

Array Patterns Synthesizing Using Genetic Algorithm

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Abstract—A planar array antenna with arbitrary geometry synthesis technique based on genetic algorithm is discussed. This approach avoids coding/decoding and directly works with complex numbers to simplify computing programming and to speed up computation. This approach uses two crossover operators that can overcome premature convergence and the dependence of convergence on initial population. Simulation results show that this method is capable of synthesizing arrays whose elements are located on irregular grids, and generates quite complex shapes and can realize good sidelobe suppression at the same time.

1. Introduction

In the interests of efficiency, the shape of a footprint pattern radiated by a satellite-borne array antenna should conform precisely to the shape of the region on Earth for which coverage is required. Alternatively, in order to achieve “isoflux” illumination [1] on the earth surface through multi-beams, the size and the shape of the footprint should be precisely controlled.

Previous works on the synthesis of arrays for arbitrary footprint shapes include Chebyshev method, modified Woodward–Lawson method and a series of methods based on sampling a circular Taylor distribution [2–4]. But these methods have the drawback that they require the arrays to lie on a rectangular lattice or circular lattice. Some methods require the aperture of the array to be regular shape.

Genetic algorithm (GA) has a high ability in global optimization. It is an increasingly popular optimization method being applied to many fields of endeavor, including electromagnetic optimization problems.

Use genetic algorithm to synthesize array pattern has no limitation on lattice shapes and aperture shapes. It can synthesize planar array with arbitrary geometry and generating arbitrary patterns. Conventional GAs [5, 6] with binary coding and binary genetic operation are inefficient and inconvenient for array pattern synthesis problems to optimize complex numbers. Unlike conventional GAs, this approach avoids coding/decoding and directly deals with complex excitation vectors.

Compared with other numerical methods [7, 8], this approach has unique features to treat complicated problems (complicated arrays and complicated pattern shapes).

2. Problem Formulation

The far-field radiation pattern $F(\theta, \varphi)$ at a far-field angle (θ, φ) from array broadside is given by

$$F(\theta, \varphi) = EF(\theta) \cdot AF(\theta, \varphi) \quad (1)$$

where

$$EF(\theta) = \cos^{1.5}(\theta) \quad (2)$$

is the element radiation pattern; $AF(\theta, \varphi)$ is the array factor. For an arbitrary array, the Array Factor (AF) can be expressed by the general function:

$$AF(\theta, \varphi) = \mathbf{I} \mathbf{s}(\theta, \varphi) \quad (3)$$

$$\begin{aligned} \mathbf{I} &= [I_1, I_2, \dots, I_N], I_n \in C^n, \\ \mathbf{s} &= [e^{jkr_1 \hat{\mathbf{a}}(\theta, \varphi)}, e^{jkr_2 \hat{\mathbf{a}}(\theta, \varphi)}, \dots, e^{jkr_N \hat{\mathbf{a}}(\theta, \varphi)}]^T, \\ \mathbf{r}_n &= \hat{\mathbf{a}}_x x_n + \hat{\mathbf{a}}_y y_n + \hat{\mathbf{a}}_z z_n, \\ \hat{\mathbf{a}}(\theta, \varphi) &= \hat{\mathbf{a}}_x \sin \theta \cos \varphi + \hat{\mathbf{a}}_y \sin \theta \sin \varphi + \hat{\mathbf{a}}_z \cos \theta \end{aligned}$$

where, \mathbf{I} is the excitation vector, \mathbf{s} is the steering vector, C^n is the set or subset of all complex numbers, \mathbf{r}_n is the element location vectors, $\hat{\mathbf{a}}(\theta, \varphi)$ is unit vector of distance ray of spheric coordinate, and θ and φ are the elevation and azimuth angles respectively. GA is applied to find proper excitation coefficient vector \mathbf{I} to achieve desired pattern shape, sidelobe suppression and steering.

3. The Genetic Algorithm

The GA process could be simplified as following: 1) Initialize a random pool of Individuals. 2) Evaluate each Individual. 3) Choose couples (Mating). 4) Breed them together (Crossover). 5) Evaluate each Individual. 6) Selection. 7) Mutation. 8) If the pool has converged, or a number of pre-determined cycles have been completed, finish the cycle. If not, return to step #3.

A. Construction of Chromosomes

In this approach, chromosomes are represented directly by complex excitation vectors \mathbf{I} . N elements complex excitation coefficients are genes of the chromosome.

B. Fitness Function

Evaluation plays a very important role in the GA process. Fitness function maps all the properties of an individual to a floating-point number, essentially, giving it a rank and a place amongst the other individuals in the pool. Creating the fitness function is one of the most difficult works in the creation of a GA solution.

