Nonlinear Pulse Propagation and Modulation Instability in Periodic Media with and without Defects

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Abstract—The nonlinear propagation of EMW in periodic media is of great interest due to the possibility to accumulate energy in periodic media within the stop band and, therefore, the input intensity levels for observation of nonlinear phenomena are quite low [1]. For resonant interactions, also it is possible to realize matching conditions, which are not possible in uniform media. Both resonant multiwave interactions and self-action of EMW in nonlinear periodic media have been analyzed [1, 2]. Also, a method based on a slow variation in time was proposed, which seems more adequate than others based on coupled equations [3].

The presence of defects in periodic structure leads to narrow regions of transmission within the stop-band. In this case, the application of coupled equations for counter propagating waves becomes doubtful, and a more general approach is needed [3]. Also, the influence of defects on the dynamics of modulation instability of long pulses is of a great interest. The present paper considers numerical simulations of the pointed above phenomena. For a correct description of the nonlinear dynamics, it is necessary also to take into account the wave dissipation and possible transverse diffraction.

The results of simulations demonstrated the essential influence of defects within the periodic structure on the nonlinear propagation of EM pulses, even if the carrier frequency is chosen within the stop band of the structure with a defect. This fact can be explained by a quite wide spectrum of the input pulse, and the "tail" of such a spectrum is within the transmission region due to the defect. This situation is analogous to the nonlinear propagation of short spin-dipole waves in the vicinity of the cut-off frequency [4]. The dynamics of modulation instability also changes in the presence of the defect. The diffraction can affect essentially the modulation instability dynamics.

1. Basic Equations

The case under research is the nonlinear periodic medium (OABAB...O), where A and B are dielectrics, and O is the vacuum. Each layer is assumed as isotropic and the influence of temporal dispersion is neglected here. The last consideration is valid only if the duration of the input pulse is long (> 0.1 ps). Within the structure, the presence of a singe defect is possible (such as ...ABABBBAB...). Consider almost transversely polarized EM wave, where only a single transverse component of the electric field (for instance, E_x) is dominating. A weak dependence on the radial coordinate is taken into account, to estimate an influence of diffraction. We use a slow dependence of the wave amplitude A only respect to time (essentially its variation in space, due to periodic structure of the medium), cubic non linearity and step-like dependence of the dielectric permittivity. Different spatial harmonics (both co- and counter-propagating) are included into the wave structure within the periodic lattice:

$$\frac{\partial A}{\partial t} + \frac{i\omega}{2}\left(1 + \frac{\Delta\varepsilon}{\varepsilon(z)}\right)A + \frac{i}{2\omega\varepsilon(z)}\left(\frac{\partial^2 A}{\partial t^2} + \nabla_\perp^2 A\right) + \gamma A = 0; \quad \Delta\varepsilon = \alpha(z)|A|^2 \tag{1}$$

Here $\Delta\varepsilon(E)$ is the change on the dielectric permittivity of the periodic medium due to the cubic nonlinearity. The scale of longitudinal spatial dependence is arbitrary, because of step-like dependence of dielectric permittivity. Then, the electric field is considered as:

$$E = \frac{1}{2}A(z, \rho, t) \times \exp(i\omega t) + c.c. \tag{2}$$

In equation (1), $\varepsilon = \varepsilon(z)$ is the dielectric permittivity of the periodic medium, ω is the carrier frequency ($\omega = 2\pi c/\lambda_0$, with λ_0 the wavelength in vacuum). Also the wave dissipation γ is included in this equation.

The boundary conditions at the interfaces between the layers are taken into account in Equation (1). Additionally, it is necessary to consider the boundary conditions for the tangential components of electric and magnetic fields at the input ($z = 0$) and at the output ($z = L$) of the periodic medium:

$$z = 0: \quad A = A_{inc} + A_{refl}; \quad \frac{\partial A}{\partial z} \approx \frac{\partial A_{inc}}{\partial z} + \frac{\partial A_{refl}}{\partial z} \tag{3a}$$

where: $A_{inc}(z,\rho,t) \approx A_{i0}(\rho,t)\times\exp(-ik_0 z)$; $A_{refl}(z,\rho,t) \approx A_{r0}(\rho,t)\times\exp(+ik_0 z)$, A_{i0}, A_{r0} are the amplitudes of incident and reflected waves, respectively; $k_0=\omega/c$ is the wave number of the wave in vacuum. It is assumed that the amplitude of the incident wave is known.

