Analysis and a Novel Design of the Beamspace Broadband Adaptive Array

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Abstract—An analysis of the broadband beamspace adaptive array is provided. There are two conditions imposed on the array. First, the individual beams should have a good frequency invariant property. Second, they should be linearly independent. However, these two conditions are not independent and it is shown that there is a trade-off between them. To improve the interference cancellation capability of the array, we may need to sacrifice the frequency invariant property of the beams to some degree for more linearly independent beams. A DFT-modulated design method is also proposed, where the beam directions are uniformly distributed over the spatial domain and the linear independence of the beams is guaranteed inherently. Simulation results verified our analysis and the proposed method.

1. Introduction

Adaptive beamforming has found numerous applications in various areas ranging from sonar and radar to wireless communications [1]. For arrays to accomplish nulling over a wide bandwidth, tapped-delay lines (TDLs) are employed, resulting in an array with M sensors and TDLs of length J, as shown in Fig. 1. To perform beamforming with high interference rejection and resolution, we need to employ a large number of sensors and long TDLs, which unavoidably increases the computational complexity of its adaptive part and slows down the convergence of the system. To reduce the computational complexity of a broadband adaptive beamformer and increase its convergence speed, Many methods have been proposed, including the time-domain subband adaptive beamformer [2, 3], a combination of subband decomposition in both the temporal and spatial domains [4], and those based on frequency invariant beamforming techniques [5, 6].

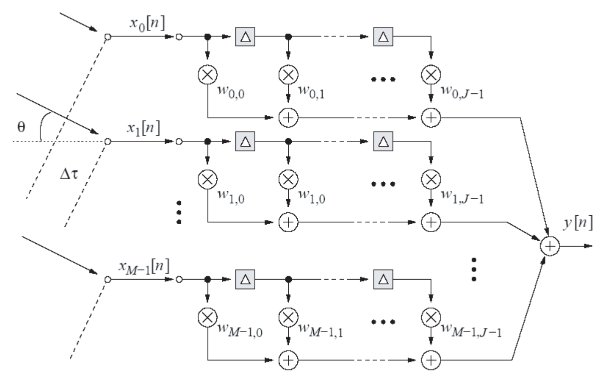

Figure 1: A signal impinges from an angle θ onto a uniformly spaced broadband linear array with M sensors, each followed by a J-tap filter.

As the broadband counterpart of the narrowband beamspace adaptive array, the beamspace broadband adaptive array was proposed in [5], where several frequency invariant beams (FIBs) are formed pointing to different directions by a fixed beamforming network with two-dimensional (2-D) filters; thereafter the outputs of these beams are combined adaptively by a single weight for each of them. Since both the number of beams and the number of selected beams are small, the total number of adaptive weights is greatly reduced.

In this paper, we will first give an analysis of the broadband beamspace adaptive array to show a trade-off between two conditions imposed explicitly or implicitly and its impact on the performance of the resultant beamformer. It can be proved that the number of independent beams formed is the same as the length N of the prototype filter for the fan filter design. Although we can design as many frequency invariant beams as we want, only N of them are independent and at most we can only null out $N-1$ interfering signals. As the array's interference cancellation ability is dependent on both the number of independent beams and the frequency invariant property of those beams, we can sacrifice the frequency invariant property to some degree to design more independent beams. As a result, the array's interference cancellation property will be improved. With the above analysis, we then propose a new design of the frequency invariant beams, where their beam directions are uniformly distributed in the spatial domain and their independence is guaranteed inherently by the special form of the prototype filters, which are derived from another prototype filter by the discrete Fourier transform (DFT) modulation with appropriately imposed zeros.

This paper is organised as follows. A brief review of the broadband beamspace adaptive array is provided in Section. An analysis of the trade-off in its design is given in. The design based on the DFT modulation is proposed in Section. Design examples and simulation results are given in Section, and conclusions drawn in Section.

2. Broadband Beamspace Adaptive Array

In a narrowband beamspace adaptive array [7], a total of N beams are formed by a beamforming network, where one is the main beam pointing to the direction of the signal of interest and the remaining $N-1$ beams are auxiliary beams pointing to the remaining directions. The output power levels of the auxiliary beams are compared to a threshold and those higher than the threshold will be chosen in the following adaptation. In this way the resultant partially adaptive array can maintain an acceptable performance with a lower computational complexity. Extend this idea to the broadband case, we can also design N broadband beams pointing to different directions to form a broadband beamspace adaptive array. To combine the outputs of the beams with one adaptive weight for each of them, their response should be frequency invariant.

