New GL Method and Its Advantages for Resolving Historical Difficulties

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Abstract—In this paper, we propose two types of new electromagnetic (EM) integral equation systems and their dual integral equation systems. Based on the EM integral equation systems, we propose the new GL EM modeling and inversion algorithms. Abstracts of our GL EM method based on the magnetic differential integral equation and the electrical integral differential equation have been published in PIERS 2005 in Hangzhuo. We used finite step iterations to exactly solve these integral equation systems or the EM and seismic differential integral equations in finite sub domains. The Global EM wave field is improved successively by the Local scattering EM wave field in the sub domains. In the FEM and FD method, the large matrix equation, inaccurate and complex absorption condition on artificial boundary, the cylindrical and spherical coordinate singularities, and ill posed in inversion are historical difficulties. The Born approximation is only used for low contrast material. The GL method is completely different from FEM, FD, and Born approximation. Our GL modeling and inversion resolved these historical difficulties. Only 3×3 or 6×6 small matrices need to be solved in the GL method; There is no artificial boundary for infinite domain in the GL method; In the GL method, the cylindrical and spherical coordinate singularities are resolved; Our GL method combines the analytic and asymptotic method and numerical method perfectly. It is more accurate than FEM and FD method and Born likes approximation. The GL method is available for all frequencies and high contrast materials. The GL solution has $O(h^2)$ convergent rate. If the Gaussian integrals are used, the GL field has $O(h^4)$ super convergence. The GL method is a high perform parallel algorithm with intrinsic self parallelization properties. The FEM and FD scheme of high order PDE are complicated. Fortunately, the GL method has very simple scheme or no scheme or half scheme such that it has half mesh and no mesh. The FEM and FD scheme only used Riemann integral. In the GL method, we can use both of Riemann and Lebesgue integral that induces a meshless method. We have developed software for 3D/2.5D EM, seismic, acoustic, flow dynamic, and QEM modeling and inversion. Our GL modeling and inversion are useful for geophysical and Earthquake exploration, environment engineering, nondestructive testing, steel and metal casting, weather radar, medical, Earth magnetic, antenna, and heating conductive imaging, space sciences and lunar and sun and stars EM and light exploration. The GL QEM modeling and inversion can be useful for studying micro optical physical and biophysical properties in nanometer materials and biophysics materials. The GL and AGILD method resolved the singularity difficulties at the poles in Navier-Stocks flow atmosphere simulation and Earth and Space EM field. We find GL numerical quanta for very high frequency EM field by GL simulation.

1. Introduction

The existing EM theory and analytical and numerical methods are published in many books and journals. However, there are historical difficulties in EM and other field modeling and inversion. The large matrix equation, inaccurate and complex absorption conditions on artificial boundary, the cylindrical and spherical coordinate singularities, and ill posed in the modeling and inversion are historical difficulties. The Born approximation can be only used for low contrast material. In this paper, we propose a new GL method "Global and Local field modeling and inversion" for resolving these historical difficulties. Our GL method is completely different from the FEM, FD, Born approximation methods.

We consider the EM, seismic, acoustic, quantum, flow and other field equations on finite inhomogeneous domain that is imbedded into an infinite domain. The analytical incident field and Green field in the background domain are called an initial global field. The inhomogeneous domain is divided into mesh or meshless sub domains. The global field is changing by local scattering field successively in each sub domain. The GL method processes will be finished when the Global field is passing through the all Local sub domains with inhomogeneous material. First in the world, the abstracts of our GL method have been published in Piers 2005 in Hangzhuo. [1, 4–9], and in the GL Geophysical Laboratorys reports [2–3].

The new GL method has the following advantages: (1) There is no large matrix to solve, only 3×3 or 6×6 small matrices need to be solved; (2) There is no artificial boundary for infinite domain; (3) The GL method combines the analytic and asymptotic method and numerical method consistently. It is more accurate than FEM and FD method and Born likes approximation; (4) The GL modeling solution has $O(h^2)$ convergent rate. In particular, if the Gaussian integrals are used, the GL solution has $O(h^4)$ super convergence; (5) The cylindrical and spherical coordinate singularities are resolved; (6) It is available for all frequencies and high contrast materials; (7) the GL method has very simple or no scheme, it has half mesh or no mesh; (8) In the GL method, we can use both Riemann and Lebesgue integrals that induce meshless methods; (9) GL method can couple consistently with AGILD, FEM, and FD method; (10) The GL method is an intrinsic self parallel algorithm in parallel T3E and PC cluster.

