Multi-level Multiplicative Schwarz Preconditioner for Solving Matrix Equations from DD-FE-BEM Formulation

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A symmetric coupling between finite and boundary elements for solving electromagnetic wave radiation and scattering problems in $\mathbb{R}^3$ has been recently proposed by Vouvakis et al., [1]. The new formulation results in a symmetric complex matrix equation which is free of internal resonances, and the spectral radius of the couplings between finite and boundary elements is bounded by 1. Moreover, the formulation is also allowing non-conformal couplings between FEM and BEM, and thus, offers great modeling versatility for solving real-life complex problems. By non-conformity, we mean that the surface triangulations of the FEM and the BEM do not need to be the same, as well as the flexibility of choosing different order of basis functions for FEM and BEM portions separately.

This paper addresses the practical issue of how to solve the resulting symmetric complex matrix equations efficiently. As is well known that in order to solve large sparse matrix equations (or even dense matrix equations, for that matter) efficiently using iterative solution, such as Conjugate Gradient (CG) methods, the most critical ingredient is the preconditioner. In the authors' group, we have developed over the years a robust preconditioner, termed p-MUS (p-type MUltiplicative Schwarz), for preconditioning the matrix equations from the application of the vector finite elements [2]. We extend the p-MUS to the current DD-FE-BEM formulation, and treating the BEM block as another "abstract" domain, and construct three possible preconditioners: an inner-outer loop domain decomposition preconditioner, an additive p-type Schwarz method, and a Multi-level MUltiplicative Schwarz (M-MUS) preconditioner for solving the resulting DD-FE-BEM matrix equations. Various numerical examples, including both radiation and scattering problems, will be presented and comparisons of the three preconditioners will be discussed as well in the presentation.

REFERENCES

1. Vouvakis, M., K. Zhao, S. Seo, and J.-F. Lee, submitted to *IEEE Trans AP*.
2. Lee, J.-F. and D. K. Sun, *IEEE Trans. MTT*, 864–870, 2004.