Design of a Metafilm-composite Dielectric Shielding Structure Using a Genetic Algorithm

J. Y. Huang[1], **M. Y. Koledintseva**[1], **P. C. Ravva**[1], **J. L. Drewniak**[1]
R. E. DuBroff[1], **B. Archambeault**[2], **and K. N. Rozanov**[3]
[1]University of Missouri-Rolla, USA
[2]IBM Co., USA
[3]Russian Academy of Sciences, Russia

Abstract—An analytical model for a shielding structure containing both bulk composite layers and planar metafilms (MFs) made of perfect electric conductors is presented, allowing for synthesis of shielding structures using the genetic algorithm (GA) optimization. MFs can be of two different types: patch or aperture. The frequency response, specifically, transmission (T) and reflection (Γ) coefficients in a plane-wave formulation, of any MF is calculated based on polarizabilities determined by the particular pattern geometry. T and Γ of a patch-type MF are derived using the generalized sheet transition conditions (GSTC) and the Babinet's duality principle is used for aperture-type MF to map the results from the complementary problem. T and Γ for a single-layered MF are represented in a unified matrix form for any angle of incidence. T-matrix approach is used for getting T and Γ for a multilayered structure. Any MF buried in a host dielectric can be decomposed into three types of basic elements: a host composite slab, interface between media, and an MF inside the homogeneous host medium. Each basic element is described by a corresponding T-matrix, and the total T-matrix of the stack is the sequential product of the each individual T-matrix. T and Γ of the stack can be easily derived from the total T-matrix. If there are two or more MFs, the distance between them justifies the condition of neglecting higher-order evanescent mode interactions. Then the GA is applied to engineer a structure with the desired frequency response. It helps to choose the best geometry of MF patterns, thickness of layers, and appropriate constitutive parameters of each composite layer.

1. Introduction

For many practical applications, it is desirable to develop shielding structures having specified frequency characteristics. Application of a robust and quickly converging genetic algorithm (GA) facilitates the engineering of composite materials, saving time and resources before manufacturing and testing real materials [1]. A shielding structure may consist of a single composite dielectric or a multilayered stack of composite dielectrics with given electromagnetic properties. However, composite dielectric layers alone may be insufficient for achieving the acceptable shielding effectiveness (SE) in a given frequency range. The presence of metafilms (MFs) buried in composite layers may increase SE in the frequency band of interest, or assure desirable frequency-selective effects.

In this work, a model of a shielding structure containing both bulk composite layers and MFs made of PEC has been developed analytically, and the approach is considered below. For simplicity, we only consider MFs constructed by square arrays (with the same periodicity along two orthogonal axes in the plane) buried inside a homogeneous host material.

2. Mathematical Model

2.1. Using T-matrix Approach to Analyze T and Γ of the Multilayered Structures

The T-matrix used in this model is similar to that defined in [2]. A wave-transmission system is modeled as a two-port network. The forward and backward waves at the input and output ports are related as

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \cdot \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}, \tag{1}$$

where a_1 and a_2 are the incoming wave and b_1 and b_2 are outgoing waves.

The total T-matrix of N cascaded 2-port networks T_1, T_2, $\ldots$, T_N is the sequential product of the corresponding T-matrices,

$$T_{tot} = \begin{bmatrix} t_{11}^{tot} & t_{12}^{tot} \\ t_{21}^{tot} & t_{22}^{tot} \end{bmatrix} = T_1 T_2 \ldots T_N. \tag{2}$$

Then T and Γ of the multilayered MF can be found as [2]:

$$T = t_{21}^{tot} = b_2/a_1 \big|_{a_2=0} \text{ and } \Gamma = t_{11}^{tot} = b_1/a_1 \big|_{a_2=0}. \tag{3}$$

Any MF buried in a host medium can be decomposed into 3 basic elements: (a) a host medium slab; (b) a medium interface, and (c) an MF inside the homogeneous host medium, as shown in Fig. 1. T-matrices will be obtained below for all these three cases. Then T and Γ of the stack can be easily obtained from (2) and (3).

