The Effective Permittivity of Inhomogeneous Objects Reconstructed by Inverse Scattering Methods

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The notion of effective permittivity originates from the effective medium theory (EMT), where a medium randomly inhomogeneous on a micro-scale is represented as a homogeneous bulk material. There exist many different formulas that give the effective permittivity in terms of the permittivities, sizes, and shapes of the microscopic constituents. Normally one tries to find an expression which would suit all possible illumination and measurement conditions. The problem is, in fact, an inverse scattering problem, and as such can be investigated using the standard inversion methodology. In particular, we have looked at it in the context of *effective inversion*, i. e. inversion with very limited scattered field data where it is impossible to reconstruct an inhomogeneous subsurface target, but may be possible to recover its effective permittivity having an approximate knowledge of the outer shape of the object. That latter shape is often the result of simple linearized imaging algorithms, such as the travel-time tomography and the sampling method. The difference with the EMT is that, strictly speaking, the reconstructed effective permittivity suits only some particular illumination and measurement configuration. Yet our results indicate that the reconstructed effective permittivities are closely related to the permittivities of the constituents. On the other hand, the inverse scattering approach is much more rigorous as far as the reconstruction of the effective permittivity is concerned, than the general but approximate techniques of the EMT. For example, no statistical assumptions are needed and the inhomogeneities do not have to be electrically small.

This time we shall talk about a very fast and efficient numerical method specifically designed for the effective inversion problem. As common in inverse scattering, the problem is nonlinear and therefore requires repetitive solution of the associated forward scattering problem. In its turn the forward problem has to be solved by one or another iterative algorithm. Hence, the solution of the inverse problem requires multiple runs of an iterative scheme. Most of the work in such a scheme is done at the stage of construction of the so-called Krylov subspace. In the effective inversion case, however, the once constructed Krylov subspace can be re-used, thereby significantly reducing the computational order of the inverse problem. With the help of this reduced-order algorithm various inhomogeneous targets can be analyzed with little computational effort. Special attention will be paid to the dispersion of the reconstructed effective permittivity.