Time-domain Statistics of Multi-layer Optical Filters

H. E. Rowe
Stevens Institute of Technology, USA

We calculate the time-response statistics for multi-layer optical filters with random errors in layer thickness or index of refraction.

Exact statistics have been determined for the reciprocal transfer function in the wavelength domain, using Kronecker product methods. Transfer function statistics are then obtained by applying perturbation theory to these exact results. This is valid because useful devices depart only slightly from their ideal design. These approximate results have been verified by comparison with exact results for special cases. First- and second-order transfer function statistics in the wavelength domain have previously been presented.

The average transfer function, varying with frequency or wavelength, is deterministic. The transfer fluctuations about the average are random. The covariance of the transfer function fluctuations in the wavelength domain has been calculated using the above methods.

The wavelength λ is a natural parameter for calculating transmission statistics. However, we require similar results as functions of frequency f in order to determine time-domain statistics. Thus for a random transfer function, the expected value of the square of the envelope of the impulse response is proportional to the Fourier transform of the covariance of the transfer function.

We separate the transfer function into deterministic and random components:

$$T(f) =< T(f) > +\Delta T(f).$$

The covariance of the random component is

$$C(f_1, f_2) =< \Delta T(f_1)\ \Delta T * (f_2) > .$$

The impulse responses for these two components are their Fourier transforms:

$$G(t) < - >< T(f) >;\ g(t) < - > \Delta T(f).$$

We use the symbol $< - >$ to denote the Fourier transform relationship. The corresponding envelopes for these time functions are $r_G(t)$ and $r_g(t)$. Then $< |r_g(t)|^2 >$, the average square envelope of the random impulse response, is calculated from the covariance C of the random component of the transfer function.

We present results for a 13-layer band-pass optical filter.

REFERENCES

1. Rowe, H. E., *Electromagnetic Propagation in Multi-Mode Random Media*, John Wiley, New York, 1999.
2. Rowe, H. E., "Second-order statistics of multi-layer optical coatings," *PIERS 2002 Proceedings*, 310, July 1–5, 2002.