On the Application of the Radiative Transfer Approach to Scattering from a Random Medium Layer with Rough Boundaries

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Abstract—For studying the problem of scattering from a random medium layer with rough boundaries the radiative transfer (RT) approach is widely used. In order to better understand this procedure we compared it with the statistical wave approach. Two such wave approaches are presented in this paper: the surface scattering operator (SSO) approach, and the unified approach. In both wave approaches two conditions are essential for arriving at RT system: the ladder approximation to the intensity operator, and the quasi-stationary approximation of fields. With these approximations one arrives at the integro-differential equations of the RT system. However, to arrive the at the RT boundary conditions, one has to impose further approximations. In the SSO approach weak surface correlation must be imposed. In the unified approach, one has to ignore the terms involving volumetric spectral densities, and consider only single scattering from the rough boundary when deriving the boundary conditions.

1. Introduction

The analysis of scattering from a random medium layer with rough boundaries is a difficult problem. This is the kind of problem one often encounters in remote sensing applications. People have used the phenomenological radiative transfer approach to study this problem (Ulaby et al., 1986; Lam and Ishimaru, 1993; Shin and Kong, 1989). This approach is conceptually simple and yet very effective for studying multiple scattering processes. Here one uses the transport equations corresponding to the random medium of the layer and then one imposes the relevant boundary conditions. Although this procedure appears to be heuristically sound it is not clear what approximations are involved, and under what conditions such a procedure may be used for the problem at hand. One way to better understand this radiative transfer approach is to compare and relate it to the statistical wave approach. For the case of unbounded random media it has been demonstrated how the ladder approximated Bethe-Salpeter equation reduces to the radiative transport equation (Barabanenkov et al., 1971). We found that this procedure can be applied to the problem of random medium layer with planar boundaries and arrive at the radiative transport system as given in Ulaby et al., (1986). However, if the boundaries are statistically rough, the problem is considerably more complicated and we need special procedures to deal with them. We have employed two different statistical wave approaches for such problems. In the first approach we assume that we know the solution of the problem without the volumetric fluctuations. The second approach is based on the solution of the problem where all the fluctuations vanish. We shall compare the results of these two approaches with those of the radiative transfer (RT) approach. This will enable us to understand and meaning and import of the radiative transfer approach as applied to our problem. To keep discussions in a simple setting we will consider the scalar problem and keep the lower boundary alone as rough.

2. Geometry of the Problem

The geometry of the problem consists of a random medium layer with a rough bottom boundary. The permittivity of the layer medium consists of a deterministic part ϵ_2 and randomly fluctuating part $\tilde{\epsilon}\epsilon_2$. $z = 0$ and $z = -d + \zeta(r_\perp)$ describe the upper and lower boundary of the layer. We assume that $\tilde{\epsilon}$ and ζ are small and smooth zero-mean stationary processes independent of each other. The medium above the layer is homogeneous, and we impose the Neumann boundary condition on the lower boundary. This layer is excited by a wave incident from above and we are interested in the scattered waves.

3. Radiative Transfer Approach

The classical equation of radiative transfer is given as

$$\hat{s} \cdot \nabla I(r, \hat{s}) + \eta I(r, \hat{s}) = \int d\Omega' P(\hat{s}, \hat{s}') I(r, \hat{s}') \tag{1}$$

where $P(\hat{s}, \hat{s}')$ is the phase function and η is the extinction coefficient. This equation was originally intended for unbounded scattering medium. However it can be applied to bounded medium with arbitrary geometry by

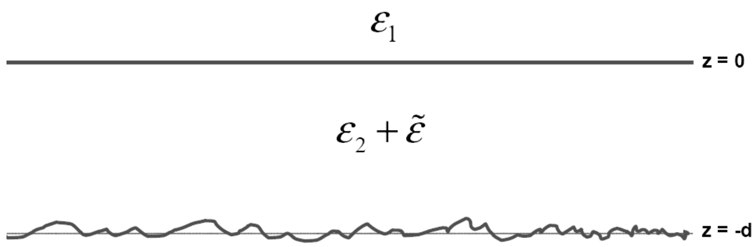

Figure 1: Geometry of the problem.

imposing appropriate boundary conditions. For layer geometry we have the following set of coupled integro-differential equations.