The plan of this paper is as follows: The introduction is presented in the section 1. In the section 2, we propose the EM integral equation systems. We propose the 3D/2D GL EM modeling based on the EM integral equation system in the section 3. In section 4, we propose the 3D/2D GL EM modeling based on the EM differential integral equation and electric and magnetic field integral equations. We propose the GL EM inversion in the section 5. In section 6, we prove the fundamental theorems of the GL method. We describe advantages of the GL method in the section 7. The GL software, applications and conclusions are described in the section 8.

2. New Electromagnetic Integral Equation Systems

In this section, we propose the new EM integral equation systems as follows:

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int\limits_{\Omega} \begin{bmatrix} E_b^J(r',r) & H_b^J(r',r) \\ E_b^M(r',r) & H_b^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} dr', \tag{1}$$

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int\limits_{\Omega} \begin{bmatrix} E^J(r',r) & H^J(r',r) \\ E^M(r',r) & H^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} dr', \tag{2}$$

where [D] is the EM material parameter variation matrix, for the isotropy materials, [D] is 6×6 diagonal matrix with variance materials $(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)$ and $i\omega(\mu - \mu_b)$, for anisotropy materials the [D] will be 6×6 full matrix. $E(r)$ is the electric field, $H(r)$ is the magnetic field, $E_b(r)$ is incident electric field in the background medium, $H_b(r)$ is incident magnetic field in the background medium, $E_b^M(r',r), \ldots, H_b^M(r',r)$ are electric or magnetic background field Green tensors exciting by the electric or magnetic dipole source respectively, The integral equations (1) and (2) are the dual system of each other.

3. The 3D/2D New GL EM Modeling Based on the Electromagnetic Integral Equation System

We propose the GL EM modeling based on the EM integral equation system in this section.

(3.1) The domain Ω is divided into a set of n mesh or meshless sub domains $\{\Omega_k\}$, $\Omega = \bigcup\{\Omega_k\}$.

(3.2) In each Ω_k, we solve the EM Green tensor integral equation system based on the equations (1) and (2). By dual curl operation, the equation systems are reduced into a 6×6 matrix equations. By solving the 6×6 equations, we obtain Green tensor field E_k^J and H_k^M.

(3.3) We improve the Global EM field $[E_k(r), H_k(r)]$ by the Local scattering field

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_k = \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_{k-1} + \int\limits_{\Omega_k} \begin{bmatrix} E_k^J(r',r) & H_k^J(r',r) \\ E_k^M(r',r) & H_k^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr', \tag{3}$$

$k = 1, 2, \ldots n$, successively. The $[E_n(r), H_n(r)]$ is the GL solution of the EM integral equations (1) and (2).

4. The 3D/2D GL EM Modeling Based on the EM Differential Integral Equation

4.1. The GL EM Modeling Based on the Magnetic Differential Integral Equation

Since 1995, we have proposed the magnetic field differential integral equation (MDI) in the frequency and time domain [10-13]. In this section, we propose the dual magnetic field differential integral equation of our MDI [10-13],

$$H(r) = H_b(r) + \int\limits_{\Omega} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{\sigma + i\omega\varepsilon} E^M(r',r) \cdot \nabla \times H_b(r') dr'. \tag{4}$$

Based on the equation (4), the GL magnetic field modeling is as follows:

(4.1) The step (4.1) is the same as (3.1).

(4.2) In each Ω_k, $k = 1, 2, \ldots n$, we solve the magnetic field differential integral equation to find $E_k^M(r', r)$ successively. By the dual curl operation, only 3×3 matrix equations need to be solved.

(4.3) We improve the Global EM field $H_k(r)$ by the Local scattering field

$$H_k(r) = H_{k-1}(r) + \int\limits_{\Omega_k} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{\sigma + i\omega\varepsilon} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r') dr', \tag{5}$$

$k = 1, 2, \ldots n$, successively. $H_n(r)$ is the GL magnetic field solution of (4).

4.2. GL EM Modeling Based on the Electric Differential Integral Equation

We propose the GL electric field modeling based on the dual electric field differential integral equation of our EDI in 1995[10-13],

$$E(r) = E_b(r) + \int\limits_{\Omega} \frac{\mu - \mu_b}{\mu} H^J(r', r) \cdot \nabla \times E_b(r') dr'. \tag{6}$$

4.3. GL EM Modeling Based on the Electric and Magnetic Integral Equation

We propose the GL method based on the electric integral equation and the magnetic integral equation. Since the electric and magnetic integral equations have divergent Green kernel, a special approach for resolving the divergent singularity is developed.

