Cones, Spirals, and Möbius Strips in Multiply Scattered Light

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Laser light scattered by a multiple scatterer invariably emerges elliptically polarized. In general, the orientations of the ellipses in elliptically polarized light vary throughout space. In three dimensions the orientation of an ellipse may be described by a 3-frame in which one frame axis is along the major axis of the ellipse, a second frame axis is along the minor axis of the ellipse, and the third frame axis is along the normal to the ellipse. These three axes are shown to generate cones, spirals, and Möius strips, characterized by a total of 27 different topological indices.

For ordinary ellipses (the vast majority) that are not on singular lines of circular or linear polarization, the Möbius strips have one full twist, and there are a total of 21 indices that are non zero. These indices, if independent, could collectively divide the field into $2^{21} = 2,097,152$ structurally different grains separated by grain boundaries on which an index becomes undefined. Selection rules, however, reduce the number of independent configurations to 140,608, while within a linear approximation for the local field surrounding an ellipse there exist degeneracies that further reduce the number of distinguishable configurations to 17,360. Of these, 1,728 are of first order, and should be readily accessible to experiments using recently developed optical near field methods.

Analytical expressions have been obtained for all indices in terms of the 20 parameters needed to define a general field of ellipses within the linear approximation, and more than 10,000 different configurations have been harvested in a simulated multiply scattered random field (speckle pattern), demonstrating that large numbers of configurations can be expected appear in practice.

This previously unsuspected, indeed unprecedented, structural proliferation is intrinsic to spatially varying elliptically polarized light, and in addition to random fields, is found in the fields of wave guides that support a small number of modes (2–3), as well as in the highly ordered fields of optical lattices. Other systems described locally by spatially varying 3-frames, such as liquid crystals, or the dielectric constants of continuous random media, can be expected to show a similar degree of structural proliferation.