Two-dimensional Randomly Rough Surfaces that Act as Gaussian Schell-model Sources

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We consider the scattering of a scalar Gaussian beam of frequency ω incident normally on a twodimensional randomly rough surface defined by $x_3 = \zeta(\mathrm{x}_\parallel)$, where $\mathrm{x}_\parallel = (x_1, x_2, 0)$. The region $x_3 > \zeta(\mathrm{x}_\parallel)$ is vacuum while the region $x_3 < \zeta(\mathrm{x}_\parallel)$ is the scattering medium. We assume that the Dirichlet boundary condition is satisfied on the surface $x_3 = \zeta(\mathrm{x}_\parallel)$. We denote the scattered field in the vacuum region by $\Phi(\mathrm{x}|\omega)_{sc}$, and its value on the plane $x_3 = 0$, by $\Phi(\mathrm{x}_\parallel|\omega)_{sc}$. We seek the surface profile function $\zeta(\mathrm{x}_\parallel)$ for which $\Phi(\mathrm{x}_\parallel|\omega)_{sc}$ satisfies the condition $\langle\Phi(\mathrm{x}_\parallel|\omega)_{sc})\Phi^*(\mathrm{x}_\parallel|\omega)_{sc}\rangle = A^2\exp(-\mathrm{x}_\parallel^2/4\sigma^2)\exp[-(\mathrm{x}_\parallel - \mathrm{x}'_\parallel)^2/2\sigma_\mathrm{g}^2]\exp(-\mathrm{x}'^2_\parallel/4\sigma_\mathrm{s}^2)$, where the angle brackets denote an average over the ensemble of realizations of $\zeta(\mathrm{x}_\parallel)$. Such a surface is a Gaussian Schell-model source of radiation. The field scattered from the resulting surface, although it is only partically coherent, has the intensity distribution of a fully coherent laser beam whose intensity in the plane $x_3 = 0$ has the form $A_L^2\exp(-2\mathrm{x}_\parallel^2/\delta_\mathrm{L}^2)$, where $\delta_L = 2\sigma_s\sigma_g/(\sigma_g^2 + \sigma_s^2)^{1/2}$ and $A_L\delta_L = 2A\sigma_s$.

Two approaches are used to determine the surface profile function that acts as a Gaussian Schellmodel source. Both are based on the geometrical optics limit of the phase perturbation theory expression for the scattered field. In the first approach the surface profile function $\zeta(\mathrm{x}_\parallel)$ is represented as a continuous array of triangular facets. The joint probability density function of two orthogonal slopes of each facet is determined from the condition that the field scattered from the resulting surface has the desired correlation property in the plane $x_3 = 0$. In the second approach it is shown that a surface profile function $\zeta(\mathrm{x}_\parallel)$ that is a stationary, zero-mean, isotropic, Gaussian random process, can also act as a Gaussian Schell-model source, when the rms height and transverse correlation length of the surface roughness are suitably chosen. Each of these two approaches is validated by the results of numerical simulation calculations of the intensity distribution of the scattered field, which show that it indeed has the form of a laser beam.