4.4. GL Modeling for Quantum Field and QEM Field

We propose the GL Schordinger modeling for two hydrogen atoms and interaction between QEM field and atoms that is useful for QEM field in nanometer materials. We find GL numerical quanta for very high frequency EM field by GLQEM simulation.

5. The New GL EM Inversion

The formal logic system and experiments are base of the sciences. Most equations are forward equations. Maxwell equation and elastic equation are forward equation and are not for inversion. The EM integral equation systems (1) and (2) and equations (4) and (6) can be used for both forward and inversion. They are well posed for forward and ill posed for inversion. From essential formal logic in physics, these equations are well posed for forward and ill posed for inversion. How to build a well posed inverse equation is the main project of scientific inversion. Our new idea of the inverse formal logic and inverse experiment in physics motivates us to propose the GL inversion that is a new explicit inversion.

5.1. The GL EM Inversion GLEMI1 for Determining $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, and $\boldsymbol{\mu}$

The following EM integral equation is for increments of EM parameters $\delta\sigma$, $\delta\varepsilon$, $\delta\mu$,

$$\begin{bmatrix} \delta E(r) \\ \delta H(r) \end{bmatrix}_k = \int\limits_{\Omega_k} \begin{bmatrix} E_k^J(r', r) & H_k^J(r', r) \\ E_k^M(r', r) & H_k^M(r', r) \end{bmatrix} [\delta D]_k \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr'. \tag{7}$$

5.2. The GL EM Inversion GLEMI2 for Determining $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$

The following magnetic field differential integral equation is for increments of parameters $\delta\sigma$, $\delta\varepsilon$,

$$\delta H_k(r) = -\int\limits_{\Omega_k} \frac{(\delta\sigma + i\omega\delta\varepsilon)}{(\sigma + i\omega\varepsilon)^2} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r') dr'. \tag{8}$$

5.3. The GL EM Inversion GLEMI3 for Determining $\boldsymbol{\mu}$

The following electric field differential integral equation is for increment of EM parameter $\delta\mu$,

$$\delta E_k(r) = -\int\limits_{\Omega_k} \frac{\delta\mu}{\mu^2} H_k^M(r', r) \cdot \nabla \times E_{k-1}(r') dr'. \tag{9}$$

The suitable strong and weaker regularizing should be added to (7), (8), and (9) to control inversion being stable and reasonable resolution. In our GL EM inversion, only smaller matrices need to be solved. The resolution is dependent on the data configuration, quality and the regularizing parameter.

6. The Fundamental Theorems of the GL Method

Theorem 1. The GL EM field $[E_n(r), H_n(r)]$ from (3.1)–(3.3) is convergent to exact EM field that satisfies the EM integral equation systems (1) and (2). The GL EM field $[E_n(r), H_n(r)]$ is convergent to exact EM field that satisfies the MAXWELL EM equation in 3D or 2D.

Theorem 2. The GL Magnetic field, $H_n(r)$ from (4.1)–(4.3) is convergent to the exact magnetic field, $H(r)$ that satisfies the magnetic field differential integral equation (4). The GL EM field, $H_n(r)$ is convergent to the exact magnetic field $H(r)$ that satisfies the exact MAXWELL EM equation.

Theorem 3. By Riemann division, the GL EM field $[E_n(r), H_n(r)]$ from (3.1–3.3) and the GL magnetic field $H_n(r)$ from (4.1–4.3) have $O(h^2)$ convergent if the trapezoid and mid point integrals are used. In particular, if the Gaussian integrals are used, the GL EM field has $O(h^4)$ super convergent rate. Proof: The theorem 1–3 have proved in [2].

7. Advantages of the GL Method

We have summarized the advantages of the GL method in the introduction. By reviewing the GL modeling and inversion in section 3, 4, and 5, we present several advantages as follows. We consider EM modeling in infinite domain that involves the finite inhomogeneous boundary domain. When we use FEM or implicit FD method to solve the problem, we need the radiation or absorption boundary condition on the artificial boundary with large enough domain. Solving the large matrix is difficult. The radiation and absorption boundary condition is complicated and inconvenience. In the EM inversion, the FEM and FD EM modeling is used in iterations. The absorption boundary errors will propagate into the internal domain, the noise is enhancing to damage the inversion. In the section 2, we propose the EM integral equation systems (1) and (2) that are equivalent to the 3D and 2.5D Maxwell EM equation in infinite domain with finite inhomogeneous domain for isotropic and anisotropic materials. Our GL EM modeling does not need any artificial boundary for solving the EM integral equation and the magnetic differential integral equation. Our GL EM modeling only needs to solve 3×3 or 6×6 small matrices, it does not need to solve any large matrix. There are $1/\rho^2$ singularity in the cylindrical coordinate and $1/r^2$, $1/\sin^2\phi$ singularities in the spherical coordinate system for Maxwell equation. These coordinate singularities are historic difficulties in FEM and FD method. In the EM integral equations (1–3) and electric and magnetic differential integral equations (4–6) for the cylindrical and spherical coordinate, the coordinate singularities are resolved. There is no coordinate singularity in the GL method. The GL modeling combines analytical and numerical methods consistent together and has super convergence. The GL method resolve many historical difficulties in traditional FEM, FD, and Bron approximation methods.

