Some Applications of the High-mode-merging Method

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Abstract—Waves guided along dielectric step discontinuity can be described by a multi-port network [1] and it is simplified as a two-port network with the influence of high-modes retaining [2]. These results can be used for treat dielectric strip waveguide, even more complicated structures. Some numerical results are got for a strip and a groove dielectric waveguide some kind of resonant phenomena also is obtained. Some comments on this method and some suggestions are given furthermore.

1. Introduction

A dielectric strip waveguide can be seen as a system constituted by 2 step discontinuous structure as shown in Fig. 1. As a symmetric system, it also can be treated by so-called bisection method, namely, it can be see equivalently as the result of superposition of 2 networks with short circuit(sc) and open circuit(oc) separately at the terminals of transmissions of length $l/2$ (Fig. 2) [1].

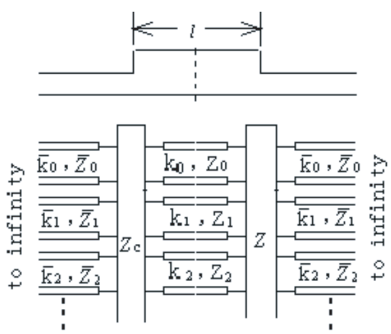


Figure 1: Network of a dielectric strip waveguide.

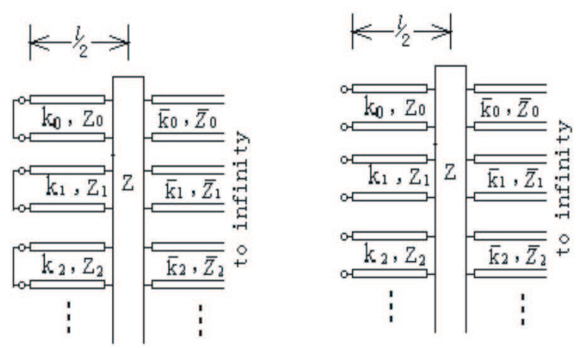


Figure 2: Bisection treatment of the dielectric strip waveguide.

This problem can be simplified by high-mode-merging method provide in [2], a two-port network in which the influence of high modes is considered has been obtained, but in present case, all Z_m in E, F, G and H of [2] ((17a-d) in [2] respectively) are replaced by

$$jZ_m tan\, k_0(l/2) \qquad \text{(for sc)} \tag{1}$$

$$-jZ_m cot\, k_0(l/2) \qquad \text{(for oc)} \tag{2}$$

There are three methods to treat the problem of strip waveguide.

2. Equivalent Circuit Method

As most of microwave engineers are more familiar to the circuit language, the network is realized by a simple T circuit generally, (see Fig. 3), impedances of which, Z_a, Z_b and Z_c are related to elements of Z-matrix, $Z_{11}Z_{12}Z_{21}$ and Z_{22} by

$$Z_a = Z_{11} - Z_{12} \quad Z_b = Z_{22} - Z_{12} \quad Z_c = Z_{12} (= Z_{21}) \tag{3}$$

(see appendix) and also be distinguished for sc and oc. In present case, we have Fig. 4(a), the impedances Fig. 4(a), the impedances looking left into the network at 2-2' plane in two cases are

$$Z_{in}^{sc} \;=\; Z_b^{sc} + \frac{Z_c^{sc}(Z_a^{sc} + Z_{in,0}^{sc})}{Z_c^{sc} + Z_a^{sc} + Z_{in,0}^{sc}} \quad \text{(for sc)} \tag{4a}$$

$$Z_{in}^{oc} \;= Z_b^{oc} + \frac{Z_c^{oc}(Z_a^{oc} + Z_{in,0}^{oc})}{Z_c^{oc} + Z_a^{oc} + Z_{in,0}^{oc}} \quad \text{(for oc)} \tag{4b}$$

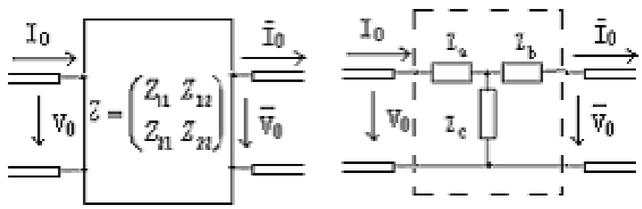
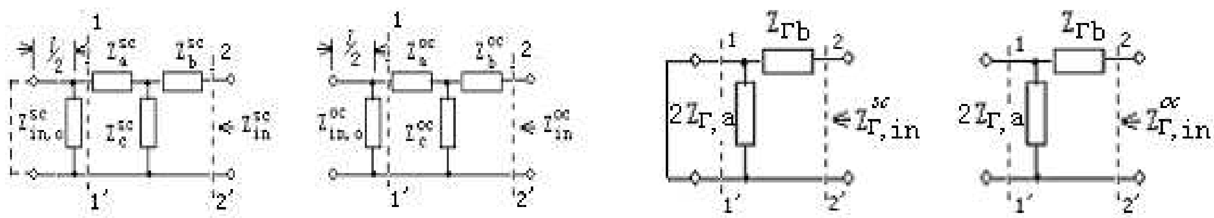


Figure 3: The comparison of a network and a circuit.

where $Z_{in,0}^{sc}$ and $Z_{in,0}^{oc}$ are the input impedances at 1-1' plane looking left into transmission line with length $l/2$ in sc and oc cases, which can be got by (1a) and (1b) respectively.