Equation (3a) can be reduced as it is shown in the following boundary conditions:

$$z=0:\quad \frac{\partial A}{\partial z}-ik_0A\approx -2ik_0A_{i0}(\rho,t)\qquad\qquad z=L:\quad \frac{\partial A}{\partial z}+ik_0A\approx 0. \tag{3b}$$

Equation (1) and boundary conditions (3) are valid in the case of transversely wide pulses. Otherwise, it is impossible to use the approximation of transversely polarized EM wave.

2. Method of Simulations

The splitting with respect to physical factors is applied. The problem of stability of simulations is very important because the possible modulation instability is under research. The full explicit three-layer scheme is used for nonlinearity, dissipation, and longitudinal transport fractional step. Two-layer implicit scheme is utilized for the diffraction fractional step. In simulations, the one-dimensional representation of Equation (1) has been used:

$$\frac{\partial\bar A}{\partial\bar t}+\frac{i\bar\omega}{2}\left(1+\frac{\Delta\varepsilon}{\varepsilon(\bar z)}\right)\bar A+\frac{i}{2\bar\omega\varepsilon(\bar z)}\left(\frac{\partial^2\bar A}{\partial\bar z^2}+\frac{1}{\bar\rho}\frac{\partial}{\partial\bar\rho}\left(\bar\rho\frac{\partial\bar A}{\partial\bar\rho}\right)\right)+\bar\gamma\bar A=0;\quad \Delta\varepsilon=b(\bar z)|\bar A|^2 \tag{4}$$

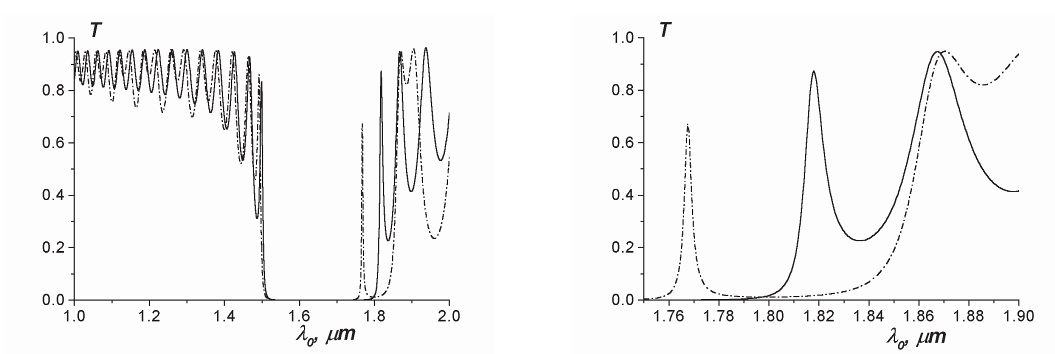
where $\bar z=z/l_n,\ \bar t=t/t_n,\ t_n=l_n/c;\ \bar\rho=\rho/l_n$.

The value of $l_n=1\,\mu$m has been chosen; thus, $t_n=3.15\times10^{-15}$ s. Below, the lines over the one-dimensioned quantities ($t,\ z,\ \rho,\ A$) are omitted, because we use one-dimensional representation. The shape of the input pulse is:

$$A_i(t,\rho)=A_{i0}\times\exp\left(-\left(\frac{t-t_1}{t_0}\right)^4\right)\times\exp\left(-\left(\frac{\rho}{\rho_0}\right)^4\right) \tag{5}$$

3. Results of Simulations

The structures under simulation include 48 layers, their dielectric permittivities are $\varepsilon_A=3.5$, $\varepsilon_B=2.0$. A-layers are nonlinear ($b=-0.1$), whereas B-layers are assumed as linear ($b=0$). Each layer has a length of $0.25\,\mu$m. A single defect is replacement of the 23^{rd} A-layer by a B-layer. The central frequency has been chosen in the region of the stop-band. In Fig. 1, the linear transmission coefficients are given for the cases of the