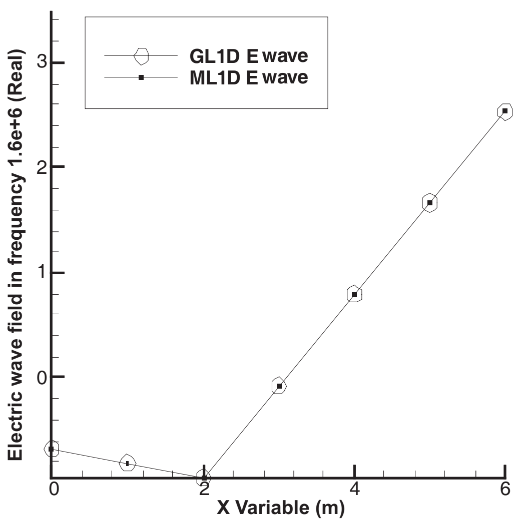

Figure 1: GL and ML Electric wave with freq. $1.6e^6$ Hz

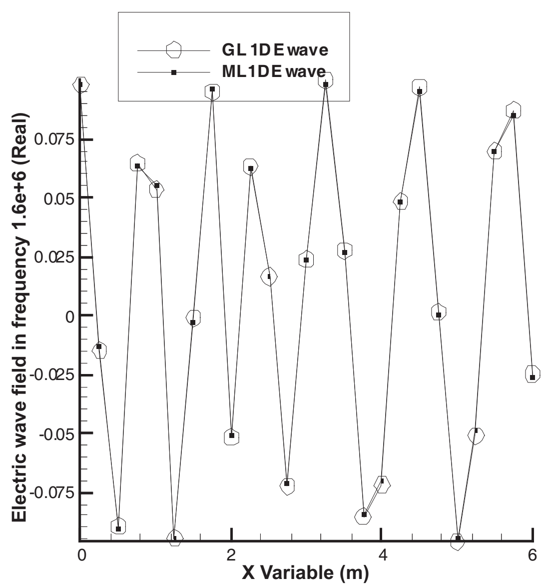

Figure 2: GL and ML Electric wave with freq. $1.6e^8$ Hz

We have created the GL method Since 2002. We have developed the seismic, EM, acoustic, flow, and Quantum field GL modeling and inversion algorithms and software. Many simulations show that the GL seismic and EM wave field has no any boundary error reflection. We have made several GL seismic and EM wave propagation movies that show the wave excited by internal sources is out going propagation perfectly without any error reflection on the boundary. Because the page limitation, we only use one dimension wave propagation to compare GL method and FEM method in the frequency domain. The absorption boundary condition is used for FEM. The numerical results show that GL wave is very accurate to match the multiple layer analytic wave for the high frequency 1.6×10^6 (Figure 1) and frequency 1.6×10^8 (Figure 2). The Figures 3 and 4 show that the FEM is fail to approximate the exact wave in the high Frequency. Our GL method and AGILD method have used in the EM stirring magnetic field simulation and obtained very accurate EM field. The GL, ML, and FEM total and scattering electric wave are shown in Figure 5 and figure 6 respectively. They show that the GL electric wave is very accurate to match to multiple layer wave, but FEM wave is not. Many 2.5D and 3D GL EM and seismic Wave show that GL modeling is accurate, fast and stable. The GL inversion is reasonable high resolution.

