An Efficient *hp* Adaptive Finite Element Solver for Time-harmonic Electromagnetic Fields

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Thanks to its great flexibility in modelling geometry and material properties, the finite element (FE) method is a widely used tool for the numerical analysis of electromagnetic devices. With the FE method, there are two different ways of improving the accuracy of numerical solutions. In case of p enrichment, the degrees of the basis functions are increased whereas, in case of h refinement, the element sizes are decreased. When the fields are smooth, p enrichment yields exponential convergence, whereas h refinement is always limited to algebraic rates of convergence. On the other hand, when singularities are present, the performance of p schemes is poor whereas nonuniform h methods succeed in keeping the rate of convergence unchanged. Since real world configurations typically involve both regions of smooth fields and localized areas of rapid field variations or even singularities, p enrichment and h refinement should be viewed as complementary rather than competing techniques.

The FE method we propose in this paper combines hp adaptivity with fast solution techniques. As for h refinement, we construct sequences of nested tetrahedral meshes which allow for subregions of greatly varying refinement levels, and impose special restriction operators to make the FE basis functions maintain proper continuity conditions. The resulting FE spaces are perfectly nested, which makes them very well-suited for advanced geometrical multi-grid solvers exploiting local sub-meshing techniques.

Regarding p enrichment, a set of hierarchical $\mathcal{H}(\text{curl})$ conforming basis functions developed by one of the authors is employed. It is of the incomplete order type, features explicit basis functions for higher order gradients as well as increased sparsity within the stiffness matrix, and possesses interpolation properties that greatly simplify h refinement and hence bridge the gap to the before mentioned method. Our basic building block for a fast solver in the p domain is a multiplicative Schwarz method.

One challenge with hp schemes is that the aspects of multi-grid schemes for h refinement and Schwarz methods for p enhancement can no longer be considered separately. In fact, there are many ways to cycle back and forth between a low-order FE space over a coarse mesh and a high-order space over a fine discretization, and it turns out that the corresponding algorithms differ greatly in their computational complexity.

In our talk we will give the details of the proposed hp adaptive FE solver. We will demonstrate the efficiency of the new approach by a number of numerical examples.