Homogenisation Theory for Resonant Nonlinear Optical Metamaterials

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Metamaterials for nonlinear optics are constructed from embeddings of resonant particulate or stratified planar materials within a matrix of homogeneous other material, which may itself be electro- magnetically linear or nonlinear. Examples of such materials that have been successfully constructed in this way include semiconductor-doped glass (SDG), rare-earth doped silica fibre, similarly doped glass or silica planar waveguides, quantum-well superlattices and quantum-dot media. In all these ma- terials, the nonlinearities may be passive or even active with suitable coupling to an external energy pump. The resonance in the embedded materials (between the carrier frequency of the electromagnetic radiation and transition energies between quantum electronic states of the embedded atoms) results in saturation effects which are intrinsic sources of electromagnetic nonlinearity.

Here the problem of homogenising such materials into effective medium parameters is examined in detail, using quantum-electronic models to describe the embedded resonances in the background medium. It is very well known that, when the optical carrier frequency Ω is far from all resonant frequencies, a nonlinear inhomogeneous medium can be homogenised to an effective homogeneous medium through the use of nonlinear susceptibility tensors $\chi^{(n)}_{k_1 k_2 \ldots k_n}$ for order-n nonlinearities. The situation when resonant interactions occur is much less clear, because the nonresonant nonlinear sus- ceptibilities have no real meaning due to singularities in their description at the resonant frequencies. In particular, proximity effects and local fields play a significant role in the resonant nonlinear regime, making the homogenisation problem considerably more difficult. It turns out that in the fully resonant case a simplified Maxwell-Bloch model is the appropriate homogenised model, with averaged parameters for dipole moments etc. of the averaged effective medium. The averaged medium parameters are obtained from a full quantum-electronic model of the medium by an averaged Lagrangian method.

This theory permits the definition of, for example, the optical gain g of a pumped electronic medium as if g were an effective single parameter of the medium, along with a homogenised model of snonlinear saturation effects.