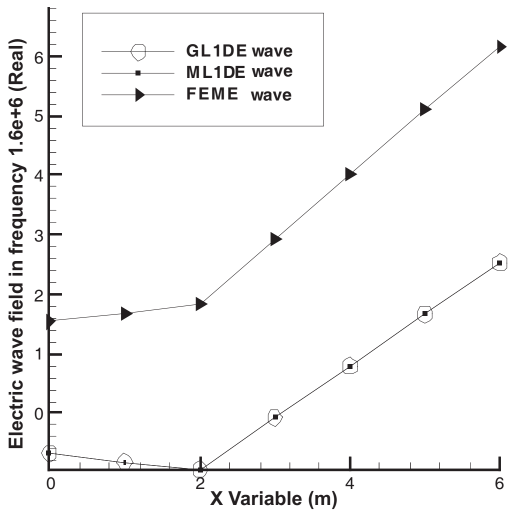

Figure 3: GL, ML and FEM Electric wave with freq. $1.6e^6$ Hz

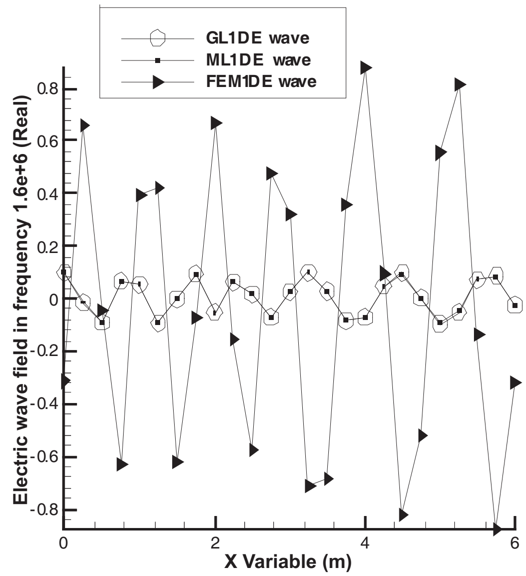

Figure 4: GL, ML and FEM Electric wave with freq. $1.6e^8$ Hz

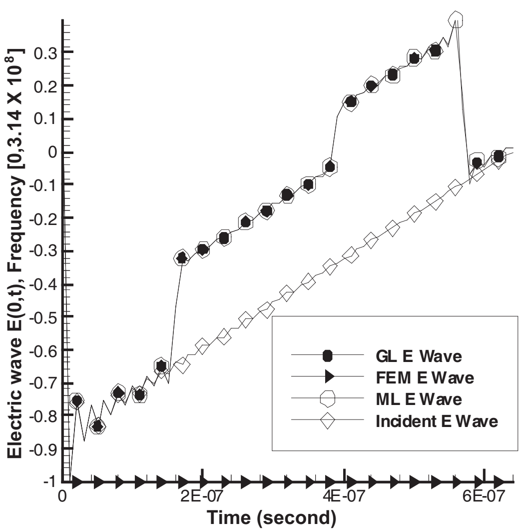

Figure 5: GL, ML and FEM Electric wave E(0,t) in time domain

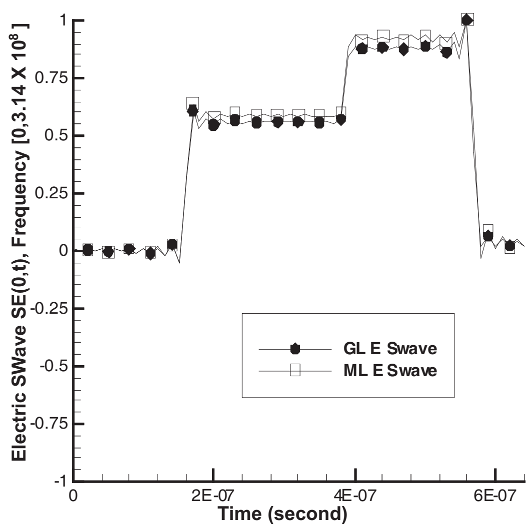

Figure 6: GL and ML Scat. Electric Swave SE(0,t) on time

8. GL Software and Applications and Conclusions

We develope many 3D and 2.5D GL EM, seismic, acoustic, flow, QEM modeling software and some GL EM and seismic inversion software. These GL EM softwares are useful for geophysical EM and seismic exploration; Earthquake EM and seismic exploration; Forest EM and seismic exploration; Environment; EM field in nanometer materials and superconductivity [6]; nondestructive testing imaging [5]; Airborne EM exploration; The stress and displacement analysis in dam, rock, underground structure; the EM Stirring and flow for caster [7]; GPR, radar, and weather imaging; Naiver Stocks weather simulation, etc.. Many applications show that the GL modeling is very fast, low cost and accurate. The GL inversion is stable and high resolution. The GL EM field is fast convergent to exact EM field for high frequency and contrast, while FEM method fails to simulate wave field in the high frequency. The GL method is breakthrough novel method and resolve historical difficulties. GL Geophysical Laboratory and authors have reserved all copyright and patents of 3D/2.5D/2D GL EM, seismic, flow, acoustic QEM modeling and inversion algorithms and have reserved all copyright and patents of the GL software.

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