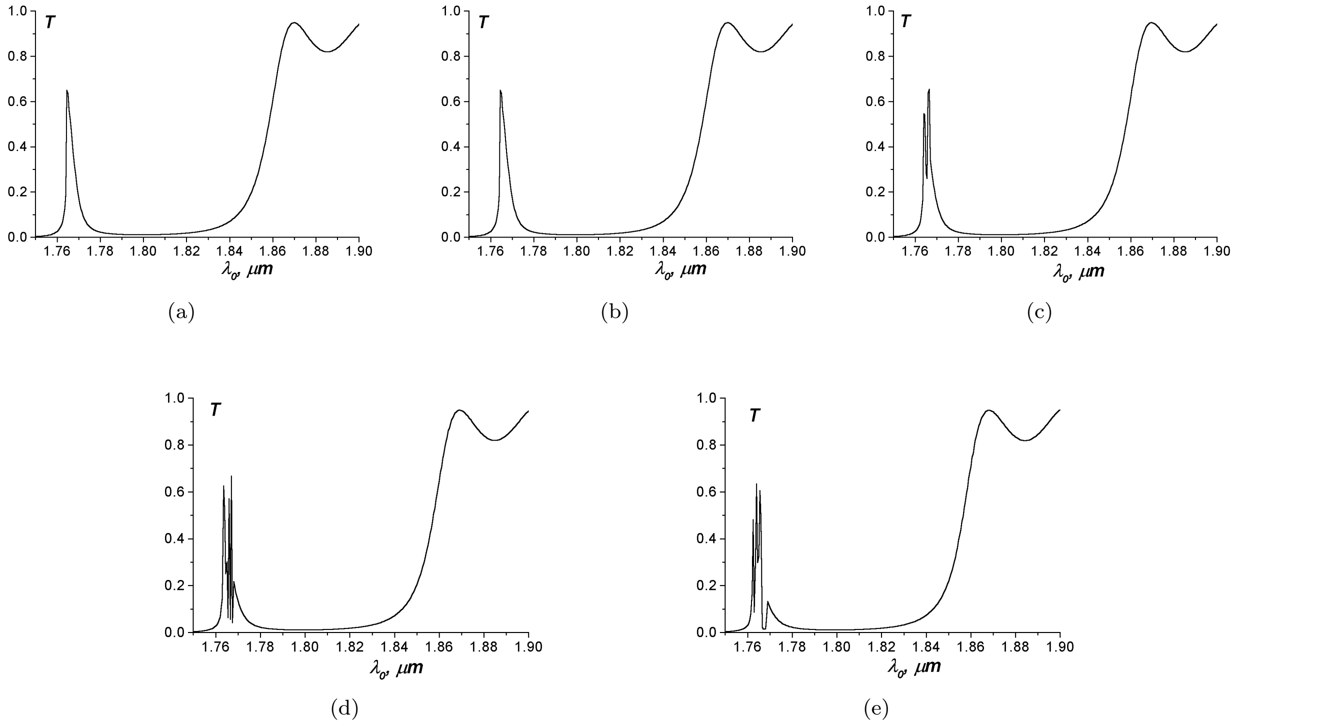


(a) (b)

Figure 1: Linear transmission coefficients. a) The general picture, b) A section only. Note that the solid lines correspond to the case without defect and the dotted line is for with defect.

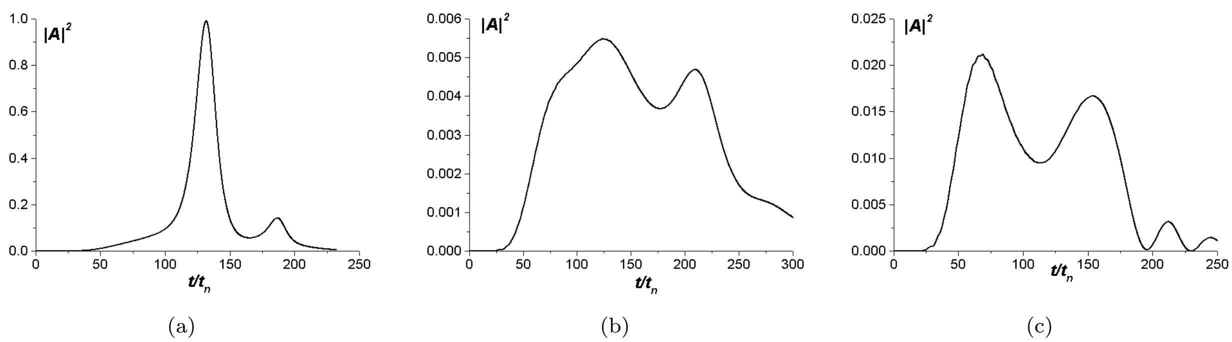
periodic structure without and with the single defect. The presence of the single defect causes a narrow region of transparency and a shift of the limits of the stop-band. Therefore, the most interesting regions of the central wave numbers (or frequencies) are localized near the upper limit of wave numbers ($\lambda_0\sim1.80\,\mu$m). Note that in the linear case the total reflection takes place for the monochromatic input EM wave.

