Light Scattering on 2D Nanostructured Resonant Gratings

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Abstract—This paper studies nanostructured gratings made up by silver nanoparticles embedded in the dielectric, which are capable of maintaining quasi-static modes. The special emphasis is devoted to following specified types of gratings: row of periodical cylinders and square grating of spheres. The problem of a diffraction of a plane electromagnetic wave on such structures has been solved within the dipolar-interaction approximation. The frequency dependences of the refraction and absorption coefficients on the grating parameters have been obtained, analyzed and compared.

1. Introduction

Recently, a significant success has been achieved in the areas related to creation of metamaterials based on resonance metal elements, specifically, films with embedded metal nanoparticles [1–3] that are capable of sustaining high-Q-factor quasi-static modes. At the resonance frequency, the scattering cross-section of such particles exceeds their geometric sizes significantly, which yields a number of new collective optical properties when they join up in nanostructures. The most preferable, in terms of practical applications (both from the standpoint of their chemical stability and resonance characteristics), are the nanoparticles of silver and gold. The coherent effects of light scattering on plane gratings formed by cylindrical and spherical nanometer silver objects have been analyzed.

2. Diffraction on a Periodical Structures

This work studies the diffraction of a plane P-polarized wave with the form:

$$\begin{aligned} H_y &= H_o \mathtt{exp}(-i\omega t - ik_o \mathtt{cos}\varphi z - ik_o \mathtt{sin}\varphi x) \\ E_x &= -E_o \mathtt{cos}\varphi \mathtt{exp}(-i\omega t - ik_o \mathtt{cos}\varphi z - ik_o \mathtt{sin}\varphi x) \\ E_z &= -E_o \mathtt{sin}\varphi \mathtt{exp}(-i\omega t - ik_o \mathtt{cos}\varphi z - ik_o \mathtt{sin}\varphi x) \end{aligned} \quad (1)$$

that falls from the vacuum onto a plane grating formed by silver nanostructures (see Figure 1). Two simplest and at the same time, evidently, basic configurations of the grating are considered: the first (unidimensional) is a periodic row of cylinders with their axes lying in the plane $z = 0$ and oriented along the y-axis (see Figure 1a), and the second (two-dimensional) is a periodic (both along x and y) square grating of spheres with their centres in the plane $z = 0$ (see Figure 1b).

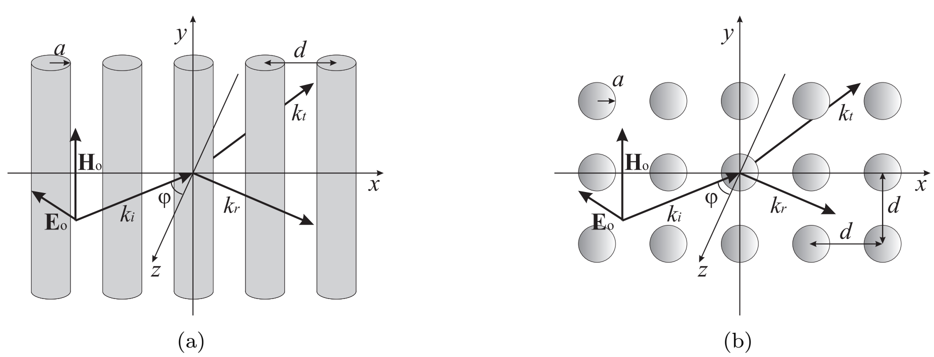
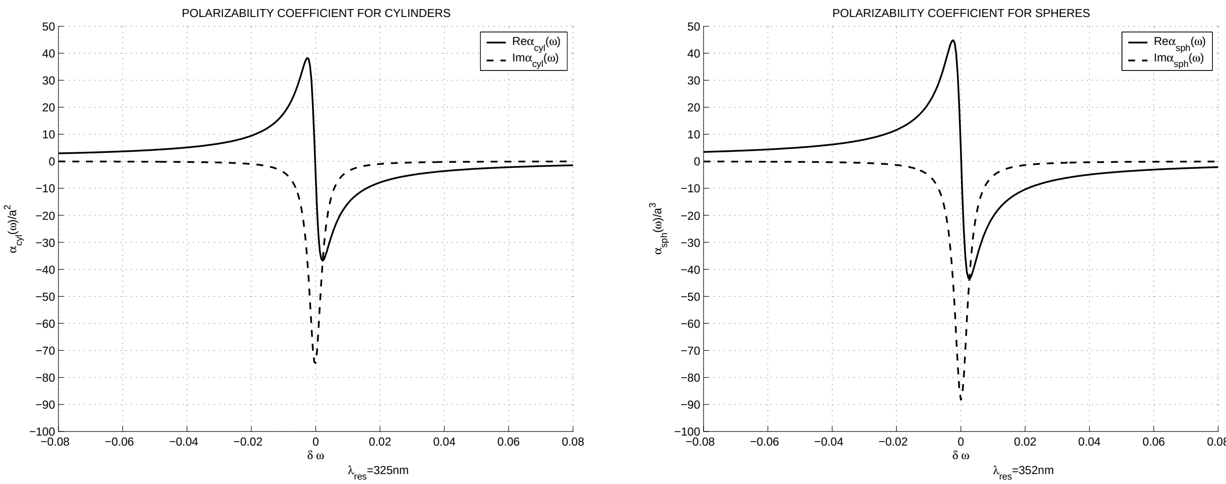