(a) (b)

Figure 4: a) T-circuit b) Γ-circuit.

The system in Fig. 4(a) also can be changed to a Γ-circuit as shown in Fig. 4(b). Obviously,

$$Z_{\Gamma,in}^{sc} = Z_{\Gamma,b} \tag{5a}$$

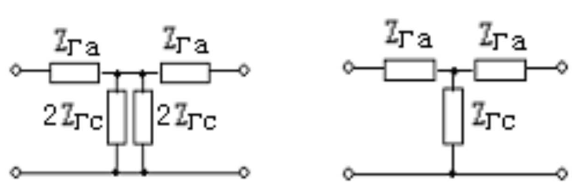
$$Z_{\Gamma,in}^{oc} = Z_{\Gamma,b} + 2Z_{\Gamma,m} \tag{5b}$$

Let them be equivalent, i.e., we have $Z_{T,in}^{sc} = Z_{\Gamma,in}^{sc}$ and $Z_{T,in}^{oc} = Z_{\Gamma,in}^{oc}$, then,

$$Z_{\Gamma,b} = Z_{\Gamma,in}^{sc} = Z_{T,in}^{sc} \tag{6a}$$

$$Z_{\Gamma,m} = (Z_{\Gamma,in}^{oc} - Z_{\Gamma,b})/2 = (Z_{T,in}^{oc} - Z_{T,in}^{sc})/2 \tag{6b}$$

where $Z_{T,in}^{sc}$ and $Z_{T,in}^{oc}$ are given by (4a) and (4b) respectively. A strip dielectric waveguide can be seen as a combination of two Γ-circuits which connected back to back as shown in Fig. 5(a), then it can be reformed as Fig. 5(b). Going back to network, the elements of a strip waveguide can be got by (3a) and (3b) in opposite way.



(a) (b)

Figure 5: (a) Combination of two Γ-circuits (b) Its deformation.

3. Equivalent Network Method

A little bit different approach is called equivalent network method (EN method).

The elements of Z-matrix can be normalized as follows.

$$Z'_{11} = Z_{11}/Z_0 \qquad Z'_{12} = Z_{12}/\sqrt{Z_0\overline{Z}_0} \tag{7a}$$

$$Z'_{21} = Z_{11}/\sqrt{\overline{Z}_0 Z_0} \qquad Z'_{22} = Z_{22}/\overline{Z}_0 \tag{7b}$$

Considering the high-mode=merging method, we have

$$Z'_{11} = (\frac{F}{H})/Z_0 \qquad Z'_{12} = (E + \frac{GF}{H})/\sqrt{Z_0\overline{Z}_0} \tag{8a}$$

$$Z'_{21} = (\frac{1}{H})/\sqrt{\overline{Z}_0 Z_0} \qquad Z'_{22} = (\frac{G}{H})/\overline{Z}_0 \tag{8b}$$

Where E, F, G and H are given in [2], and the determinate of normalized Z-matrix

$$\text{Det} \quad \mathrm{Z}' = \mathrm{Z}'_{11}\mathrm{Z}'_{22} - \mathrm{Z}'_{21}\mathrm{Z}'_{12} = \frac{\mathrm{E}}{\mathrm{H}}/(\mathrm{Z}_0\overline{\mathrm{Z}}_0)$$

then we can get the transfer matrix for right step discontinuity of the strip waveguide

$$A'_r = A' = \frac{1}{Z'_{21}}\begin{bmatrix} Z'_{11} & |Z'| \\ 1 & Z_{22} \end{bmatrix} = \begin{bmatrix} F\sqrt{\overline{Z}_0/Z_0} & -E/\sqrt{Z_0\overline{Z}_0} \\ H\sqrt{Z_0\overline{Z}_0} & G/\sqrt{Z_0/\overline{Z}_0} \end{bmatrix} \tag{9}$$

Let voltage and current in both sides of the right discontinuity be normalized as

$$V'_0 = V_0/\sqrt{Z}_0,\ I'_0 = I_0\sqrt{Z}_0,\ \overline{V}'_0 = \overline{V}_0/\sqrt{\overline{Z}}_0,\ \overline{I}'_0 = \overline{I}_0\sqrt{\overline{Z}_0}$$