In [5], such a broadband beamspace adaptive array was proposed for an equally spaced linear array. With the recent development in the design of frequency invariant beamformers for one-dimensional (1-D), two-dimensional (2-D) and three-dimensional (3-D) arrays [8], we can easily extend the idea of a beamspace adaptive array to the 2-D and 3-D cases. Here we will focus on the case of a linear array and first we give a brief review of the proposed beamspace approach.

Suppose a signal with an angular frequency ω and an angle of arrival θ impinges on the uniformly spaced linear array of Fig. 1, then its output in continuous form can be written as

$$y(t) = e^{j\omega t} \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\omega\Delta\tau} \cdot e^{-jk\omega T_s} \tag{1}$$

with $\Delta\tau = \frac{d}{c}\sin\theta$, where T_s is the delay between adjacent samples in the TDL, d is the array spacing, and c is the wave propagation speed. Then the array's response can be written as

$$\tilde{R}(\omega,\theta) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\omega\Delta\tau} \cdot e^{-jk\omega T_s}. \tag{2}$$

With the normalised angular frequency $\Omega = \omega T_s$, we obtain the response as a function of Ω and θ

$$R(\Omega,\theta) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jm\mu\Omega\sin\theta} \cdot e^{-jk\Omega} \quad \text{with} \quad \mu = \frac{d}{cT_s}. \tag{3}$$

With the substitution of $\Omega_1 = \Omega$ and $\Omega_2 = \mu\Omega\sin\theta$ in (3), we obtain a 2-D digital filter response

$$R(\Omega_1,\Omega_2) = \sum_{m=0}^{M-1} \sum_{k=0}^{J-1} w_{m,k} \cdot e^{-jk\Omega_1} \cdot e^{-jm\Omega_2}. \tag{4}$$

We see that the spatio-temporal spectrum of the received signal lies on the line $\Omega_2 = \mu\Omega_1\sin\theta$. Suppose the desired frequency invariant response of the array is $P(\sin\theta)$. By the substitution $\sin\theta = \left(\frac{\Omega_2}{\mu\Omega_1}\right)$, we can obtain the response $R(\Omega_1,\Omega_2)$ with nominal parameters Ω_1 and Ω_2. Sample the function $R(\Omega_1,\Omega_2)$ at the (Ω_1,Ω_2) plane and then apply an inverse discrete Fourier transform (DFT) to the resultant 2-D data, we will then find the corresponding coefficients $w_{m,k}$. To fit the spatial and temporal dimensions of the array, we may need to truncate the result from the inverse DFT [5, 8].

For the desired response $P(\sin\theta)$, it can comes from a 1-D digital filter $H(e^{j\Omega})$ by the substitution $\Omega = \pi\sin\theta$. If $H(e^{j\Omega})$ is a lowpass filter [5], then signals from the directions around $\theta = 0$ will correspond to its passband, and a beam is formed pointing to the direction $\theta = 0$. If we want to steer this beam to the direction $\theta = \theta_0$ with the same low pass filter $H(e^{j\Omega})$, we can vary it into the form $\tilde{H}(e^{j\pi\sin\theta}) = H(e^{j(\Omega-\pi\sin\theta_0)})$ and consider $\tilde{H}(e^{j\pi\sin\theta})$ as the new desired frequency invariant response.

As the sampling frequency is in general twice the highest frequency component of the signal and array spacing is half the wavelength of the highest frequency component, we have $d = \frac{1}{2}\cdot c\cdot(2T_s) = cT_s$ and $\mu = 1$. Therefore, without loss of generality, we will only consider the case with $\mu = 1$ in the design and simulations.

Moreover, we also assume the signal of interest comes from the broadside, then the main beam will point to the direction of $\theta = 0$. For the auxiliary beams, their directions are decided in such a way that the main direction of a beam should ideally coincide with nulls (zero responses) of all other beams, as mentioned in the simulation part of [5].