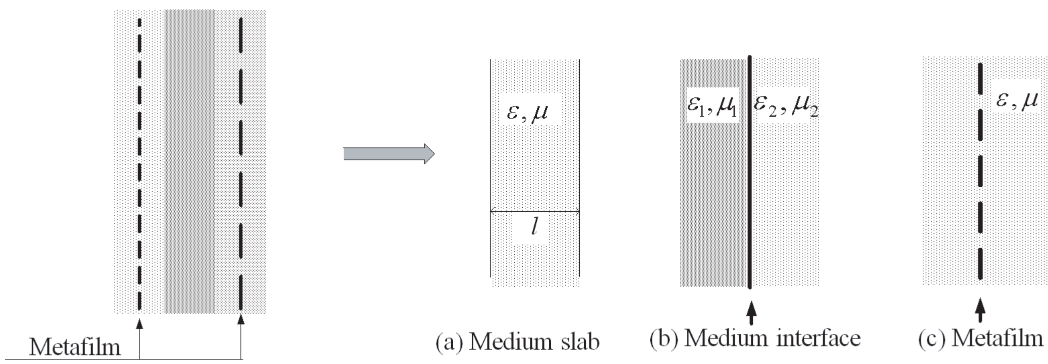


Figure 1: Decomposition of multilayered MF.

2.2. Formulation for a Composite Layer

The Maxwell Garnett (MG) effective medium formulation can serve as a basis for engineering composite microwave materials [1],

$$\varepsilon_{eff} = \varepsilon_b + \frac{\frac{1}{3}\sum_{i=1}^{n} f_i(\varepsilon_i - \varepsilon_b) \sum_{j=1}^{3} \frac{\varepsilon_b}{\varepsilon_b + N_{ij}(\varepsilon_i - \varepsilon_b)}}{1 - \frac{1}{3}\sum_{i=1}^{n} f_i(\varepsilon_i - \varepsilon_b) \sum_{j=1}^{3} \frac{N_{ij}}{\varepsilon_b + N_{ij}(\varepsilon_i - \varepsilon_b)}}, \tag{4}$$

where $\varepsilon_b(j\omega) = \varepsilon_{\infty b} + \chi_b(j\omega)$ and $\varepsilon_i(j\omega) = \varepsilon_{\infty i} + \chi_i(j\omega)$ are the relative permittivity of a base dielectric and of the i-th type of inclusions, respectively; $\varepsilon_{\infty b,i}$ are the high-frequency permittivities for the base material and inclusions of the i-th type, respectively; and $\chi_{b,j}(j\omega)$ are the corresponding dielectric susceptibility functions. f_i is the volume fraction occupied by the inclusions of the i-th type; N_{ij} are the depolarization factors of the i-th type of inclusions, and the index $j = 1, 2, 3$ corresponds to x, y, and z Cartesian coordinates. The effective permittivity of a mixture might have complex-shaped frequency dependence. As shown in [1], it can be approximated by a series of Debye-like terms with real relaxation constants τ_k and complex (in general case) coefficients A_k. The coefficients A_k can be found using the genetic algorithm (GA) optimization technique [3],

$$\varepsilon_{eff}(j\omega) = \varepsilon_{\infty eff} + \chi_{eff}(j\omega) = \varepsilon_{\infty eff} + \sum_{k=1}^{N} \frac{A_k}{1 + j\omega\tau_k}. \tag{5}$$

The T-matrix of the homogenized composite slab with a thickness of l is as in [4]:

$$T_S = \begin{bmatrix} e^{jkl\cos\theta} & 0 \\ 0 & e^{-jkl\cos\theta} \end{bmatrix}, \tag{6}$$

where $k = \omega\sqrt{\varepsilon_{eff}\mu}$ is the wave number of the effective composite medium, and θ is the angle of incidence.

2.3. T-matrix for an Interface of Two Media

Let two media have permittivites ε_1, ε_2 and permeabilities μ_1, μ_2, respectively. The T-matrix [4] is

$$T_I = \frac{1}{\tau_{T1}} \begin{bmatrix} 1 & \rho_{T1} \\ \rho_{T1} & 1 \end{bmatrix}, \tag{7}$$

where

$$\tau_{T1} = \frac{\eta_{T2} - \eta_{T1}}{\eta_{T2} + \eta_{T1}},\ \rho_{T1} = \frac{2\eta_{T2}}{\eta_{T2} + \eta_{T1}},\ \eta_{T1,2} = \begin{cases} \sqrt{\mu_{1,2}/\varepsilon_{1,2}}/\cos\theta & \text{for } TE \text{ plane wave;} \\ \sqrt{\mu_{1,2}/\varepsilon_{1,2}} \cdot \cos\theta & \text{for } TM \text{ plane wave.} \end{cases} \tag{8}$$