$$\cos\theta \frac{d}{dz} I_u(z,\Omega) + \eta I_u(z,\Omega) = I_u^c + \frac{|k|^4}{4\pi} \int d\Omega' \left\{ \Phi(\theta,\theta';\phi-\phi') I_u(z,\Omega') + \Phi(\theta,-\theta';\phi-\phi') I_d(z,\Omega') \right\} \tag{2}$$

$$\cos\theta \frac{d}{dz} I_d(z,\Omega) - \eta I_d(z,\Omega) = -I_d^c + \frac{|k|^4}{4\pi} \int d\Omega' \left\{ \Phi(-\theta,\theta';\phi-\phi') I_u(z,\Omega') + \Phi(-\theta,-\theta';\phi-\phi') I_d(z,\Omega') \right\} \tag{3}$$

Eqs. (2) and (3) follow from (1) noting that the problem is translationally invariant in azimuth. I_u and I_d represent the incoherent part of radiant intensities corresponding to upward and downward travelling waves inside the layer. I_u^c and I_d^c represent the corresponding contributions due to coherent intensities. Φ represents the spectral density of the volumetric fluctuations. Eqs. (2) and (3) are solved using the following boundary conditions.

$$I_d(0,\Omega) = |R_{12}(\Omega)|^2 I_u(0,\Omega) \tag{4}$$

$$I_u(-d,\Omega) = \int d\Omega' \langle |R_{32}(\Omega,\Omega')|^2 \rangle I_d(-d,\Omega) \tag{5}$$

The extinction coefficient η is readily derived from the differential scattering cross section of the random medium. R_{12} is the reflection coefficient at the upper boundary for waves incident from below. R_{32} is the reflection coefficient at the lower boundary for waves incident from above. Thus we see that the formulation in the radiative transfer approach is simple and straight forward, and can be applied to a variety of different geometries. The fundamental quantity in this approach is the radiant intensity and hence is not suitable to represent wave phenomena such as diffraction, interference, etc. A more general approach to this problem is the statistical wave approach. In this paper we will describe two such approaches and compare them with that of radiative transfer.

4. Surface Scattering Operator Approach

We start with the following equations governing the Green's functions of the problem.

$$\begin{aligned} \triangle G_{12} + k_1^2 G_{12} &= 0 \\ \triangle G_{11} + k_1^2 G_{11} &= -I \\ \triangle G_{21} + k_2^2 G_{21} &= -qG_{21} \\ \triangle G_{22} + k_2^2 G_{22} &= -I - qG_{22} \end{aligned} \tag{6}$$

where $q = \omega^2 \mu \tilde{\epsilon} \epsilon_2$ represents the volumetric fluctuations. We write the above system as

$$LG = -I - QG \tag{7}$$

where $G \equiv \{G_{ij}\}$, $L = \mathrm{diag}\{L_1, L_2\}$, $L_j = \triangle + k_j^2$, $Q = q\mathrm{diag}\{0,1\}$. For multiple scattering analysis it is convenient to convert (7) into the following integral equation.

$$G = \breve{G} + \breve{G} Q G \tag{8}$$

where $\breve{G}$ is the Green's function of the problem without volumetric fluctuations. In principle, one can construct such Green's functions using surface scattering operators (Voronovich, 1994; Soubret et. al., 2002). First average (8) w. r. t. volumetric fluctuations.

$$\langle G_v\rangle \simeq \breve{G} + \breve{G}\langle Q\langle G\rangle_v Q\rangle\langle G\rangle_v \tag{9}$$

On operating this by L we obtain

$$L\langle G\rangle = -I - \langle Q\langle G\rangle_v Q\rangle\langle G\rangle_v \tag{10}$$

From this we find that

$$L_1\langle G_{11}\rangle_v = -I \tag{11a}$$
$$L_2\langle G_{22}\rangle_v = -I - \langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v \tag{11b}$$

Next average (11) w. r. t. surface fluctuations

$$L_1\langle G_{11}\rangle_{vs} = -I \tag{12a}$$
$$L_2\langle G_{22}\rangle_{vs} = -I - \langle\langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v\rangle_s \tag{12b}$$

We infer from (12a) that the mean propagation constant in Region 1 is unaffected by the fluctuations of the problem. To interpret (12b) we approximate $\langle\langle G_{22}\rangle_v\langle qq\rangle\langle G_{22}\rangle_v\rangle_s$ as $\langle G_{22}\rangle_{vs}\langle qq\rangle\langle G_{22}\rangle_{vs}$. As we shall see, this kind of approximation is essential for arriving at the RT system as given in the previous section. Thus

$$(\triangle + k_2^2)\langle G_{22}\rangle_{vs} = -I - \langle G_{22}\rangle_{vs}\langle qq\rangle\langle G_{22}\rangle_{vs} \tag{13}$$

This implies that

$$k_{2m}^2 = k_2^2 + \langle G_{22}\rangle_{vs}\langle qq\rangle \tag{14}$$

This is the operational definition for the mean propagation constant in the layer region. With this we can proceed to construct the mean Green's functions.

