Higher Order Hierarchical FEM Solutions with Enhanced Efficiency and Practicality

B. M. Notaroš, M. M. Ilić, and A. Ž. Ilić
University of Massachusetts Dartmouth, USA

Recently, computational techniques based on using electrically large curved elements for geometrical modeling (large domains) and higher order basis functions for field modeling have been employed within the framework of the finite element method (FEM), with an objective to significantly reduce the number of unknowns and computational resources for a given (high) accuracy when compared to low-order small-domain solutions. However, these advantages become evident and convincing only if the large-domain FEM approach is carefully planned and implemented. This paper addresses several numerical aspects of the higher order implementation and presents some new advancements in the context of hierarchical curl-conforming polynomial vector basis functions on generalized hexahedral finite elements [1], which are all crucial for making this approach an efficient and practical analysis and design tool for engineers.

Hierarchical curl-conforming vector basis functions enable using different orders of field approximation in different elements for efficient selective discretization of the solution domain. We demonstrate very effective higher order FEM models of complex structures consisting of both very large and very small elements of very different shapes. We also discuss some of the algorithms for the higher order hexahedral mesh generation. Although hierarchical polynomials are inherently ideal for p-refinement of solutions, for general structures it must be combined with an h-refinement. We show excellent convergence properties for several hp-refined meshes and discuss possible further improvements of the technique.

Hierarchical basis functions generally have poor orthogonality properties, which results in FEM matrices with large condition numbers. The ill-conditioning is principally caused by a strong mutual coupling between the pairs of higher-order functions defined on the same (electrically large) generalized hexahedron, which become increasingly similar to one another as the polynomial degrees increase. In order to reduce this coupling, basis functions with better orthogonality properties have to be utilized. We show that higher order large-domain hierarchical curl-conforming FEM vector basis functions constructed from standard orthogonal polynomials and their modifications on generalized curvilinear hexahedral elements exhibit a very slow increase of the condition number of the FEM matrix with increasing the field-approximation orders and a very dramatic reduction of the condition number for high orders as compared to the technique in [1] using field expansions based on simple power functions (the reduction is as large as fourteen orders of magnitude in some cases).

To ensure that the CPU time per unknown in higher order solutions is comparable to that in low-order solutions, rapid and accurate recursive procedures are needed for evaluation of elements of FEM matrices. We show how important for the efficiency of the solution is that computation algorithms avoid redundant operations related to the indices for basis and testing functions and for geometrical representations within all of the interactions in the FEM solution, as well as the summation indices in the Gauss-Legendre integration formulas. In addition, it is crucial that the topological analysis of the problem and assembly of the connectivity matrix are also done in an optimal way that minimizes the total number of nonzero elements and ensures a similar level of sparsity of system matrices as for low-order solutions.

REFERENCES

1. Ilić, M. M. and B. M. Notaroš, "Higher order hierarchical curved hexahedral vector finite elements for electromagnetic modeling," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 51, 1026–1033, March 2003.