In this approach, we desired that the magnitude $|F(\theta, \varphi)|$ of the far-field pattern remain bounded between some specified limits as

$$|F_{\min \text{ limit}}(\theta, \varphi)| \leq |F(\theta, \varphi)| \leq |F_{\max \text{ limit}}(\theta, \varphi)| \quad (4)$$

A cost measure to be minimized is the sum of the squares of the excess far field magnitude outside the specified bounds. This can be written as

$$\begin{aligned} f_1 = & c_1 \sum_{j=1}^J \sum_{k=1}^K \max(|F(\theta_j, \varphi_k)| - |F_{\max \text{ limit}}(\theta_j, \varphi_k)|, 0)^2 \\ & + c_2 \sum_{j=1}^J \sum_{k=1}^K \max(|F_{\min \text{ limit}}(\theta_j, \varphi_k)| - |F(\theta_j, \varphi_k)|, 0)^2 \end{aligned} \quad (5)$$

where, $|F(\theta, \varphi)|$ are the far field pattern values from (1) evaluated at $J \times K$ far-field angles (θ_j, φ_k) . θ_j and φ_k are spaced in some rules. This will be decided by the beam-pointing angle. c_1, c_2 are weights.

If the dynamic range ratio $|I_{\max}/I_{\min}|$ is too large, the excitation will not easy to realize. We can limit it using the following fitness function:

$$f = f_1 + c_3(|I_{\max}/I_{\min}|) \quad (6)$$

where, c_3 is a weight parameter. Lower values of f indicate better fitness.

C. Mating Scheme

Yeo [9] discussed three mating schemes and thought if one or more near-solutions were added to an initial population of random individuals, EMS scheme usually yields the best chromosome among these three methods. However, this scheme always results in prematurity. In this approach, we make use of a stochastic mating scheme. All individuals have chance to mate and no one can mate two times.

D. Crossover

The crossover operator is the most important operator and it is the operator that combines two individuals to create (a) new individual(s), which will, it is hoped, become better than his/their parents. This might and can work because the selection operator chooses the better individuals.

Real coded GAs usually use interpolate cross operator to breed offspring. Its operating process can be described as follows,

\mathbf{I}^1 and \mathbf{I}^2 are parents. The chromosomes of them have N genes. The offspring of them can be written as

$$I_i^{1'} = cI_i^1 + (1-c)I_i^2 \quad (7)$$

$$I_i^{2'} = cI_i^2 + (1-c)I_i^1 \quad (8)$$

where, $c \in [0, 1]$, $i=1, 2, \dots, N$.

Extrapolate cross operator is another real number cross operator. The offspring genes of \mathbf{I}^1 and \mathbf{I}^2 can be written as

$$I_i^{1'} = I_i^1 - (I_i^2 - I_i^1)c \quad (9)$$

$$I_i^{2'} = I_i^2 + (I_i^2 - I_i^1)c \quad (10)$$

The two real number operator can work with complex number as well. Interpolate cross operator has advantages of fast convergence. Extrapolate cross operator expand the search space. Combining two operators can overcome premature convergence and the dependence of convergence on initial population.

The range of genes is $|I_i| \leq 1$. If genes generated by crossover are out of the bound, then

$$I_i = I_i / |I_i| \quad (i = 1, 2, \dots, N) \quad (11)$$

E. Selection

The selection operator distinguishes the better individuals from the worse individuals using their fitness. In this approach, both the child and parent populations are ranked together in ascending order. Then, based on the principle of survival of the fittest, those producing superior output survive, while those producing inferior output die off. Please note that the competitors for survival selection include both parents and their children so that the members of next generation may include members of the previous generation. This guarantees that the new generation performs no worse than older ones. In other words, the cost f versus generation curve decreases monotonically. This selection scheme includes optimum maintaining strategy.

F. Mutation

The mutation operator plays a secondary role with respect to crossover operators. It can maintain the diversity of the population. Mutation is a minor change to the genes of an individual, in a hope to find an even better solution, or rather, to expand the search space to a point where normal breeding might not reach. Mutation effectively slows down convergence, but might yield better and closer-to-best individuals. If an individual is “pushed” to a different peak area, a higher one, it might “pull” other individuals with the crossover process to the new peak, thus climbing a better and higher peak, and achieving a better solution. Both the probability and the range of mutation can affect convergence [10].

Nevertheless, mutation is required to prevent an irrecoverable loss of potentially useful information that occasionally reproduction and crossover can cause.

The fitness of mutated individual usually has low value. If put mutation in front of selection. The mutated individuals would die off because of the child and parent competition. In this approach mutation is put after selection.