We next turn our attention to the second moments of the fields. Taking the tensor product of (8) with its complex conjugate and performing volumetric averaging leads to

$$\langle G\otimes G^*\rangle_v = \langle G\rangle_v \otimes \langle G\rangle_v^*\{I + K\langle G\otimes G^*\rangle_v\} \tag{15}$$

where K is the intensity operator corresponding to the volumetric fluctuations. Hence the equation for field correlation is

$$\langle\psi\otimes\psi^*\rangle_v = \langle\psi\rangle_v\otimes\langle\psi\rangle_v^* + \langle G\rangle_v\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v \tag{16}$$

Averaging this over surface fluctuations we have

$$\langle\psi\otimes\psi^*\rangle_{vs} = \langle\langle\psi\rangle_v\otimes\langle\psi\rangle_v^*\rangle_s + \langle\langle G\rangle_v\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v\rangle_s \tag{17}$$

Now we employ the following two approximations essential for arriving at the radiative transfer system.

$$\langle\langle G\rangle\otimes\langle G\rangle_v^* K\langle\psi\otimes\psi^*\rangle_v\rangle_s \simeq \langle\langle G\rangle_v\otimes\langle G\rangle_v^*\rangle_s K\langle\psi\otimes\psi^*\rangle_{vs} \tag{18a}$$
$$K \simeq \langle Q\otimes Q^*\rangle \tag{18b}$$

The first is the weak surface correlation approximation. The second is called the ladder approximation. Thus we arrive at the following equation for the second moment of the fields inside the layer

$$\langle\psi_2\otimes\psi_2^*\rangle_{vs} = \langle\langle\psi_2\rangle_v\otimes\langle\psi_2\rangle_v^*\rangle_s + \langle\langle G_{22}\rangle_v\otimes\langle G_{22}\rangle_v^*\rangle_s K\langle\psi_2\otimes\psi_2^*\rangle_{vs} \tag{19}$$

Observe that $\psi_2 = \langle\psi_2\rangle_{vs} + \tilde{\psi}$ and $\langle\psi_2\rangle_v = \langle\psi_2\rangle_{vs} + \widetilde{\langle\psi_2\rangle}_v$ where tilde is used to denote the fluctuating part. Using these relations in (19) we obtain

$$\langle\tilde{\psi}_2\otimes\tilde{\psi}_2^*\rangle_{vs} = \langle\widetilde{\langle\psi_2\rangle}_v\otimes\widetilde{\langle\psi_2\rangle}_v^*\rangle_s + \langle\langle G_{22}\rangle\otimes\langle G_{22}\rangle_v^*\rangle_s\langle q\otimes q^*\rangle\langle\psi_2\otimes\psi_2^*\rangle_{vs} \tag{20}$$

We next introduce Wigner transforms of the wave functions and the Green's functions in (20) and obtain

$$\tilde{\mathcal{E}}(z,k) = \tilde{\mathcal{E}}^s(z,k) + \frac{|k_2|^4}{(2\pi)^6}\int dz_1\int d\alpha\int d\beta\,\mathcal{G}(z,k;z_1,\alpha)\Phi_v(\alpha-\beta)\mathcal{E}(z_1,\beta) \tag{21}$$

where $\tilde{\mathcal{E}}$, $\tilde{\mathcal{E}}^s$, $\mathcal{E}$ and $\mathcal{G}$ are the Wigner transforms of $\langle\tilde{\psi}_2\otimes\tilde{\psi}_2{}^*\rangle_{vs}$, $\langle\widetilde{\langle\psi_2\rangle}_v\otimes\widetilde{\langle\psi_2\rangle}_v^*\rangle_s$, $\langle\psi_2\otimes\psi_2^*\rangle_{vs}$ and $\langle\langle G_{22}\rangle_v\otimes\langle G_{22}\rangle_v^*\rangle_s$, respectively. Φ_v is the spectral density of volumetric fluctuations. Boundary conditions relate radiant intensities arriving at and departing the boundary. Therefore, we need to split $\tilde{\mathcal{E}}$ into upward and downward travelling components. Assume that the fields are quasi-stationary and hence only waves travelling over similar paths will be correlated. This leads to the following approximation.