Figure 1: Configurations of considering gratings.

Let us assume, for the sake of simplicity, that the dielectric permittivity of the substrate, which the grating is mounted on, is close to unity, such that the environment is actually vacuum everywhere. Let the radii of the cylinders and the spheres are small as compared with the length of the incident wave λ ($a << \lambda$). Then the field scattered by these structural elements is the field of a linear dipole with its dipole momentum (per length unit) $\mathbf{P}_{cyl} = \alpha_{cyl}\mathbf{E}_D$ for the cylinder, and the field of a point dipole with its dipole momentum $\mathbf{P}_{sph} = \alpha_{sph}\mathbf{E}_D$ for the sphere, where $\mathbf{E}_D$ is the effective field. When radiation losses are neglected, the polarizability coefficients look as follows (see Figure 2):



(a) (b)

Figure 2: Polarizability coefficients of considering elements as a function of relative frequency shift $\delta\omega = \frac{\omega-\omega_{res}}{\omega_{res}}$ ($\lambda_{res} = 325nm$ for cylinders, $\lambda_{res} = 352nm$ for spheres).

$$\alpha_{cyl} = \frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+1}a^2 \qquad \alpha_{sph} = \frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}a^3 \tag{2}$$

Here $\varepsilon(\omega)-$ is the dielectric permittivity of the object. For silver, which is interest for us, it is described, in the range $\lambda \sim 300 - 500nm$, with good accuracy as [4]:

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega - i\gamma)} \tag{3}$$

where $\varepsilon_\infty = 4.7, \omega_p = 1.38 \cdot 10^{16}s^{-1}, \gamma = 2.7 \cdot 10^{13}s^{-1}$.

The effective field is the sum of the incident field and the fields of all other dipoles at the location of some segregated dipole in its absence. We propose that the following procedure should be used to find that field, which is somewhat different from the traditional procedure and, in our opinion, seems to be convenient. Taking into account that a polarized medium can be described by means of polarization currents $\mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} = -i\omega\mathbf{P}$, let us pass over from dipoles to currents. The density of such currents is represented as:

$$\begin{aligned} \mathbf{J}_{cyl}(x) &= -i\omega\alpha_{cyl}(\omega)\mathbf{E}_D^{cyl}\delta(z)\sum_{n=-\infty}^{+\infty}\delta(x-nd) \\ \mathbf{J}_{sph}(x,y) &= -i\omega\alpha_{sph}(\omega)\mathbf{E}_D^{sph}\delta(z)\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\delta(x-nd)\delta(y-md) \end{aligned} \tag{4}$$

where $\delta(\cdot)$ is the Dirac δ function.

Then, from the shown system of discrete currents, using the Poisson formula [5] we pass over to continuous surface dummy currents of spatial harmonics:

$$\begin{aligned} \mathbf{J}_{cyl}(x) &= -i\omega\alpha_{cyl}(\omega)\mathbf{E}_D^{cyl}\frac{\delta(z)}{d}\sum_{n=-\infty}^{+\infty}\mathtt{exp}(i\frac{2\pi}{d}nx) \\ \mathbf{J}_{sph}(x,y) &= -i\omega\alpha_{sph}(\omega)\mathbf{E}_D^{sph}\frac{\delta(z)}{d^2}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathtt{exp}(i\frac{2\pi}{d}nx)\mathtt{exp}(i\frac{2\pi}{d}my) \end{aligned} \tag{5}$$