2.4. Plane Wave Formulas for Single-layered MFs

2.4.1. Extended GSTC for MFs

For an MF buried inside a homogeneous host medium with permittivity ε_{eff} and permeability μ, suppose the microscopic polarizability tensor $\bar{\bar{\alpha}}$ of the pattern is

$$\bar{\bar{\alpha}} = \begin{bmatrix} \bar{\bar{\alpha}}_{ee} & \bar{\bar{\alpha}}_{em} \\ \bar{\bar{\alpha}}_{me} & \bar{\bar{\alpha}}_{mm} \end{bmatrix}. \tag{9}$$

Extending the GSTC [5] for the case when there is cross-coupling between electric and magnetic polarizations of individual scatterers of MFs, the following boundary conditions for any metafilm in (xy) plane can be derived:

$$\begin{aligned} \hat{z} \times \vec{H} \Big|_{z=0^-}^{0^+} &= j\omega \left[\bar{\bar{\alpha}}_{EE,t} \quad \bar{\bar{\alpha}}_{EM,t}\right] \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} - \hat{z} \times \nabla_t \left(\left[\bar{\bar{\alpha}}_{ME,z} \quad \bar{\bar{\alpha}}_{MM,z}\right] \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} \right); \\ \vec{E} \Big|_{z=0^-}^{0^+} \times \hat{z} &= -j\omega\mu \left[\bar{\bar{\alpha}}_{ME,t} \quad \bar{\bar{\alpha}}_{MM,t}\right] \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} - \frac{1}{\varepsilon_{eff}} \nabla_t \left(\left[\bar{\bar{\alpha}}_{EE,z} \quad \bar{\bar{\alpha}}_{EM,z}\right] \cdot \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}_{av} \right) \times \hat{z}, \end{aligned} \tag{10}$$

where the tensors are sub-components of the macroscopic polarizability, determined by the microscopic polarizability $\bar{\bar{\alpha}}$ and periodicity of the metafilm pattern. Assuming that the pattern periods are equal, $D_x = D_y = D$,

$$\bar{\bar{\alpha}}^{mac} = \begin{bmatrix} \bar{\bar{\alpha}}_{EE} & \bar{\bar{\alpha}}_{EM} \\ \bar{\bar{\alpha}}_{ME} & \bar{\bar{\alpha}}_{MM} \end{bmatrix} = \left[D^2 \bar{\bar{I}} + \bar{\bar{\alpha}} \cdot \bar{\bar{G}} \right]^{-1} \cdot \bar{\bar{\alpha}}. \tag{11}$$

The matrix $\bar{\bar{G}} = Diag \left[-\frac{1}{4R\varepsilon_{eff}} \quad -\frac{1}{4R\varepsilon_{eff}} \quad \frac{1}{2R\varepsilon_{eff}} \quad -\frac{1}{4R} \quad -\frac{1}{4R} \quad \frac{1}{2R} \right]$, where $R \approx 0.6956D$, according to [5].

2.4.2. T and Γ for Single-layered Patch-type MFs at the Oblique Plane Wave Incidence

Consider the TE or TM plane waves as in Fig. 2.

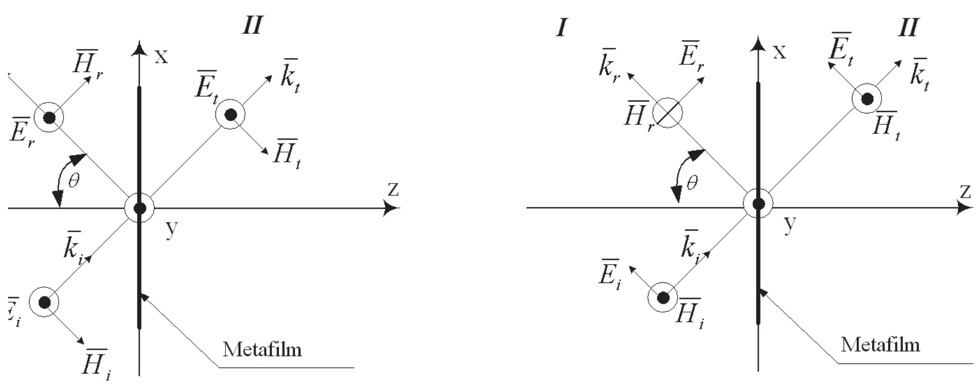


Figure 2: TE and TM polarized plane waves.