$$\mathcal{G} = \mathcal{G}^o + \mathcal{G}_{uu} + \mathcal{G}_{ud} + \mathcal{G}_{du} + \mathcal{G}_{dd} \tag{22}$$

$\mathcal{G}^o$ is the Wigner transform of $G^o \otimes G^{o*}$ where G^o is the singular part of $\langle G_{22}\rangle$. $\mathcal{G}_{uu}$ is the Wigner transform corresponding to that part of $\langle G_{22}\rangle$ involving the surface scattering operator $\langle S_{uu}\rangle$ and so on. Using this decomposition we split (21) as follows.

$$\begin{aligned}\tilde{\mathcal{E}}^u(z,k) = \tilde{\mathcal{E}}^{su}(z,k) &+ \frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{z} dz_1 \int d\alpha \int d\beta \mathcal{G}^{>}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\\ &+\frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{0} dz_1 \int d\alpha \int d\beta \{\mathcal{G}^{uu} + \mathcal{G}^{ud}\}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\end{aligned} \tag{23a}$$

$$\begin{aligned}\tilde{\mathcal{E}}^d(z,k) = \tilde{\mathcal{E}}^{sd}(z,k) &+ \frac{|k_2|^4}{(2\pi)^6}\int_{z}^{0} dz_1 \int d\alpha \int d\beta \mathcal{G}^{<}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\\ &+\frac{|k_2|^4}{(2\pi)^6}\int_{-d}^{0} dz_1 \int d\alpha \int d\beta \{\mathcal{G}^{du} + \mathcal{G}^{dd}\}(z,k;z_1,\alpha)\Phi(\alpha-\beta)\mathcal{E}(z_1,\beta)\end{aligned} \tag{23b}$$

On using the expressions for $\mathcal{G}$'s the above pair of equations can be represented as the following integro-differential transport equation system

$$[dz + 2\eta''] \tilde{\mathcal{E}}_u(z,k_\perp) = \mathcal{E}_u^c + \frac{|k_2|^4}{16\pi^2|\eta|^2}\int d\alpha_\perp \{\Phi_v(k_\perp - \alpha_\perp; \eta' - \eta'_\alpha)\tilde{\mathcal{E}}_u(z,\alpha_\perp) + \Phi_v(k_\perp - \alpha_\perp; \eta' + \eta'_\alpha)\tilde{\mathcal{E}}_d(z,\alpha_\perp)\} \tag{24a}$$

$$[dz - 2\eta''] \tilde{\mathcal{E}}_d(z,k_\perp) = -\mathcal{E}_d^c + \frac{|k_2|^4}{16\pi^2|\eta|^2}\int d\alpha_\perp \{\Phi_v(k_\perp - \alpha_\perp; -\eta' - \eta'_\alpha)\tilde{\mathcal{E}}_u(z,\alpha_\perp) + \Phi_v(k_\perp - \alpha_\perp; -\eta' + \eta'_\alpha)\tilde{\mathcal{E}}_d(z,\alpha_\perp)\} \tag{24b}$$

Here $\mathcal{E}_u^c$ and $\mathcal{E}_d^c$ are the contributions due to coherent intensities. The associated boundary conditions are obtained as

$$\tilde{\mathcal{E}}_d(0,k_\perp) = |R_{12}(k_\perp)|^2 \tilde{\mathcal{E}}_u(0,k_\perp) \tag{25a}$$

$$\tilde{\mathcal{E}}_u(-d,k_\perp) = \langle |R_{32}(k_\perp,k'_\perp)|^2\rangle \tilde{\mathcal{E}}_d(-d,k_\perp) \tag{25b}$$

where R_{12} and R_{32} are the reflection coefficients at the lower and upper boundaries for waves in the layer. In the process of obtaining (25) we had to impose the following approximation

$$\langle [R_{32}\otimes R_{32}^*][(I+S_{dd})\otimes(I+S_{dd})^*]\rangle \simeq \langle R_{32}\otimes R_{32}^*\rangle\langle (I+S_{dd})\otimes(I+S_{dd})^*\rangle \tag{26}$$

This is similar to the weak surface correlation approximation in the sense that we assume that the influence of the boundary fluctuations result in local relations. On observing that $I(z,\Omega) = \frac{\epsilon c k_2^2}{(2\pi)^2}\mathcal{E}(z,k_\perp)\cos\theta$ we find that the system of integro-differential Eqs. (24) and (25) is identical to the radiative transfer equation system (2)–(5). The conditions under which this has been possible are:
1. ladder approximation to the intensity operator
2. quasi-stationary approximation for fields
3. weak surface correlation

For unbounded random media and random medium layer with planar boundaries we find that the first two conditions are sufficient. But for random media with rough boundaries we need in addition the third approximation.

