# One-dimensional Inverse Scattering: Localization of Planar Interface

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The problem of localization of a planar interface in a layered medium from the knowledge of the scattered electromagnetic field is set as an inverse problem within the frequency domain.

This problem has practical interest in many disciplines, as civil engineering and geophysics, because in many applications the subsurface can be considered as a planarly layered medium and the unknown position of interfaces are to be determined. In particular, in civil engineering, an important application is the localization and the determination of the depth of voids embedded in masonries of ancient buildings. Other promising applications concern the assessment of the widths of asphalts or the evaluation of the position of metal inclusions, e.g., sheets or fine meshes reinforcement. In geophysics, the determination of underground stratification is very important.

The problem is often considered in time domain by the search for successive reflections within received reflected field; however sometimes, because of the finite available bandwidth, later reflections due to deeper interfaces are 'buried' by oscillations of earlier responses. We choose to deal with the problem in the frequency domain and found that multiple reflections allow to improve the resolution in the localization. In the formulation we assumed that the dielectric properties of the layered medium are known.

Consider first the localization of a single interface within a layered medium. Since the relation between the scattered field and the position of the unknown interface is non linear; we reformulate the problem by introducing a Dirac distribution [1] centred on the unknown position and derive a linear operator linking such an unknown distribution to the scattered electric field. Next, we invert this operator by the Singular Value Decomposition (SVD) and use the truncated SVD (TSVD) as regularizing algorithm.

In the case of two-layered media, the operator to be inverted is a finite Fourier transform with limited data whose SVD has been well studied and is known in closed form. The resolution [1] is independent of the threshold of truncation of the SVD, that is of the uncertainties on data, due to the typical step-like behaviour of the singular values.

In the case of a three-layered losseless medium, in the search for the second interface, the operator can be recast as the summation of an infinite number of Fourier transforms in correspondence of successive bouncing of the waves inside the layer and a numerical investigation of the SVD of the relevant operator shows the curve of the singular values as successive steps corresponding to each reflection. When only the first two contributions are taken into account [2], an analytical estimate can be performed in some circumstances. The main results is that the achievable resolution can be improved with respect to the single interface case, according to the available uncertainty level on data.

Numerical experiments confirm the main conclusions. In addition the approach has been successfully employed to experimental data, in order to find the position and the width of a void layer inside a masonry of tuff. A procedure amounting to iteratively locating successive interfaces has been established. Calibration data collected in reference geometries make it available the scattered field at each step needed for the inversion.

## REFERENCES

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