In Fig. 2, the nonlinear transmission coefficients of monochromatic waves are given in the structures with and without defect. In the structure with defect the transmission coefficient gets more broken dependence, in comparison with the linear case. More over, in the vicinity of the narrow transmission region, the shape of such dependence becomes chaotic-like. This fact can be explained by the accumulation of the energy of EM oscillations at the defect of the lattice.



(a) (b) (c) (d) (e)

Figure 2: Nonlinear transmission coefficients through the structure with the defect for the monochromatic wave: a) with $A_0 = 0.01$ (linear case), a) $A_0 = 0.1$, b) $A_0 = 0.12$, c) $A_0 = 0.15$, d) $A_0 = 0.2$. For a comparison, e) is for $A_0 = 0.2$ (the regular structure).

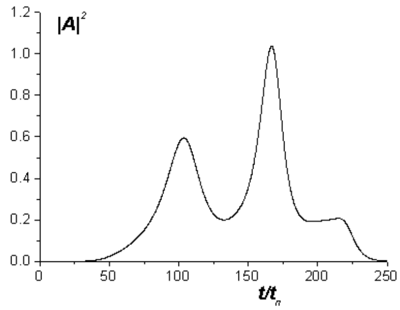


(a) (b) (c)

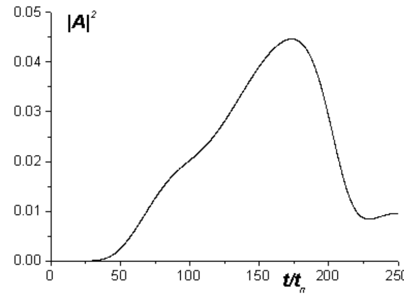
Figure 3: Propagation of the short transversely wide pulses. The carrier frequency corresponds to the wave number $\lambda_0 = 1.80\,\mu$m. Part a) is the nonlinear propagation though the regular structure ($A_0 = 1$); b) the same as a), but the input amplitude is 3 times smaller ($A_0 = 0.31$); c) the same input pulse as a) ($A_0 = 1$) but the propagation through the structure with defect.

Two different situations can occur due to the dependence on the duration of the incident pulse. The first one corresponds to relatively short pulses. The typical results of simulations are given in Figs. 3 and 4. Here the intensities of transmitted pulses are presented in the figure captions; $t_1 = 100$, $t_0 = 80$. In the case of the regular periodic structure, the nonlinear transparency phenomenon occurs at lower amplitudes of the input pulse, compare Fig. 3(a) and 3(c). In the case of the structure with the defect, the amplitude of the transmitted pulse is quite small even in nonlinear case, Fig. 3(c). The simulations of nonlinear propagation of transversely narrower pulses have demonstrated that an influence of diffraction is not expressed for the maximum of the transmitted pulse but changes weakly the rear part of it.

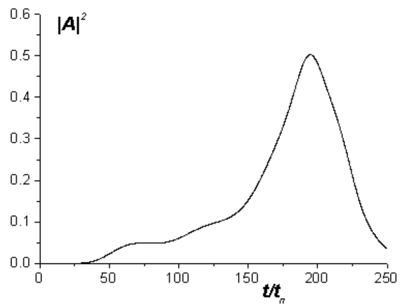
A comparison of Figs. 3 and 4 has demonstrated that a relatively small shift of the central frequency of the pulse changes the nonlinear dynamics essentially. Moreover, the regular periodic structures and ones with the defect possess different frequency regions for the manifestation of the nonlinear transparency phenomenon.



