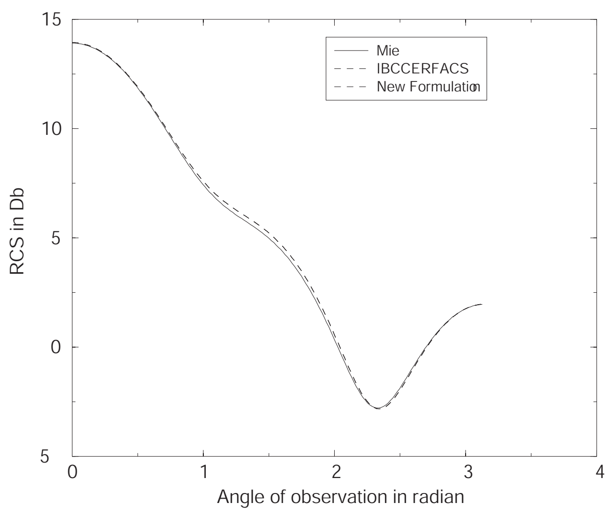

Figure 2: RCS obtained by 3 methods.

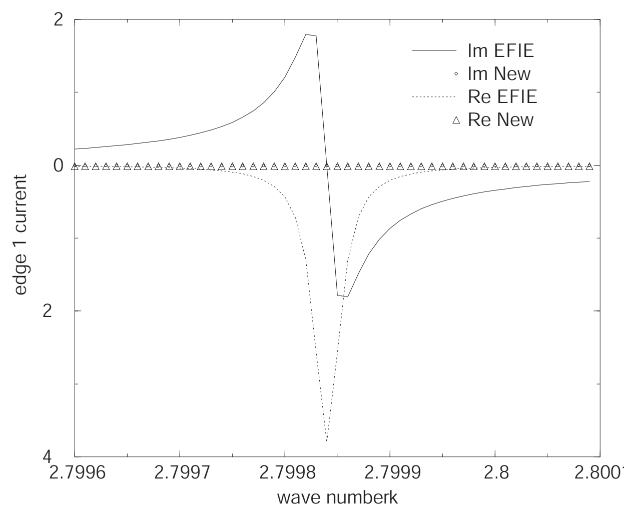

Figure 3: Real and imaginary part of current value computed by EFIE and new formulation.

4. Conclusion

The proposed formulation provides practitioners in computational electromagnetism with a general well posed method to deal with all kinds of impedant bodies, from usual IBC medium up to very good and even perfect conductors without any risk of spurious solution. Interior problem local operators are very easy to implement using RWG elements. They generate a negligible extra computation compared to the one needed for the exterior problem. Since magnetic currents must always be taken into account, even for PEC/PMC, the main drawback is the doubling of the number of degrees of freedom compared to [4]. In addition, it worth noting that the well posedness condition which states that the complement problem must be non singular could be extended to all forms of PMCHW formulations.

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