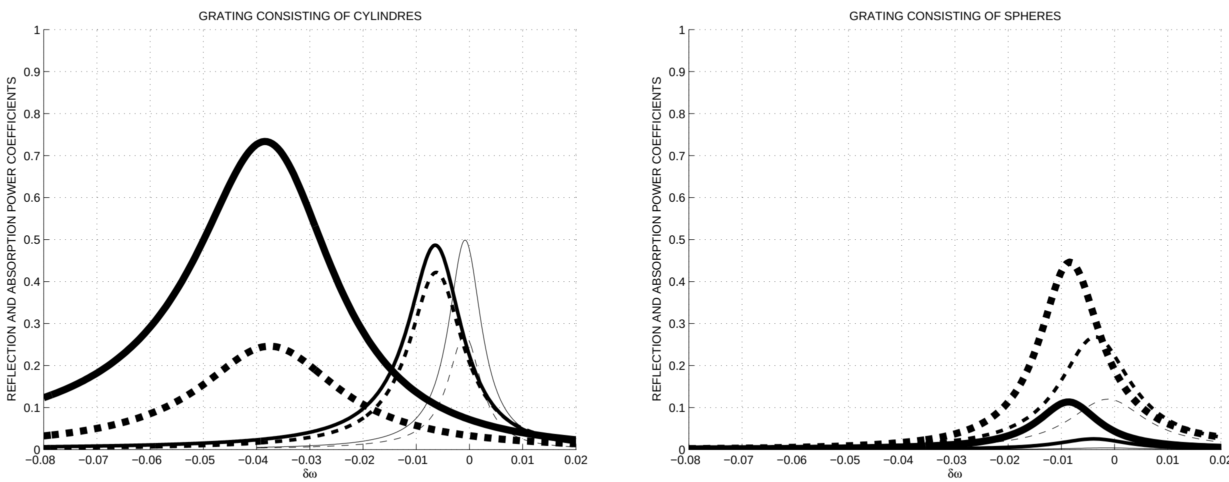
Finding the field of the individual spatial harmonic is elementary for the tangential component of the current, and somewhat more difficult for the normal one. Note that the normal component of the electric field is actually equivalent to the tangential of the magnetic current. The effective field is further obtained by subtracting the field of the segregated dipole situated, e. g., at the origin of coordinates. Representing the latter as an integral over the same harmonics, we obtain finally the following self-consistent expressions:

$$\mathbf{E}_D^{cyl}(0) = \mathbf{E}_o(x=z=0) + \sum_{n=-\infty}^{+\infty} \tilde{\mathbf{E}}(\varkappa_x = \frac{2\pi}{d}n) - \frac{d}{2\pi}\int_{-\infty}^{+\infty} d\varkappa_x \tilde{\mathbf{E}}(\varkappa_x)$$
$$\mathbf{E}_D^{sph}(0,0) = \mathbf{E}_o(x=z=0) + \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} \tilde{\mathbf{E}}(\varkappa_x = \frac{2\pi}{d}n, \varkappa_y = \frac{2\pi}{d}m) - \frac{d^2}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\varkappa_x d\varkappa_y \tilde{\mathbf{E}}(\varkappa_x, \varkappa_y) \quad (6)$$

from which the effective field is extracted as a function of the incident field. It should be noted that the second and third term in formula (6) have singularities at zero, which are mutually compensated. This should be taken into account when performing numerical calculations.

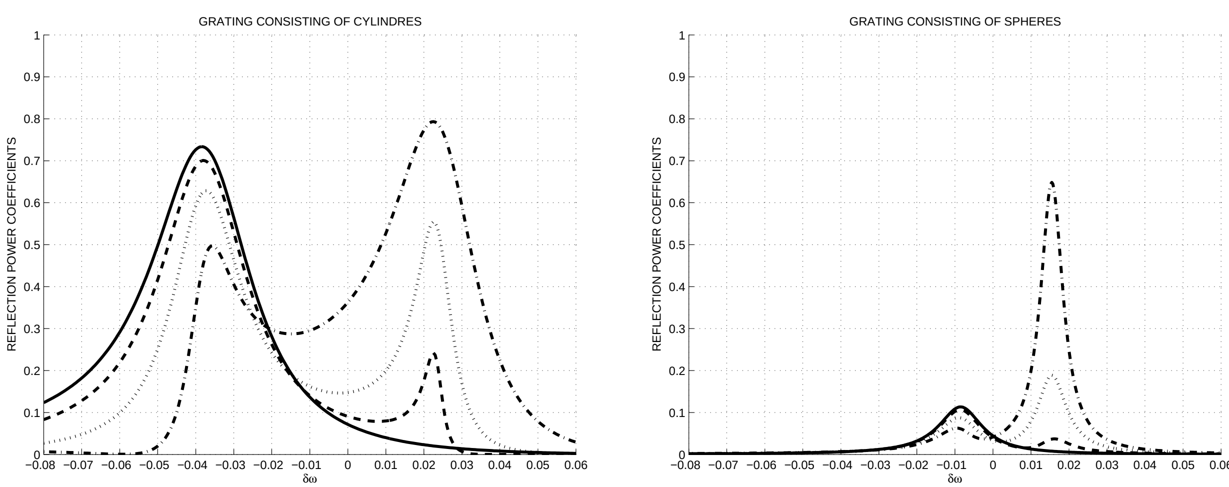
Granted that the effective field is known, it is possible to solve the set diffraction problem.