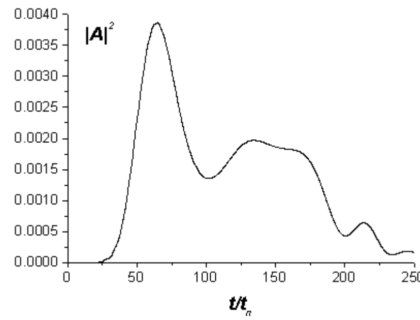
(a)



(b)

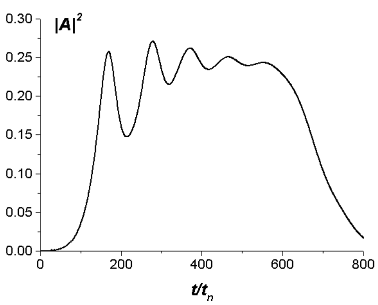


(c)

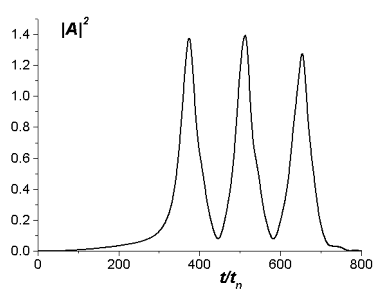


(d)

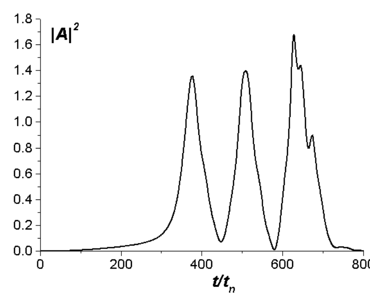
Figure 4: Propagation of the short, transversely wide pulses. The carrier frequency corresponds to the wave number $\lambda_0 = 1.82\,\mu$m. Part a) is the nonlinear propagation though the regular structure ($A_0 = 1$); b) the same as a), but the input amplitude is 3 times smaller ($A_0 = 0.31$); c) and d) are the same input pulses as a) and b) ($A_0 = 1$, $A_0 = 0.31$) but the propagation is through the structure with defect.



(a)



(b)



(c)

Figure 5: Modulation instability with and without defects. The carrier frequency corresponds to the wave number $\lambda_0 = 1.82\,\mu$m. The maximal input amplitude is $A_0 = 1.00$. Part a) is for the regular structure, b) is for the structure with defect ($\rho_0 = 60$), c) is for the structure with defect (with an influence of diffraction, $\rho_0 = 24$).

The case of much longer incident pulses corresponds to the occurrence of modulation instability (MI). Here the general picture is somewhat different from the previous case, see Figs. 5 and 6. The parameters of the input pulses are $t_0 = 340$, $t_1 = 400$. At the output of the structure the multipeak signal occurs. Also, the role of diffraction is essentially expressed; compare Fig. 6(d) and 6(e). In the case of MI developed, the influence of temporal dispersion can be essential, and the used approximation cases its validity.

An interesting result of simulations is also the fact that the regular structure and one with the defect possess different frequency regions of observing modulation instability within the stop-band.