A single adaptive weight is applied to each of the auxiliary beams by minimizing the variance of the error signal between the main beam and the auxiliary beams. In the adaptation, some of the auxiliary beam outputs are active and some others are simply discarded if their output signals are below some prescribed level. Fig. 2 shows the diagram of such a broadband beamspace adaptive array, where $\mathbf{x}[n]$ is the vector containing the received signals $x_0[n], \ldots, x_{M-1}[n]$ and $w_1, \ldots, w_{P-1}$ are the adaptive weights attached to each of the beam outputs.

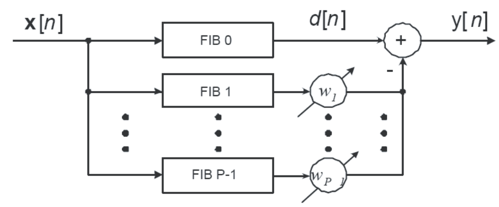

Figure 2: A broadband beamspace adaptive array with P frequency invariant beams (FIBs).

3. Analysis of the Broadband Beamspace Adaptive Array

For the beamspace array to work, the frequency invariant beamforming network needs to meet two conditions, which are imposed explicitly or implicitly.

First, the beams formed should have a satisfactory frequency invariant property for the interested frequency band, which is dependent on the required shape $P(\sin\theta)$ of the beam and the temporal/spatial dimension of the corresponding 2-D filter. The more complicated the shape, the more coecients we need in each of the frequency invariant beams, i.e., a larger M and J.

From the discussion of the last section, the desired beam response can be derived from the corresponding prototype filter $H(e^{j\Omega})$. Suppose the length of filter is N. As the shape is decided by the prototype filter, the dimension M and J of the 2-D fan filter (frequency invariant beamformer) should be at least 3 times that of the prototype filter to maintain the shape of the response of the prototype filter, that is, $N \leq \min\{\frac{M}{3}, \frac{J}{3}\}$ [5].

Secondly, the beams formed should not be linearly dependent. Otherwise, some of the beam outputs will be a linear combination of the others, which leads to a waste of resources and also reduces the number of effective beams. As a result, we will not be able to null out the desired number of interfering signals. This second condition is not mentioned explicitly in [5], but it is a necessary condition to fully exploit the potential of the beamspace adaptive array. We will see later that the beam direction arrangement in [5] guarantees the linear independence of the beams.

These two conditions are not independent and there is a close relationship between them. In the following, we will show that the number of independent beams formed N_{ind} cannot exceed the length N of the prototype FIR filter. We prove this by contradiction.

Suppose we can have $P > N$ independent beams formed by some prototype filters with a length N. These beams have a response of $H_p(e^{j\pi\sin\theta})$, $p = 0, 1, \ldots, P-1$. Each of them is derived from the corresponding prototype filter $H_p(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$, with an impulse response of $\mathbf{h}_p = [h_{p,0}, h_{p,1}, \ldots, h_{p,N-1}]^T$, $p = 0, 1, \ldots, P-1$. These prototype filters $H_p(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$ can further be derived from the same lowpass filter as discussed in the last section, or they can simply be some different filters.

Now consider the linear combination of the following form

$$\mathbf{0} = \alpha_0\mathbf{h}_0 + \alpha_1\mathbf{h}_1 + \cdots + \alpha_{P-1}\mathbf{h}_{P-1}, \tag{5}$$

where $\alpha_0, \cdots, \alpha_{P-1}$ are scalars to be found for this equation to hold. Taking the transpose of both sides and then multiplying the equation with the vector $[1\; e^{j\pi\sin\theta} \ldots e^{j(N-1)\pi\sin\theta}]^T$, we arrive at

$$0 = \alpha_0 H_0(e^{j\pi\sin\theta}) + \alpha_1 H_1(e^{j\pi\sin\theta}) + \cdots + \alpha_{P-1} H_{P-1}(e^{j\pi\sin\theta}), \tag{6}$$

where $H_p(e^{j\pi\sin\theta})$, $p = 0, 1, \ldots, P-1$ is exactly the response of those independent beams. Since they are independent, all the scalars $\alpha_0, \cdots, \alpha_{P-1}$ must be zero for (6) to hold, and then for (5) to hold, which means that $\mathbf{h}_p$, $p = 0, 1, \ldots, P-1$ are independent. However, as P is larger than the length of each vector $\mathbf{h}_p$, the rank of the $N \times P$ matrix formed by $H = [H_0, H_1, \ldots, H_{P-1}]$ cannot be larger than N, that is, it is impossible for all of the vectors $\mathbf{h}_p$ to be independent. Thus, we reach a contradiction.