Using the GSTC and the approach in [6], let us introduce the forward and backward vectors $\bar{C}^{\,+}_{TE(TM)}$ and $\bar{C}^{\,-}_{TE(TM)}$ as

$$\bar{C}^{\,\pm}_{TE} = [0 \quad 1 \quad 0 \quad \mp\cos\theta/\eta \quad 0 \quad \sin\theta/\eta]^T; \quad \bar{C}^{\,\pm}_{TM} = [\cos\theta \quad 0 \quad \mp\sin\theta \quad 0 \quad \pm 1/\eta \quad 0]^T, \tag{12}$$

where $\eta = \sqrt{\varepsilon_{eff}/\mu}$.

The following linear system can be derived for solving $T_{TE(TM)}$ and $\Gamma_{TE(TM)}$:

$$\begin{bmatrix} A_{1,TE(TM)} & A_{2,TE(TM)} \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} T_{TE(TM)} \\ \Gamma_{TE(TM)} \end{bmatrix} = \begin{bmatrix} A_{3,TE(TM)} \\ 1 \end{bmatrix}, \tag{13}$$

$$\begin{aligned} A_{1,TE} &= \left([U_{TE(TM)}] - [V_{TE}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{+}_{TE}; & A_{1,TM} &= \left([U_{TM}] + [V_{TM}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{+}_{TE}; \\ A_{2,TE} &= -\left([U_{TE}] + [V_{TE}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{-}_{TE}; & A_{2,TM} &= \left(-[U_{TM}] + [V_{TM}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{-}_{TM}; \\ A_{3,TE} &= \left([U_{TE}] + [V_{TE}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{+}_{TE}; & A_{3,TM} &= \left([U_{TM}] - [V_{TM}] \cdot \bar{\bar{\alpha}}^{mac}\right) \cdot \bar{C}^{+}_{TE}; \end{aligned} \tag{14}$$

where the elements $A_{i,TE(TM)}$ are

$$[U_{TE}] = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]; \qquad [V_{TE}] = [0 \quad jk/(2\varepsilon_{eff}) \quad 0 \quad 0 \quad 0 \quad jk\sin\theta/2];$$
$$[U_{TM}] = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]; \qquad [V_{TM}] = [j\omega/2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]. \tag{15}$$

2.4.3. T and Γ for Single Layered Aperture-type MFs

It is found that T and Γ cannot be calculated by directly applying the corresponding polarizabilities to the GSTC. However, this obstacle can be bypassed by solving the corresponding patch-type complementary problem, and then, using Babinet's duality principle [7, 8], mapping the results into T and Γ of the aperture-type MFs [9]. The relations between T and Γ for two complementary arrays at oblique incidence are

$$T_{TE} = -\tilde{\Gamma}_{TM} \quad and \quad \Gamma_{TE} = -\tilde{T}_{TM}. \tag{16}$$

In (16), the tilde refers to the complementary structure.

T-matrix of a metafilm buried in a homogeneous host material is

$$T_{M_{TE(TM)}} = \frac{1}{T_{TE(TM)}} \begin{bmatrix} 1 & -\Gamma_{TE(TM)} \\ \Gamma_{TE(TM)} & T^2_{TE(TM)} - \Gamma^2_{TE(TM)} \end{bmatrix} \tag{17}$$

2.5. Requirement for Distance d between Neighboring MFs

The distance d between two neighboring MFs must be large enough for the evanescent modes to sufficiently decay and not interfere with the propagating mode. Given the ratio δ of the amplitude of the most intense high-order mode to the amplitude of the main propagating mode, the following inequality must fulfill:

$$\left| e^{jd\left(k-\sqrt{k^2-(2\pi/D)^2}\right)} \right| < \delta. \tag{18}$$

Numerous simulations have shown that the ratio δ <10% is sufficient for neglecting the higher-order modes.

