Uniform Signal Contribution of the Step Function Modulated Sine Wave

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A canonical problem of central importance in the theory of ultrawideband pulse propagation through temporally dispersive, absorptive materials is the propagation of a step function modulated signal through a dielectric material that exhibits anomalous dispersion. One method in which a closed-form approximation to the propagated pulse may be obtained is application of asymptotic expansion techniques to the Fourier integral representation. For a linearly-polarized plane-wave pulse traveling in the positive z-direction with the temporal behavior on the plane $z = 0$ given by

$$E(0,t) = u(t)\sin(\omega_c t), \tag{1}$$

the propagated pulse on the plane $z > 0$ is given exactly by the Fourier-Laplace integral

$$E(z,t) = \frac{1}{2\pi}\Re\left\{ i \int_{ia-\infty}^{ia+\infty} \frac{i}{\omega-\omega_c} \exp\left[\frac{z}{c}\phi(\omega,\theta)\right] d\omega \right\}. \tag{2}$$

Here, $u(t)$ denotes the step function, ω_c is the *carrier frequency* of the input pulse, $\phi(\omega,\theta)$ is the *complex phase function*

$$\phi(\omega,\theta) \equiv i\frac{c}{z}\left(\tilde{k}(\omega)z - \omega t\right) = i\omega\left[n(\omega) - \theta\right], \tag{3}$$

where $\tilde{k}(\omega) = (\omega/c)n(\omega)$ is the *complex wavenumber*, $\theta = ct/z$ is a dimensionless space-time parameter, and a is a real constant greater than the abscissa of absolute convergence for the initial field $E(0,t)$. The problem then is to evalute the contour integral (2) for all $\theta > 1$, the field given by (2) vanishing identically for all $\theta \leq 1$.

Asymptotic expansion techniques were first applied to (2) by Brillouin in 1914 [Brilluoin, *Ann. Phys.* **44** (1914)]. His analysis showed that the propagated field is comprised of two precursors and the main signal as

$$E(z,t) = E_S(z,t) + E_B(z,t) + E_c(z,t), \tag{4}$$

where $E_S(z,t)$ is the Sommerfeld precursor, $E_B(z,t)$ is the Brillouin precursor and $E_c(z,t)$ is the signal contribution. Later analysis by Oughstun and Sherman [*Pulse Propagation in Causal Dielectrics*, Springer-Verlag (1984)] using uniform expansion techniques showed that Brillouin's results were quantitatively incorrect in several important space-time regions. However, their expansion exhibited several discontinuities in its description of the propagated signal. Here, corrections to this previous research (in particular, to the signal contribution) are presented which results in a completely uniform asymptotic description of the propagated pulse.