Radiative Transport in Rotated Reference Frames

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We have proposed a novel method for solving the linear radiative transport equation (RTE) in a three-dimensional macroscopically homogeneous medium. The method utilizes the concept of locally rotated reference frames and can be used with an arbitrary phase function of a random medium consisting of spherically-symmetrical microscopical scatterers. The angular dependence of the specific intensity written in the spatial Fourier representation is obtained as an expansion into spherical functions defined in reference frames whose z-axes coincide with the direction of the Fourier vector k. In practice, this expansion is truncated at the maximum order $l_{\max}$. Boundary conditions have been considered in the slab and half-space geometries.

We have applied the method to (i) calculation of the RTE Green's function for different values of the absorption and scattering coefficients, μ_a and μ_s, and the asymmetry parameter g within the Henyey-Greenstein model for the phase function [1, 2], and (ii) to generation of forward data for an inverse problem of optical tomography [3]. In particular, we have demonstrated in [3] that the spatial resolution of images obtained in optical tomography is not limited to the fundamental length scale of one transport mean free path.

Angular dependence of the specific intensity I due to a unidirectional point source is illustrated in the figure (see caption for numerical values of relevant parameters). An accurate value of the specific intensity at $\theta \approx \pi/2$ ($\approx 10^{-3}$ relative error) was obtained at $l_{\max} = 34$. Note that for smaller values of $l_{\max}$, such as $l_{\max} = 10$, the result is still grossly inaccurate and can even be negative.

REFERENCES

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