Then we have

$$\begin{bmatrix} V'_0 \\ I'_0 \end{bmatrix} = A'_r \begin{bmatrix} \overline{V'_0} \\ \overline{I'_0} \end{bmatrix}$$

Considering symmetry of the strip, the matrix of left step discontinuity is just the inverse matrix of one of the right step:

$$A'_l = (A'_r)^{-l}(= (A')^{-l}) = \begin{bmatrix} G\sqrt{\overline{Z}_0/Z_0} & H/\sqrt{Z_0\overline{Z}_0} \\ -E\sqrt{Z_0\overline{Z}_0} & F/\sqrt{Z_0/\overline{Z}_0} \end{bmatrix} \tag{10}$$

The uniform structure between two step discontinuities corresponds a segment of an uniform transmission line with length l, the transfer matrix of which is

$$A'_m = \begin{bmatrix} \cos k_0 l & j\sin k_0 l \\ j\sin k_0 l & \cos k_0 l \end{bmatrix} \tag{11}$$

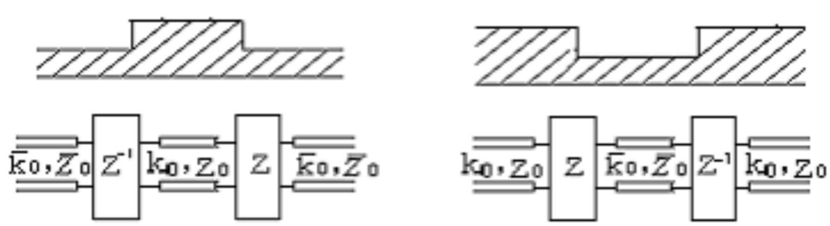
Finally, the transfer matrix of the whole strip can be got as the continued-multiplication product:

$$A'_{strip} = A'_l A'_m A'_r = (A')^{-1} A'_m A' \tag{12}$$

This procedure can be shown in Fig. 6(a). For a rectangular groove dielectric waveguide, corresponding matrix, then, is

$$A'_{groove} = A' A'_m (A')^{-1} \tag{13}$$

(see Fig. 6(b)).



(a) (b)

Figure 6: (a) Strip waveguide (b) Groove waveguide .

4. Effective Dielectric Constant Method

Besides there is also a rather rough method, in which the influence of all high-modes is neglected, that we only take the A'_m as the transfer matrix of whole strip (or groove waveguide):

$$A'_{strip} = A'_m \tag{14}$$

It's so-called effective dielectric constant (EDC) method.

5. Numerical Examples

For comparing these 3 methods, some numerical calculations have been done for some characteristics of some kinds of waveguides: Fig. 7(a)–(c) show plots of reflection and transmission coefficients (including the argument and modulus both of them) vs width of waveguide; Fig. 7(d) gives ones for loss.