As the maximum rank of $\mathbf{H}$ is N, we can see from the proof that the maximum number of independent beams formed will be equal to the length N of the prototype FIR filter. Clearly, although we can design as many frequency invariant beams as we want, only N of them are independent and at most we can only null out $N-1$ interfering signals. As the array's interference cancellation ability is dependent on both the number of independent beams and the frequency invariant property, there is trade-off between these two factors for a fixed M and J. We may choose a prototype filter with $N = \min\{\frac{M}{3}, \frac{J}{3}\}$ for a good frequency invariant property, but when the number of interferences increases and becomes larger than $(\min\{\frac{M}{3}, \frac{J}{3}\} - 1)$, the array will not be able to null out the additional interferences. Therefore we may need to sacrifice the frequency invariant property a little to increase N and design more independent beams. The loss in frequency invariant property can be compensated by the gain in the increasing number of independent beams. As a result, the interference cancellation ability of the array is improved. We will give some results to show this trade-off in our simulations.

The next question is, provided the length of the prototype filter N, how to design N independent frequency invariant beams. We will propose a DFT-modulated method in the next section with the beam directions uniformly distributed in the spatial space and their independence guaranteed inherently.

4. DFT-modulated Design of the Frequency Invariant Beamformers

Before we proceed further, we want to give a sufficient condition with which the P beams formed by P general prototype filters $\mathbf{h}_p$, $p = 0, 1, \ldots, P-1$ are linearly independent. This condition is stated as follows.

• **As long as for the $\hat{p}-th$ frequency response $H_{\hat{p}}(e^{j\Omega})$, $\hat{p} = 0, \ldots, P-1$, there exists a point $\Omega = \Omega_{\hat{p}}$, where $H_{\hat{p}}(e^{j\Omega_{\hat{p}}}) \neq 0$ and all the remaining frequency responses $H_{p\neq\hat{p}}(e^{j\Omega_{\hat{p}}}) = 0$, the set of frequency responses $H_p(e^{j\Omega})$, $p = 0, 1, \ldots, P-1$, and hence the set of beams formed by them will be linearly independent**.

The proof is given in the following. Consider the equation (5) again. Taking the transpose of both sides and then multiplying the equation with the vector $[1\, e^{j\Omega} \ldots e^{j(N-1)\Omega}]^T$, we arrive at

$$0 = \alpha_0 H_0(e^{j\Omega}) + \alpha_1 H_1(e^{j\Omega}) + \cdots + \alpha_{P-1} H_{P-1}(e^{j\Omega}), \tag{7}$$

For $\hat{p} = 0$, put the value $\Omega = \Omega_0$ into the above equation, we have

$$0 = \alpha_0 H_0(e^{j\Omega_0}) + \alpha_1 H_1(e^{j\Omega_0}) + \cdots + \alpha_{P-1} H_{P-1}(e^{j\Omega_0}) = \alpha_0 H_0(e^{j\Omega_0}) + 0 + \cdots + 0. \tag{8}$$

As $H_0(e^{j\Omega_0}) \neq 0$, we have $\alpha_0 = 0$. Similarly, we have $\alpha_p = 0$, $p = 0, 1, \ldots, P-1$. Therefore, for (7) to hold, all the P scalars must be zero, that is, both the vectors $\mathbf{h}_p$ and frequency responses $H_p(e^{j\Omega})$ are linearly independent. The proof is complete.

In [5], the main direction of a beam was arranged to coincide with nulls (zero responses) of all other beams. From the above proof, clearly, this arrangement guarantees the independence of the beams. However, in [5], the authors were simply using the existing nulls of the prototype filter, so the direction of the auxiliary beams can not be controlled by the designer and they can point to anywhere depending on the chosen lowpass prototype filter. Here we propose a DFT-modulated method for the design of the independent frequency invariant beamformers, where the beam directions are uniformly distributed in the spatial domain and their independence is guaranteed inherently.