Assuming P_m is the mutation probability, the mutation is as follows: P real numbers are generated in $[0, 1]$ randomly. Each number corresponds to an individual of the population. Set the number corresponding to current optimum to “1” to avoid being mutated. An individual whose corresponding number less than P_m will be muted use (12).

$$\mathbf{I}' = \mathbf{I} + \mathbf{d} \quad (12)$$

where, \mathbf{d} is a vector dimension same as \mathbf{I} .

G. Convergence Observation

For fast convergence, the initial population can include approximate excitations by other techniques (such as Fourier expansion method [11], etc.) We care about not only the shape of main beam, but also the sidelobe level. In order to obtain the required sidelobe level rapidly, we put the optimization result of adjacent beams into initial population which can reduce optimization time largely.

In this approach, two crossover operators are used to generate offspring. Adjusting proportions act on population of two operators, i. e., 40% population use interpolate cross operator to generate offspring and 60% population use extrapolate cross operator to generate offspring, can makes the algorithm has a good performance.

4. Simulation Results

This subsection presents a shaping example based on a Low Earth Orbit (LEO) satellite-borne antenna array. As shown in Figure 1, a 61 elements antenna array with hexagonal (or equilateral triangular) grid is used.

In order to achieve “isoflux” illumination on the earth surface, a circularly symmetric cell layout was decided as shown in Figure 2 after calculation. Wedge shaped cells are arranged in rings about nadir. There are 23 beams requiring shape, and 30 dB sidelobe suppression is needed.

Figure 3 is the pattern of beam 1. Gain in main beam edge is higher than that in beam center. This can compensate the path loss due to the slant range differences from satellite to earth. Figures 4, 5 and 6 are the patterns of beam 2, 3 and 8. It is observed that the main lobe satisfy the requirement, and side lobe level suppress reach 30 dB which is outstanding the results of [7, 8].

If the dynamic range ratio $|I_{\max}/I_{\min}|$ is too large, the excitation will not easy to realize. For beam 1 shown as Figure 2, $|I_{\max}/I_{\min}| = 20$. For beams in ring 2 and a sidelobe level of -30 dB, an $|I_{\max}/I_{\min}| < 40$ can be reached. For beams in ring 3 and a side lobe level of -30 dB, an $|I_{\max}/I_{\min}| < 50$ can be reached. These excitations are easily to realize.

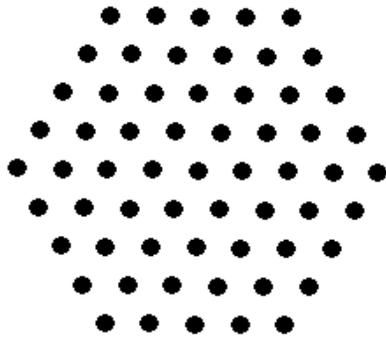


Figure 1: 61 elements antenna array.

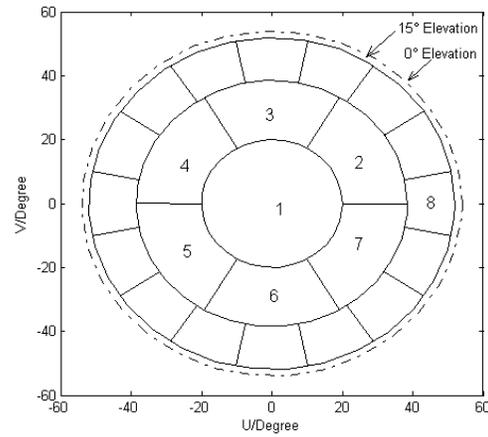


Figure 2: Cells arranged in rings, $U = \theta \cos \varphi$, $V = \theta \sin \varphi$.

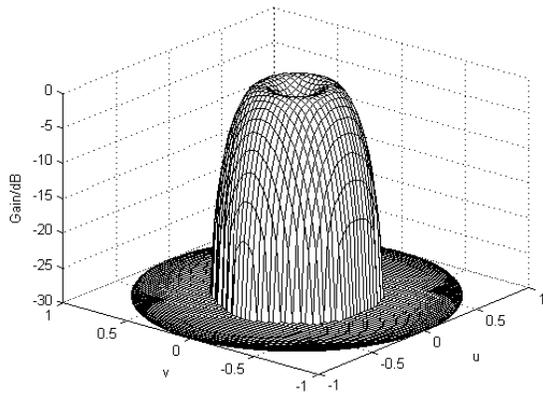


Figure 3: Pattern of beam 1, $u = \sin \theta \cos \varphi$, $v = \sin \theta \sin \varphi$.