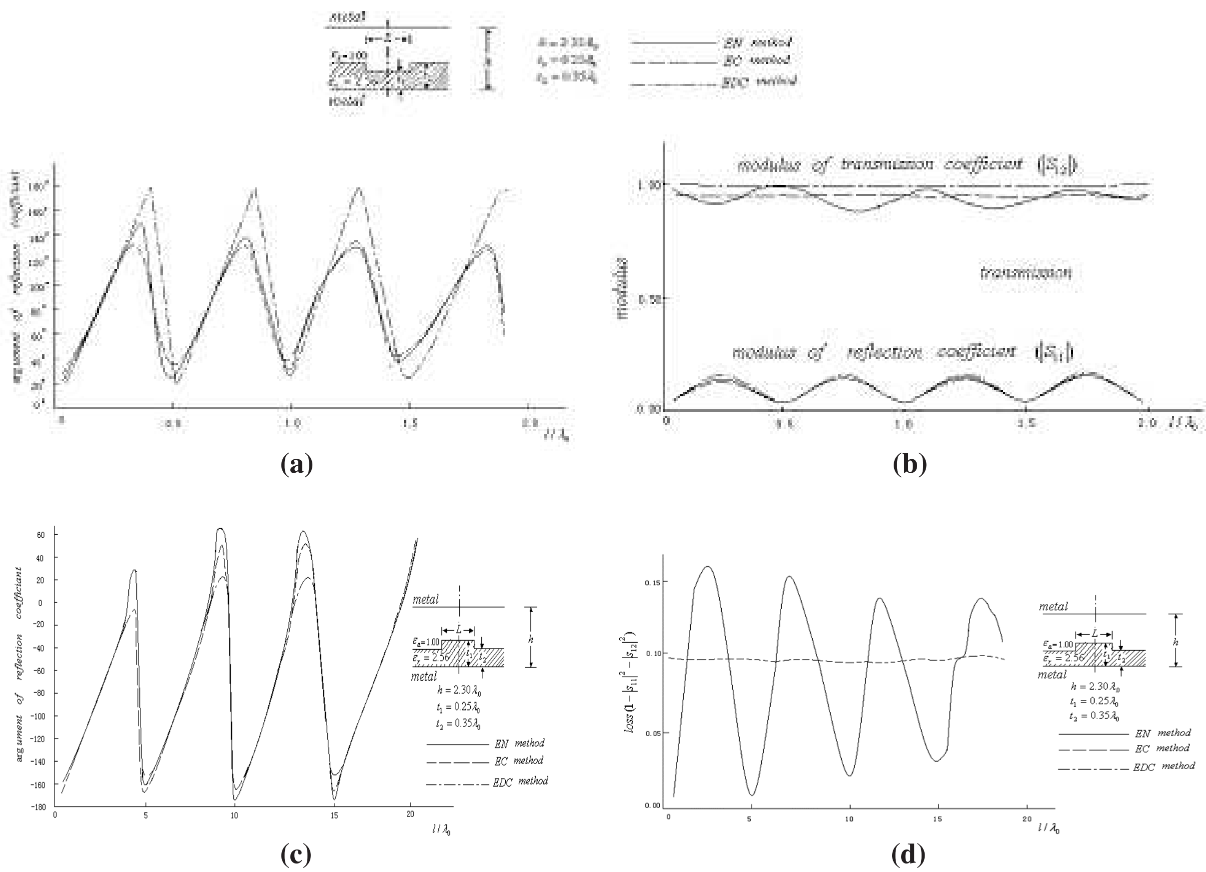


Figure 7: Some numerical results.

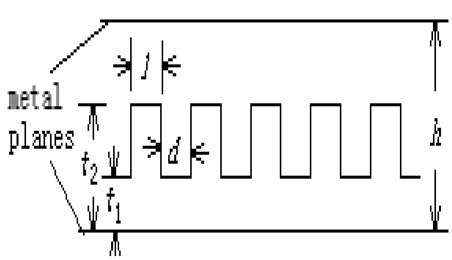


Figure 8.

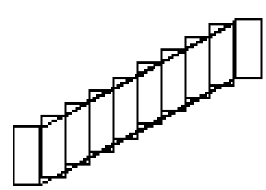


Figure 9.

6. Conclusion

(1) The results for all methods give similar tendency and accord one another pretty good in certain accuracy; especially results of EN method and EC method are more closed.

(2) The EDC method is still useful in some cases because it is rather simple and easy for calculations and with clear physical meaning. A significant defect is that it can't give the loss.

(3) There is some kind of periodic phenomena existing. The reflection coefficients, both argument and modulus, and loss are varying with the width of waveguide periodically. It is coincide with the conclusion of [3]. This phenomena can be seen as resonance, but the mechanism of it is remained to be explained further. It is also indicated that the high-mode-merging method is correct.

(4) In high-mode-merging method, the coupling between TE modes and TM-modes has not been considered. It is also one of the defects of this theory.

(5) In the original theory of [1], two parallel perfect conductive planes are needed. So, the waveguide discussed here is not open absolutely. If the upper one of them moved far enough, it almost can be seen as an open one approximately. If we want to remove the upper conductive plane, we'll get infinite number of continuous high modes it is a problem of continuous spectrum and is out of the topic of this paper.

(6) The cascade network method is not confined to solve only symmetric system like single strip dielectric waveguide but also can be extended to treat some more complicated structures, such as finite periodic strip(groove) dielectric waveguide (Fig. 8), the curved surface dielectric waveguide (Fig. 9) etc.

Appendix

Consider a T-circuit as shown in Fig. 3. The transfer matrixes of Fig. 3 devices are

$$A_a = \begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix}, \quad A_c = \begin{bmatrix} 1 & 0 \\ 1/Z_c & 1 \end{bmatrix}, \quad A_b = \begin{bmatrix} 1 & Z_b \\ 0 & 1 \end{bmatrix}$$

As a whole, the transfer matrix then is

$$A = A_a A_c A_b = \begin{bmatrix} 1 + Z_a/Z_c & Z_a + Z_b + Z_a Z_b/Z_c \\ 1/Z_c & 1 + Z_b/Z_c \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $a = 1 + Z_a/Z_c, b = Z_a + Z_b + Z_a Z_b/Z_c, c = 1/Z_c$, and $d = 1 + Z_b/Z_c$ with $|A'|= ad\text{-}bc=1$. Then, changing it to impedance matrix equivalently, we get

$$Z = \frac{1}{c} \begin{bmatrix} a & |A| \\ 1 & d \end{bmatrix} = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Namely, we get

$$Z_{11} = Z_a + Z_c \qquad Z_{12} = Z_{21} = Z_c \qquad Z_{22} = Z_b + Z_c$$

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