Assume the impulse response of a lowpass filter is $h[n]$, $n = 0, 1, \ldots, N-1$. Based on $h[n]$, we can obtain the response $\mathbf{h}_p$ of the $p-th$ prototype filter for the $p-th$ beam shape design by the following DFT modulation

$$h_{p,n} = h[n] e^{j\frac{2pn\pi}{P}}. \tag{9}$$

In the frequency domain, this modulation shifts the response of original prototype filter $h[n]$ along the frequency axis by $\frac{2p\pi}{P}$. If the z-transform $H(z)$ of $h[n]$ can be expressed as

$$H(z) = \prod_{p=1}^{P-1} (1 - e^{j\frac{2p\pi}{P}} z^{-1}), \tag{10}$$

then after modulation, the main direction of the P resultant beams will coincide with the nulls of the other beams, hence these beams will be independent. Note in this case, we have $P = N$, i.e., the number of independent beams formed will be the length of the prototype filter.

For the main directions of these beams, we have

$$\pi \sin\theta = \begin{cases} \frac{2p\pi}{P} & \text{for } \frac{2p\pi}{P} < \pi \\ \frac{2p\pi}{P} - 2\pi & \text{for } \frac{2p\pi}{P} \geq \pi \end{cases} \Rightarrow \sin\theta = \begin{cases} \frac{2p}{P} & \text{for } \frac{2p}{P} < 1 \\ \frac{2p}{P} - 2 & \text{for } \frac{2p}{P} \geq 1 \end{cases}, \tag{11}$$

for $p = 0, 1, \ldots, P-1$. They are uniformly distributed in the $\sin\theta$ domain, where the first beam point to the direction $sin\theta = 0$ will be the main beam and the others will be the auxiliary beams. Fig. 3 gives an example of the desired beam shapes with $P = N = 5$, where it can be seen clearly that each of the five beam directions coincides with the nulls of the other beams. Once we obtain the P desired beam responses $H_p(e^{j\pi \sin\theta})$, we can follow the procedures given in [8] to obtain the coefficients of the corresponding beamformers.

One point to note is, in general, the $H_p(e^{j\pi \sin\theta})$ obtained by DFT modulation is of complex value for different θ that is,

$$H_p(e^{j\pi \sin\theta}) = A_p(\theta) e^{jB_p(\theta)} \tag{12}$$

where $A_p(\theta)$ and $B_p(\theta)$ are some real functions. The change of both $A_p(\theta)$ and $B_p(\theta)$ with respect to different θ will lead to a more complicated (Ω_1, Ω_2) pattern for the design, which will require more coefficients in the temporal domain and therefore larger dimension of the array. As $A_p(\theta)$ contains enough information about the shape of the beam response, we can ignore the phase part $B_p(\theta)$ and our results show that in this way we can significantly improve the frequency invariant property of the beams with the same array dimensions.

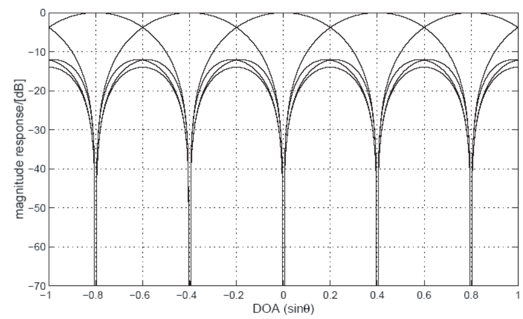

Figure 3: The desired beam shapes with $P = N = 5$ formed by DFT modulations.

5. Simulations

To show the trade-off between the frequency invariant property and the number of linear independent beams, the spatial and temporal dimensions of the frequency invariant beams are fixed as $M = 14$ and $J = 16$. According to [5], ideally we should use a prototype filter of length $\lfloor 14/3 \rfloor = 4$ for the design of the 4 FIBs. Fig. 4 shows the pattern of the main beam based on a 4-tap filter over the bandwidth $[0.4\pi; 0.9\pi]$.

