On Effective Parameters of Periodical Metamaterials

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Various metamaterials, very actively studied in recent years, usually consist of metal inclusions of complex shapes periodically arranged in space. If the period of the lattice and the dimensions of the inclusions are small compared with the wavelength, the material is usually considered as an effectively homogeneous material characterized by effective material parameters (permittivity and permeability). The shape and dimensions of inclusions define the electromagnetic response, which may be rather exotic (negative material parameters, for example).

It is well known that the effective medium description looses its physical meaning when any of the material dimensions (period, inclusion size) becomes comparable with the wavelength. Retrieved effective parameters cannot be anymore used as effective parameters, as they depend on the sample shape and size, as well as on the wave vector of the incident field. Moreover, in this regime these effective parameters are not response function, so that they do not obey causality, for example. There is another complication in modeling metamaterials — the resonant nature of the inclusions. Due to complex inclusion shape, inclusion resonance takes place at frequencies, where the overall inclusion dimension is still much smaller than the wavelength.

Formal retrieval of material parameters from computed or measured reflection and transmission coefficients of metamaterial slabs often leads to results that clearly violate causality and energy conservation laws, indicating that these parameters do not have the usual physical meaning. This can happen in the frequency regions quite far from the lattice resonance, where usually the effective medium description is meaningful. A typical example can be seen in Fig. 2 (left) of paper [1]. Even at low frequencies, quite far from the resonance, where the losses are small, the retrieved permittivity has a negative derivative with respect to the frequency (violating the reactance theorem), and in the resonance region the energy conservation law does not hold. In the literature, these problems have been attributed to the effect of the periodicity of the medium, and it has been suggested that although the meaning of the effective parameters is quite limited, they may be still useful in understanding the material response (e.g., [2]). This shows that the problem of effective parameter description and parameter retrieval calls for further study.

In this presentation we will discuss this problem using the approach of equivalent periodically loaded transmission lines [3](152–154). Extending this method to resonant inclusions, we will study how the impedance calculated from reflection-transmission data relates with the impedance defined as the ratio of averaged electric and magnetic fields. Limitations for the effective medium description coming from material periodicity and from resonant nature of the inclusions will be discussed.

REFERENCES

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