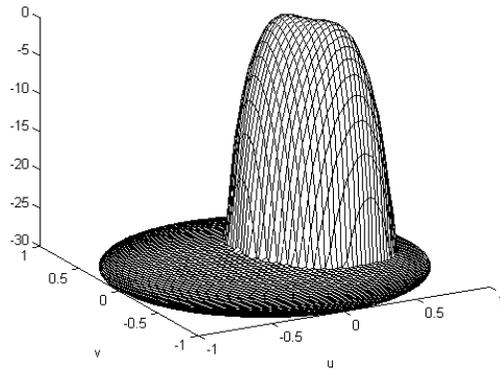


Figure 4: Pattern of beam 2.

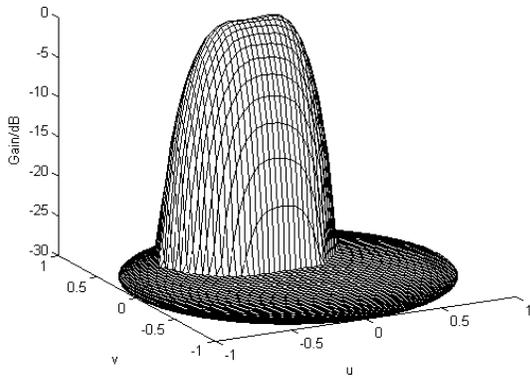


Figure 5: Pattern of beam 3.

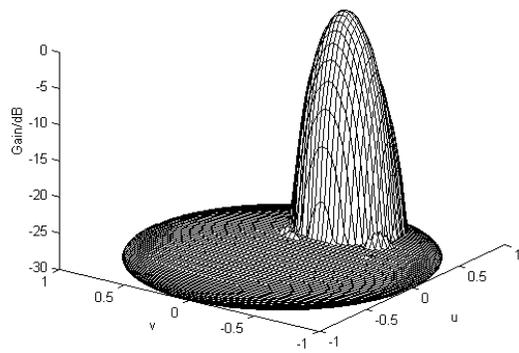


Figure 6: Pattern of beam 8.

If Woodward-Lawson method was used, it requires the elements to lie on a rectangular lattice and require the aperture of the array to be rectangle. And the $|I_{\max}/I_{\min}|$ of the solution will reach 800 or even higher [3].

The -3dB contour of each beam after being shaped is shown in Figure 7. Notice the shape of beams, for example beams in ring 2, for perfection beamforming the footprints shape should wedge-shaped. Of course, it is achievable only by an infinitely large array. Due to the limitation of aperture size and restriction of elements number, the footprint shapes are kidney-shaped. However, it can satisfy the requirements of isoflux illumination.

Figure 8 shows the gain of beams along $U = 0^\circ$ of the satellite coverage. Path loss due to the slant range

differences from satellite to earth is considered. From the slice figure we can see that gain is higher than 13 dBi, and ripples lower than 3 dB.

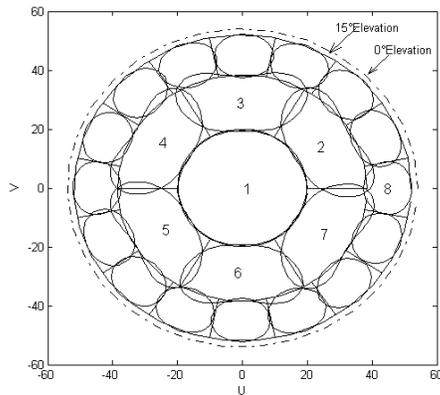


Figure 7: -3 dB contour of each beam.

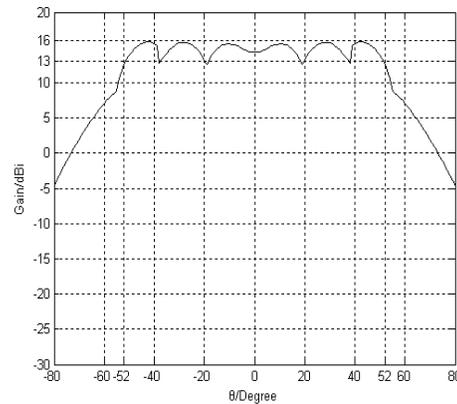


Figure 8: Slice figure of $U = 0^\circ$.

5. Conclusion

A complex coded GA based method is discussed for the synthesis of planar arrays with arbitrary geometry that generate footprints of arbitrary shape. This approach is capable of synthesizing quite complex shapes of 3D patterns for main lobe and can realize good sidelobe suppression at the same time. The method has been proved to be useful for the synthesis of large array antennas whose elements are located on irregular grids.

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