A PDE-constrained Optimization Algorithm for Frequency Domain Optical Tomography

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Diffuse Optical Tomography (DOT) is a fast developing new medical imaging modality that use near-infrared light to probe various sections of the human body. The light from laser diodes is deliver through optical fibers to several locations on the surface of the body part under investigation. Measurements of backreflected and transmitted light intensities at other positions on the surface are recorded and analyzed. The technology for making these measurements is nowadays readily available and has mainly been applied to breast and brain imaging. However, a major challenge that still remains is the development of efficient numerical schemes that transform these data into useful cross-sectional images of the interior.

In this work, we present a novel approach to solving the inverse problems encountered in optical tomography. We have implemented a PDE-constrained optimization method that uses a finite-volume method for the discretization of the frequency-domain radiative transport equation (RTE). The finite-volume discretization gives rise to an algebraic nonlinear programming problem that is solved using the iterative augmented Lagrangian method. By simultaneously updating both radiance and optical properties, the method solves the forward and inverse problems in optical tomography all at once. In this way, the computing time is greatly reduced as compared to traditional unconstrained optimization methods, during which one has to repeatedly solve the forward problem many times. We tested and quantified the performance of the algorithm for various combinations of mesh sizes, noise, regularization parameters, initial guesses, optical properties and measurement geometries. Besides the speed of the code, we compared image qualities by defining a correlation coefficient ρ as well as a deviation factor δ.

In the cases that involve image reconstruction from synthetic measurement data we observe 10-30-fold decrease in computing time for the constrained optimization code compared to the unconstrained optimization code. The regularization parameter β has some influence on the computing time, but with reasonable values on the order of $\beta \sim 10^{-7}$ to 10^{-9}, the computational time changes less than 20%. In general, reconstruction of both absorption and scattering together took longer than reconstructions of only the scattering coefficient or only the absorption coefficient. As expected the correlation coefficients ρ and deviation factors δ worsen as the signal-to-noise ratio decreases. Similarly δ decrease substantially as the (homogeneous) initial guess is not chosen close to the optical properties of the actual background medium. Interestingly ρ is only weakly affected by the initial guess. As long as the optical properties are chosen within 50% of the actual background medium ρ changes by only 10-20%. Finally δ and ρ do not change once the mesh is fine enough so that the average size of finite volumes becomes less than the average scattering mean free path $(1/\mu_s)$. Another positive aspect of the augmented Lagrangian method is that it maintains storage requirements that are comparable to requirements encountered in unconstrained optimization methods.

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