Single Value Decomposition and Degree-of-ill-posedness Assessment in Microwave Imaging

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In all Gauss-Newton iterative microwave imaging scenarios, there are a number of parameters related to the overall configuration that can impact the resultant image quality. These include signal frequency, amount of measurement data, number of property parameters reconstructed, etc. Overall, the four most important constituents comprising the system and algorithms are the measurement data, accurate forward solution, parameter update and the sensitivity map in the form of the Jacobian matrix. The latter encodes rich information regarding system performance and algorithm efficiency.

In this presentation we explore the singular value decomposition (SVD) of the Jacobian matrix and adopt the notion of degree-of-ill-posedness, α [1], developed by Brander and DeFacio [2] for the inverse Born approximation. While these previous discussions were developed for analytical expressions of the Jacobian matrix, we have applied minor approximations to our standard imaging algorithms to generate the numerically-based nodal-adjoint expression for constructing the Jacobian matrix [3]. This has facilitated convenient computation of the degree-of-ill-posedness for multiple system parameter studies. We have also applied this only to the first iteration of the Jacobian matrix to simplify the overall discussion while being able to observe important trends with respect to parameter variation. We have also restricted this analysis to a circular antenna array geometry with the single point sources configured close to the target under investigation.

We have studied three important algorithm parameters in simulation which have important implications with respect to hardware implementation and overall system performance: signal frequency, amount of measurement data and number of property parameters reconstructed. In general, the results match previous intuitive notions; i.e., that increased frequency and increased measurement data improve α. However the results with respect to the number of inverse parameters are somewhat counter-intuitive in that α improves even well after the number of parameters has exceeded the amount of independent measurements ($\mathrm{N}_{\mathrm{sources}} \times \mathrm{N}_{\mathrm{receivers}}/2$). Historically, groups have worked to collect more independent data than parameters reconstructed at considerable system expense. This latter result confirms observations we have made with our microwave system in both phantom and clinical imaging sessions and also for the near infrared imaging system being developed at Dartmouth College [4, 5]. These analyses have provided important insights as we work toward robust clinical implementation.

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