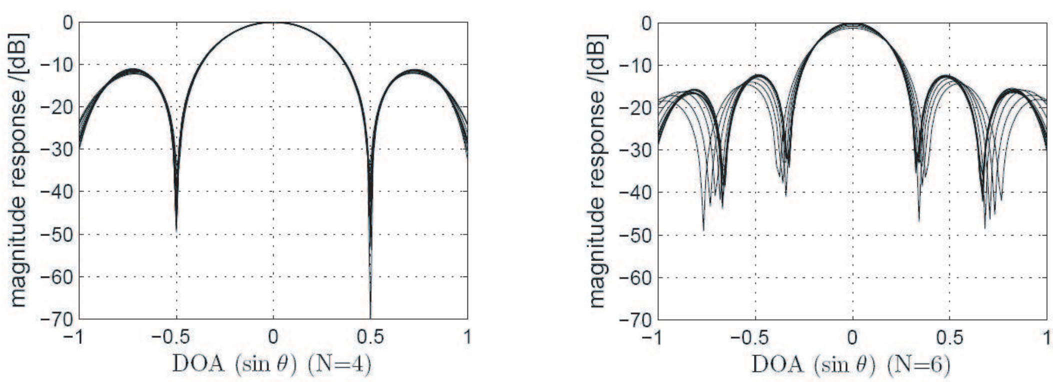

Figure 4: The magnitude response of the main beam over the bandwidth of $[0.4\pi; 0.9\pi]$, based on a 4-tap and a 6-tap prototype filter, respectively.

The signal of interest comes from broadside and with a signal to interference ratio (SIR) of -20 dB and signal to noise ratio (SNR) of 20 dB. Five interfering signals come from the angles of 20°, -25°, 45°, -50°, and -80°, respectively. Both the interfering signals and the signal of interest have a bandwidth of $[0.4\pi; 0.9\pi]$. We used a normalised LMS algorithm for adaptation. The learning curve with a stepsize of 0.01 is shown by the dashed line in Fig. 5. As the number of interfering signals are 5, which is larger than $4 - 1 = 3$, the number of auxiliary beams, the 4-beam adaptive array can not null out all of the interferences, although all of the beams have a very good frequency invariant response over the interested bandwidth $[0.4\pi; 0.9\pi]$. As a result, the learning curve only reaches a level of 15 dB. In order to improve its performance, we need to sacrifice the frequency invariant property a little. So, we increased the length N of the prototype filter to 5, and 5 independent beams were obtained. The learning curve of this new system with the same stepsize is shown by the dotted line in Fig. 5. Compared to the 4-beam array, the ensemble mean square residual error has been reduced to about 8 dB. We can further to improve the performance of the system by designing 6 independent beams based on a 6-tap prototype filter ($N = 6$). The frequency invariant property of the main beam in this case is also shown in

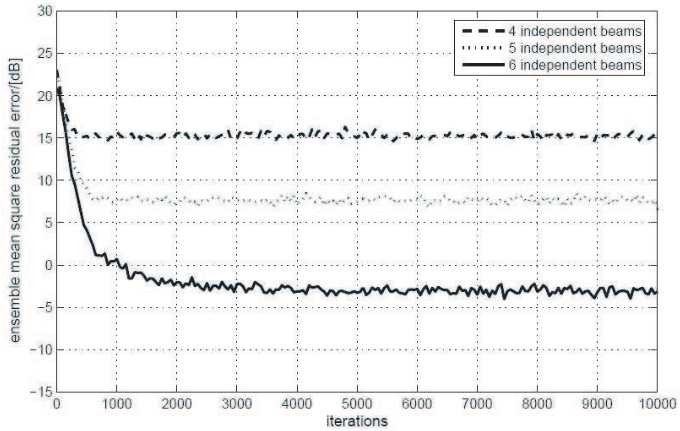

Figure 5: The learning curves for different number of independent beams.

Fig. 4, which is clearly not as good as that of $N = 4$. However, as there are more independent beams formed in this array, a further improvement of more than 10 dB has been achieved, as shown by the solid line in Fig. 5.

6. Conclusions

An analysis of the broadband beamspace adaptive array has been provided and it is shown that in order to improve the interference cancellation capability of the array, we may need to sacrifice the frequency invariant property of the beams to some degree for more linearly independent beams. We also proposed a DFT-modulated design of the frequency invariant beams employed in the broadband beamspace adaptive array, where the beam directions are uniformly distributed over the spatial domain and the linear independence of the beams is guaranteed inherently. Simulation results verified our analysis and the proposed method.

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