A Fast Solution of Combined Field Volume Integral Equation for EM Scattering

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The volume integral equation (VIE) approach is one of the most popular methods for electromagnetic scattering problems due to its flexibility in treating inhomogeneous objects. However, so far, VIE has been applied only to objects with non-magnetic materials (*i.e.*, $\mu = \mu_0$), and thus only the solution of the electric field volume integral equation (EFVIE) was considered. Materials with non-trivial permeability are often encountered in realistic applications, and hence, there exists a need to extend VIE to scatterers with arbitrary permeability (μ) and permittivity (ε).

In this paper, a combined field volume integral equation (CFVIE) is presented and solved for scattering from objects with arbitrary permittivity and permeability. Since the material has permeability (μ) that is different from the background medium, a magnetic volume current is required in addition to the electric volume current. This resulted in a coupled EFVIE and MFVIE (magnetic field volume integral equation), known as the combined field volume integral equation (CFVIE). The CFVIE is then solved using method of moments (MoM) to obtain the two unknown functions. However, as is well known, the traditional MoM cannot handle electrically large objects due to excessive memory requirement and computational complexity. To alleviate this problem, we need to leverage on recently developed fast algorithms, such as conjugate gradient fast Fourier transform (CG-FFT) method, multilevel fast multipole algorithm (MLFMA), adaptive integral method (AIM) and pre-corrected fast Fourier transform (P-FFT) method. However, all these algorithms have been restricted to FFVIE for purely dielectric objects so far. For CFVIE, the CG-FFT method is not applicable as the resultant matrix equation does not possess Toeplitz property. In contrast, the irregular-mesh-based P-FFT and AIM methods remain usable, although the extension is not trivial.

In this paper, we extended the P-FFT method to CFVIE to facilitate analysis of large inhomogeneous scatterers. The P-FFT method avoids the filling and storage of the usual coefficient square matrix. In the implementation of the P-FFT method, two sets of projection operators are constructed for the projections of the electric and magnetic sources, respectively. In addition, two sets of interpolation operators are also applied to the computation of vector/scalar potentials and the curl of vector potentials, respectively, in the support of the testing functions. The interpolation operators operate only on the Green's function, and thus are valid for any kind of basis functions. The resultant method has a memory requirement of O(N) and a computational complexity of O(Nlog$N$) respectively, where N is the number of unknowns. Due to the significant reduction in computational requirements as compared to the traditional MoM, the present method can analyze complex dielectric and magnetic objects of much larger sizes.