5. Unified Approach

The system of equations that we start here is the same as that in the surface scattering operator (SSO) approach, viz., (6) and (7). However, the integral equation representation is different. In the SSO approach we did not directly deal with the boundary conditions. The role of the boundaries are represented entirely by the SSO. Indeed the boundary conditions are essential to determine the SSO. However, in the unified approach we will directly make use of the boundary conditions. At the top surface the boundary conditions are given as

$$\begin{aligned}G_{12}(r_\perp,0;r') &= G_{22}(r_\perp,0;r')\\ \epsilon_2\partial_z G_{12}(r_\perp,0;r') &= \epsilon_1\partial_z G_{22}(r_\perp,0;r')\end{aligned} \tag{27}$$

There is a similar pair of relations at the top surface involving G_{11} and G_{21}. At the bottom surface we have

$$\partial_n G_{21}(r_\perp,\zeta;r') = \partial_n G_{22}(r_\perp,\zeta;r') = 0 \tag{28}$$

These boundary conditions are translated on the plane $z = -d$ by using the following approximation which applies when the surface fluctuations are small and smooth.

$$\begin{aligned} \partial_z G_{21}(r_\perp, -d; r') &= \mathcal{H} G_{21}(r_\perp, -d; r') \\ \partial_z G_{22}(r_\perp, -d; r') &= \mathcal{H} G_{22}(r_\perp, -d; r') \end{aligned} \tag{29}$$

where $\mathcal{H} = \nabla_\perp \zeta \cdot \nabla_\perp - \zeta \partial_z^2$. Using (29) we can convert the differential equation system of our problem into the following integral equation system.

$$G = G^o + G^o Q G \tag{30}$$

where

$$Q = Q_v + Q_s \tag{31a}$$

$$Q_v = qN \qquad\qquad Q_s = -\mathcal{H}\delta(z+d)N \tag{31b}$$

Q_v and Q_s represent the volumetric fluctuation and the surface fluctuation, respectively. G^o is the Green's function for the unperturbed problem, viz., the problem when all the fluctuations vanish. Notice that, in this approach, volumetric and surface fluctuations are treated on equal footing. Thus statistical averaging over volumetric and surface fluctuations are carried out at the same step. Therefore we do not have any subscripts for the averaging operations. This is in contrast with the SSO approach where these operations are carried out separately at different stages of the analysis and we had to use subscripts to indicate whether it is w. r. t. volumetric fluctuations or surface fluctuations.

First average (30) to get

$$\langle G \rangle = G^o + G^o \langle Q \langle G \rangle Q \rangle \langle G \rangle \tag{32}$$

This is the mean Green's function that we will use in our analysis of the second moments of the fields. Details of the analysis of (32) are given in Mudaliar (2005). We proceed to the calculation of the field correlation described by the following equation

$$\langle \psi \otimes \psi^* \rangle = \langle \psi \rangle \otimes \langle \psi \rangle^* + \langle G \rangle \otimes \langle G \rangle^* K \langle \psi \otimes \psi^* \rangle \tag{33}$$

where

$$K \simeq \langle Q \otimes Q^* \rangle = \langle Q_v \otimes Q_v^* \rangle + \langle Q_s \otimes Q_s^* \rangle \tag{34}$$

We employ the Wigner transforms in (33) as before and obtain

$$\mathcal{E}(r,k) = \tilde{\mathcal{E}}^m(r,k) + \frac{1}{(2\pi)^6} \int dr_1 \int d\alpha \int d\beta \mathcal{G}(r,k;r_1,\alpha)\{T_v + T_s\}\mathcal{E}(r_1,\beta) \tag{35}$$

where T_v and T_s are spectral representations of $\langle Q_v \otimes Q_v^* \rangle$ and $\langle Q_s \otimes Q_s^* \rangle$, respectively. As before we employ the quasi-stationary field approximation, use (22), and hence arrive at a system of integro-differential equations. The system thus obtained in identical to that in SSO approach. However, the boundary conditions are quite complicated and we have