3. Results and Discussion



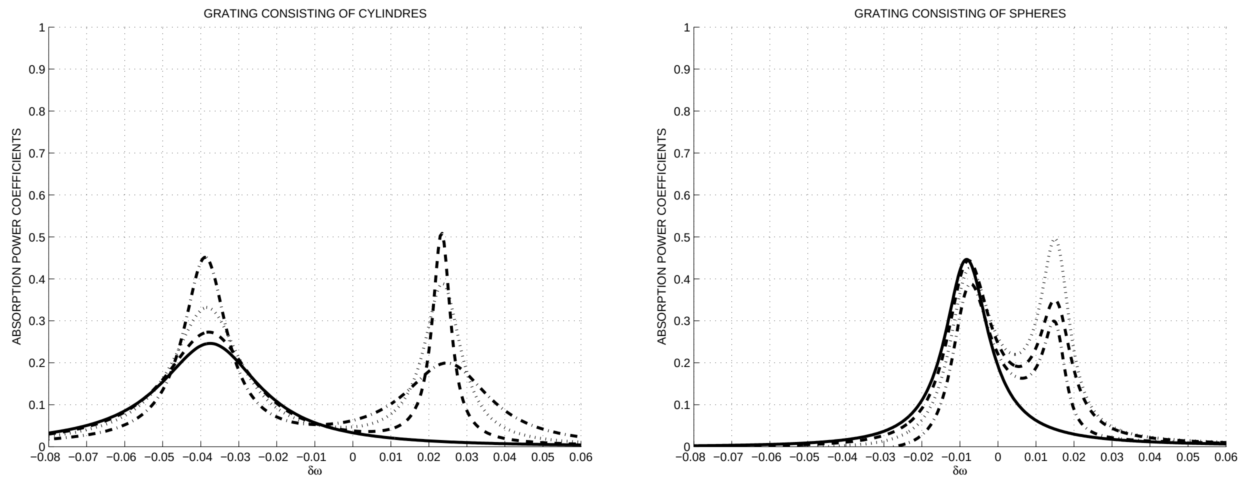
(a) (b)

Figure 3: Reflection (firm line) and absorpion (dash line) power coefficients of considered gratings as a function of relative frequency shift $\delta\omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various periods d of gratings ($a/d = 0.2$—blue line, $a/d = 0.1$ —green line, $a/d = 0.05$—red line). The normal incidence case ($\varphi = 0$).



(a) (b)

Figure 4: Reflection power coefficients of considered gratings as a function of relative frequency shift $\delta\omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various incidence angles ($\varphi = 0°$—red line, $\varphi = 30°$—green line, $\varphi = 50°$—blue line, $\varphi = 70°$—black line). Relation $a/d = 0.2$ is fixed.



(a) (b)

Figure 5: Absorption power coefficients of considered gratings as a function of relative frequency shift $\delta\omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various incidence angles ($\varphi = 0°$—red line, $\varphi = 30°$—green line, $\varphi = 50°$—blue line, $\varphi = 70°$—black line). Relation $a/d = 0.2$ is fixed.

Further the results of the numerical calculations based on the formulas (6) are presented and discussed. Reflection and absorpion power coefficients as a function of relative frequency shift $\delta\omega = \frac{\omega - \omega_{res}}{\omega_{res}}$ for various grating parameters are shown in Figures 3, 4, 5. All results are given for the fixed parameter $k_{res}a = \frac{\omega_{res}}{c}a = 0.08$. Thus, the radius of cylinders a is equal $4.1nm$ and the radius of spheres a is equal $4.5nm$. The left-hand parts of figures respond a case of the grating from cylinders, and right parts—to a case of the grating from spheres. Comparison allows to present common features and distinctivenesses.

Figure 3 convincingly shows effect of coherent interaction of separate elements at their converging. The peak value of reflection coefficients for the grating from spheres is noticeably less than for the grating from cylinders. It is stipulated by essential difference in filling factors ($f_{cyl}/f_{sph} = 4a/3d$). Different shift of frequencies, at which the maxima of reflection coefficient for considered gratings is attained, is determined, apparently, various interactions of linear dipoles and point dipoles.

In case of oblique incidence (see Figure 4) with increase of an incidence angle, one more peak occurs and gradually grows. It is stipulated by coherent interaction dipoles oriented along the z-axis.

4. Conclusion

A two-dimensional problem of plane electromagnetic wave diffraction on a gratings consisting of resonance elements is solved in dipole-interaction approximation. A novel method of obtaining effective field expression is proposed. Reflection and absorption coefficients are found for various compositions of gratings parameters.

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