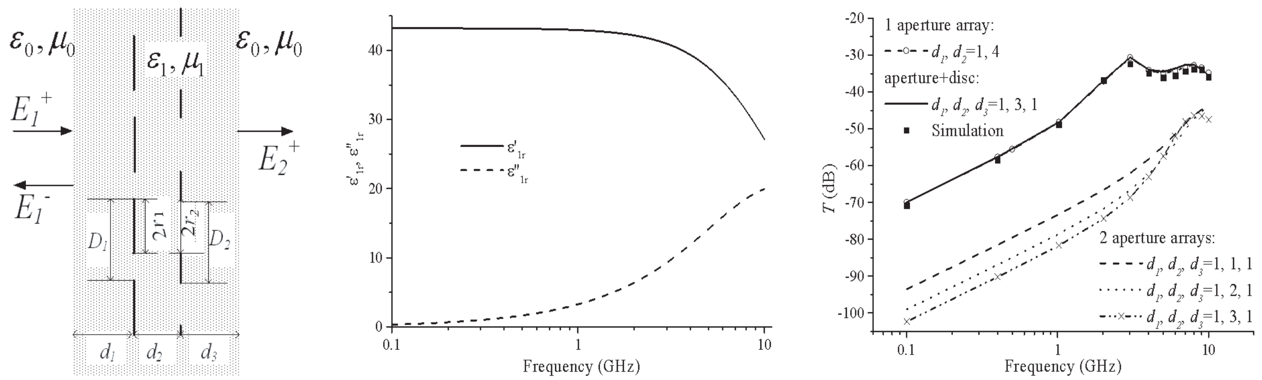
3. Genetic Algorithm for Synthesis of MF-Composite Shielding

Before the synthesis process, a designer should have some initial information based on a particular application of the shielding under design. The requirements for the desired frequency response of the shielding structure should be known, and an appropriate number of composite layers and the total maximum thickness of the structure, as well as the reasonable ranges of electromagnetic parameters of layers for the initial search pool should be specified.

The synthesis algorithm determines thickness and frequency dependence of the effective parameters of each layer. The GA yields a "recipe" of physical parameters (appropriate base material, aspect ratio, concentration, and conductivity of inclusions) for composite layers. The designer chooses the best solution (parameters characterizing the frequency dependence for composites, pattern geometry for MFs, and the order of layer disposition) for approximating the desired frequency response. This latter selection is based on a range of practically available ingredients with realistic parameters. Thus, the codes developed for the design of shielding structures with the desired frequency characteristic combine the Maxwell Garnett effective medium mixing rule, the described above analytical formulation, and the GA optimization procedure.

4. Computation Results

Consider the three-layer structure with two MFs as in Fig. 3(a). The parameters are the following: the slab thickness is $d_1 + d_2 + d_3 = 5$ mm, the cell period is $D_1 = D_2 = 2$ mm, the radius of the apertures in the left metafilm is $r_1 = 0.6$ mm, and the radius of the discs (or apertures) in the right metafilm is $r_2 = 0.6$ mm. The host dielectric is a composite containing carbon particles in a Teflon base $\varepsilon_b = 2.2$ (dispersion and loss are neglected). Carbon particles having conductivity of $\sigma = 1000$ S/m are shaped as cylinders with the aspect ratio a=*length*/*diam* = 50. Their volume fraction in the composite is 8%, while percolation threshold is higher than 9%. The best parameters of the composite from a shielding effectiveness point of view, and at the same time, practically available composite material components were chosen using the GA. The frequency characteristic of the composite dielectric with $\varepsilon_1(f)$ and $\mu_1 = \mu_0$ shown in Fig. 3(b) was modeled using (4) and approximated by one Debye term in (5). Fig. 3(c) shows the calculated transmission coefficient for the structure at different distances $d_{1,2,3}$. The best shielding effectiveness of the structure in the frequency range of interest is obtained



(a) (b) (c)

Figure 3: Multilayered structure with two different MFs buried in the composite dielectric layer.

with the thicknesses $d_1 = 1\,\text{mm}$; $d_2 = 3\,\text{mm}$; $d_3 = 1\,\text{mm}$ determined by the GA with two aperture MFs. Analytical and numerical simulation (using HFSS software) confirm this.

5. Conclusions

The shielding structures containing MFs and composite dielectrics can be engineered based on the presented analytical formulas for T and Γ and using an optimization GA. T and Γ are directly related to geometries of MF patterns, constitutive parameters, concentrations, and geometries of composite material phases. This approach provides a straightforward synthesis process for desirable frequency responses.

The effective parameters of the composites are modeled by Maxwell Garnett mixing formalism. The analytical formulas for T and Γ of multilayered MF structures are obtained using (1) the generalized GSTC, (2) the Babinet's duality principle for complementary structures—aperture-type and patch-type MFs, and (3) the T-matrix cascading.

The analytical approach in this paper has an advantage over the full-wave numerical methods, since it saves computational resources for the synthesis process, and reduces the design cost prior to manufacturing.

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