$$\tilde{\mathcal{E}}(0,k_\perp) = \tilde{\mathcal{E}}^o(0,k_\perp) + \int_{-d}^{0} dz_1 \int d\alpha_\perp W(0,k_\perp;z_1,\alpha_\perp)\tilde{\mathcal{E}}(z_1,\alpha_\perp) \tag{36a}$$

$$\tilde{\mathcal{E}}(-d,k_\perp) = \tilde{\mathcal{E}}^o(-d,k_\perp) + \int_{-d}^{0} dz_1 \int d\alpha_\perp W(-d,k_\perp;z_1,\alpha_\perp)\tilde{\mathcal{E}}(z_1,\alpha_\perp) \tag{36b}$$

where $\tilde{\mathcal{E}}^o$ is the single scattering solution, and W is a 2×2 matrix given in the appendix. Observe that the boundary conditions are not localized. Furthermore, W involves both surface scattering and volumetric scattering. Thus our system incorporates volumetric and surface scattering interactions. However, if we let $\Phi_v \to 0$ and consider only single scattering from the rough boundary, then we obtain the boundary conditions used in the radiative transfer approach, viz., (25).

6. Conclusion

Radiative transfer approach is very efficient and at the same time simple for describing multiple scattering phenomena. Quite rightly this approach is very popular and is used in a wide variety of applications. Consequently, there are several different interpretations of the meaning and domain of applicability of this approach. One good way to understand this approach is to compare and relate it to the statistical wave approach.

Its relation to the wave approach has been well established for the case of unbounded random media. The primary conditions for establishing this equivalence are: ladder approximation to the intensity operator and quasi-stationary approximation of the fields. We find these two conditions are the only requirements even for random media with plane parallel boundaries. However, if the boundaries are statistically rough we need to impose additional restrictions. To illustrate this point we considered two statistical wave approaches: the surface scattering operator approach and the unified approach. In both approaches the integro-differential equations for intensities are the identical to those used in the RT approach. However the boundary conditions are different from those in the RT approach. In the case of SSO approach we need to impose the weak surface correlation approximation to arrive at the boundary conditions of the RT approach. In the case of unified approach we had to let $\Phi_v \to 0$ and consider only single scattering from the rough surface while deriving the boundary conditions. With these additional conditions all the three approaches result in the same system of equations. This study has thus helped us to better understand the three approaches and in particular the relation between the radiative transfer approach and the statistical wave approach when applied to the problem of scattering from a random medium layer with rough boundaries.

Acknowledgment

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Appendix

$$
\begin{aligned}
W_{uu} \;&=\; \frac{1}{(2\pi)^2} e^{-2q''z}\{|k_2|^4 T^v_{uu} + T^s_{uu}\} \\
&\qquad T^v_{uu} = \mathrm{rect}\{z,-d\}|S^{>}|^2 e^{2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;\eta'-\eta'_\alpha) + |S^{uu}|^2 e^{2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;\eta'-\eta'_\alpha) \\
&\qquad\qquad +|S^{ud}|^2 e^{-2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;-\eta'-\eta'_\alpha) \\
&\qquad T^s_{uu} = \{|S^{uu}|^2 e^{-2\eta''d} + |S^{ud}|^2 e^{2\eta''d}\}\Phi_s(k_\perp-\alpha_\perp)\{(k_\perp-\alpha_\perp)\cdot\alpha_\perp - \eta'^2_\alpha\}^2 \\
W_{ud} \;&=\; W_{uu}\{\eta'_\alpha \to -\eta'_\alpha\} \\
W_{du} \;&=\; \frac{1}{(2\pi)^2} e^{2q''z}\{|k_2|^4 T^v_{du} + T^s_{du}\} \\
&\qquad T^v_{du} = \mathrm{rect}\{0,z\}|S^{<}|^2 e^{-2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;-\eta'-\eta'_\alpha) + |S^{du}|^2 e^{2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;\eta'-\eta'_\alpha) \\
&\qquad\qquad +|S^{dd}|^2 e^{-2\eta''z_1}\Phi_v(k_\perp-\alpha_\perp;-\eta'-\eta'_\alpha) \\
&\qquad T^s_{du} = \{|S^{du}|^2 e^{-2\eta''d} + |S^{dd}|^2 e^{2\eta''d}\}\Phi_s(k_\perp-\alpha_\perp)\{(k_\perp-\alpha_\perp)\cdot\alpha_\perp - \eta'^2_\alpha\}^2 \\
W_{dd} \;&=\; W_{du}\{\eta'_\alpha \to -\eta'_\alpha\}
\end{aligned}
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