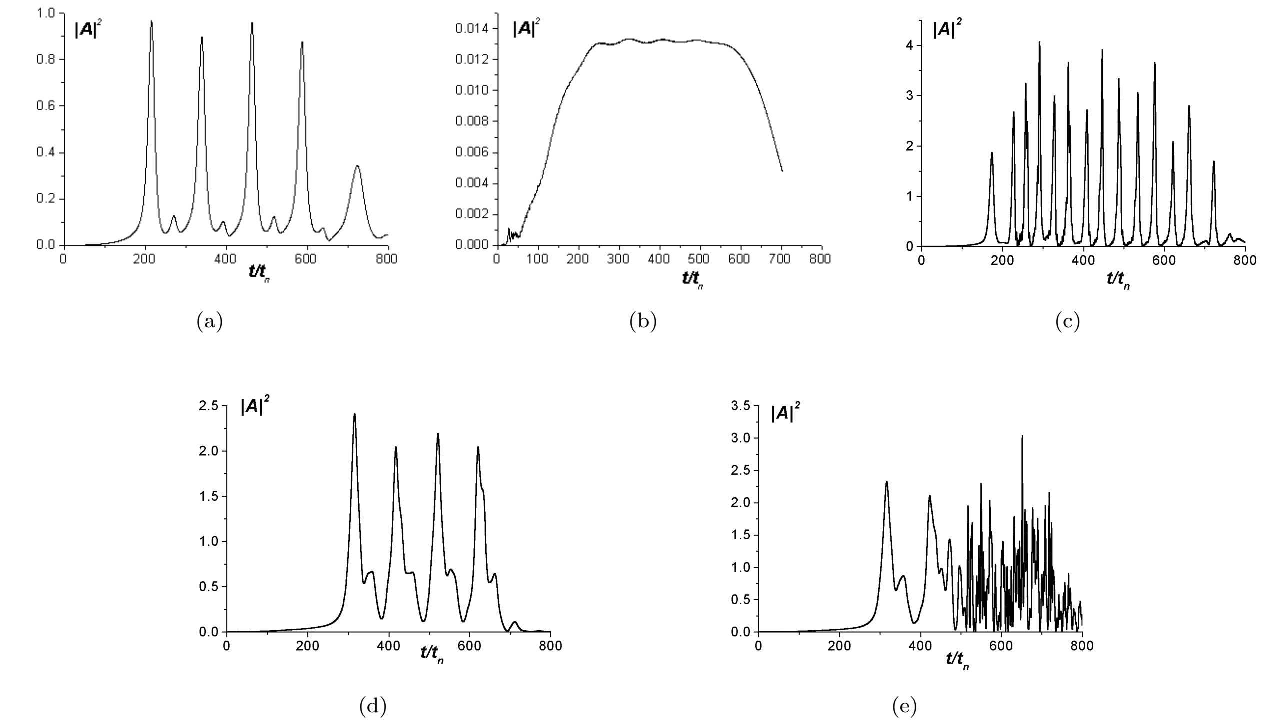


Figure 6: Modulation instability with and without defects. The carrier frequency corresponds to the wave number $\lambda_0 = 1.80\,\mu$m. Part a) is for the regular structure, $A_0 = 1.0$, (transversely wide pulse, $\rho_0 = 60$); b) is for the structure with defect, $A_0 = 1.0$ ($\rho_0 = 60$); c) is for the regular structure, $A_0 = 1.41$ ($\rho_0 = 60$); d) is for the structure with defect, $A_0 = 1.41$ ($\rho_0 = 60$); e) is for the structure with defect, $A_0 = 1.41$ (transversely narrow pulse, $\rho_0 = 24$).

4. Conclusions

The results of simulations have been demonstrated an essential influence of defects within the periodic structure on the nonlinear propagation of EM pulses, even if the carrier frequency is chosen within the stop band of the structure with the defect. This can be explained by a quite wide spectrum of the input pulse, and the "tail" of such a spectrum is within the transmission region due to the defect. The dynamics of modulation instability also changes in the presence of the defect. The diffraction can affect essentially the modulation instability dynamics.

Acknowledgment

This work was partially supported by CONACyT (Mexico) under grant 45667-F.

REFERENCES

1. Gupta, S. D., "Nonlinear optics of stratified media," *Progress in Optics*, Elsevier, Amsterdam, NL, Vol. 38, 3–84, 1998.
2. Escobedo-Alatorre, J., J. Sanchez-Mondragon, M. Torres-Cisneros, R. Selvas-Aguilar, and M. Basurto-Pensado, "A device approach to propagation in nonlinear photonic crystal," *Optical Materials*, Vol. 27, No. 7, 1260–1265, 2005.
3. Scalora, M. and M. E. Crenshaw, "A beam propagation method that handles reflections," *Opt. Commun.*, Vol. 108, 191–196, 1994.
4. Burlak, G., V. Grimalsky, S. Koshevaya, and P. Marquez-A, "Dynamics of magnetostatic wave solitons in the vicinity of frequency cut-off," *Journ. of Magn. Magn. Mater.*, Vol. 237, No. 1, 97–103, 2001.