Analysis of Cylindrical Microstrip Line with Finite Thickness of Conductor

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Abstract—Novel analytical method based on extended spectral domain approach (ESDA) is presented for cylindrical microstrip line. The method utilizes the aperture fields as the source quantities, as opposed to the conventional methods, which have used the current on the strip as the source. The whole region can be divided into sub-regions by the introduction of aperture fields, and each sub-region can be treated independently. This method makes possible the analysis both of zero and finite thickness of the strip conductor. The numerical procedure incorporates the effects of the edge singularities properly and can afford the efficient and accurate calculations for the phase constants and characteristic impedances of a microstrip line with zero- and finite-thickness conductor. The calculated results by the present method reveal the effect of conductor thickness on the characteristics of a cylindrical microstrip line.

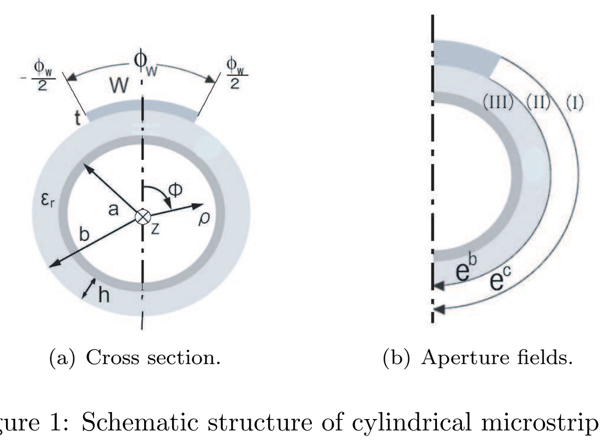
1. Introduction

Recently, curved surface substrates have attracted an attention as materials of antennas and front ends for portable terminals. A lot of analyses of the propagation characteristic of the stripline and the coplanar waveguide composed on a cylinder substrate are reported [1–6], including the moment method, the FDTD method [3], and the finite element method [5]. However, their works assumed the conductor thickness to be zero, and the report concerning the effect of the conductor thickness on the propagation characteristic has not be found. Recently, authors reported on the effect of the finite thickness of a conductor on electric characteristics of cylindrical coplanar waveguides (CCPWs) by using the extended spectral domain approach (ESDA).

In this paper, we report on the analytical method of the cylindrical microstrip line based on ESDA, and the effect of conductor thickness by numerical calculation. The present method utilizes the electric fields at the interface of the aperture as the source quantities, as opposed to the conventional methods [1, 2], which have used the current on the strip as the source. The accurate and efficient numerical procedure, which makes consideration for the field singularities near the conductor edge of zero- and finite-thickness, reveals the effect of the curvature and the finite thickness of a conductor on the characteristic impedances and the phase constants of the cylindrical microstrip line.

2. Theory

Cross section of a microstrip line on a cylindrical dielectric substrate is shown in Fig. 1(a). Curvature R of the cylindrical substrate is defined as the ratio of inner and outer diameter of substrate,



(a) Cross section. (b) Aperture fields.

Figure 1: Schematic structure of cylindrical microstrip line.

$$R = \frac{a}{b} = 1 - \frac{h}{b} \tag{1}$$

where $h = b - a$ is substrate thickness. A signal conductor of W in width is put on the substrate, which is backed by the ground conductor. Both conductors are assumed to be perfect electric conductor (PEC), but the signal conductor has the finite thickness t, as opposed to the previous reports. A single-layered substrate is assumed in the following explanation for the simplicity, although the method is applicable to multilayered and/or overlaid structure problem. The theoretical scheme is based on the ESDA [8–10]. The method has been successfully worked out to analyze the effect of the conductor thickness of the various types of planar transmission lines. Here, in this study, the method is extended further to the analysis of the effect of conductor thickness in cylindrical microstrip line. In the ESDA, first the aperture electric fields are introduced at the circumferential surfaces of dielectric substrate at $\rho = b$, $e^b(\phi)$, and the upper surface of signal conductor at $\rho = b + t$, $e^c(\phi)$ shown in Fig. 1(b). By introducing these aperture fields and utilizing the equivalence theorem, the whole region is divided into subregions, i. e., (I) the outer ($\rho > b + t$), (II) the aperture ($b < \rho < b + t$) and (III) the substrarte ($\rho < b$) subregions. After dividing the region, each subregion can be treated separately, and then the longitudinal components of electromagnetic fields in each subregion are expressed in terms of the appropriate eigenfunctions $\Phi_n^{(i)}(\phi)$, $\Psi_n^{(i)}(\phi)$, which satisfy the boundary conditions in the ϕ-direction.

$$E_z^{(i)}(\rho,\phi)e^{-j\beta z} = \sum_{n=0}^{\infty} \tilde{E}_z^{(i)}(\rho)\Phi_n^{(i)}(\phi)e^{-j\beta z} \tag{2}$$

$$H_z^{(i)}(\rho,\phi)e^{-j\beta z} = \sum_{n=0}^{\infty} \tilde{H}_z^{(i)}(\rho)\Psi_n^{(i)}(\phi)e^{-j\beta z} \tag{3}$$

$$i = I, II, III$$

where β is the unknown phase constant and $\tilde{E}_z^{(i)}$ is the transform of $E_z^{(i)}$. The transversal (ρ,ϕ) field components can be related to the longitudinal components $E_z^{(i)}$ and $H_z^{(i)}$ by utilizing the field equations. The general solution of the transform $\tilde{E}_z^{(i)}$ in region (i) can be expressed as

$$\tilde{E}_z^{(i)}(\rho) = A^{(i)}J_n(\beta_c\rho) + B^{(i)}Y_n(\beta_c\rho) \tag{4}$$

$$\beta_c = \sqrt{\omega^2\varepsilon\mu - \beta^2}$$

where $A^{(i)}$, $B^{(i)}$ is unknown constants, and they are determined by the boundary conditions at the interfaces. The continuities of electric fields are expressed as

$$E_\phi^{(III)}(\rho = a + 0, \phi) = 0, \qquad E_z^{(III)}(\rho = a + 0, \phi) = 0 \qquad \text{at} \ \ \rho = a \tag{5}$$

$$E_\phi^{(II)}(\rho = b + 0, \phi) = E_\phi^{(III)}(\rho = b - 0, \phi) = e_\phi^b \qquad \text{at} \ \ \rho = b \ \ (\frac{\phi W}{2} < \phi < \pi) \tag{6}$$

$$E_z^{(II)}(\rho = b + 0, \phi) = E_z^{(III)}(\rho = b - 0, \phi) = e_z^b \tag{7}$$

$$E_\phi^{(I)}(\rho = b + t + 0, \phi) = E_\phi^{(II)}(\rho = b + t - 0, \phi) = e_\phi^c \qquad \text{at} \ \ \rho = b + t \ \ (\frac{\phi W}{2} < \phi < \pi) \tag{8}$$

$$E_z^{(I)}(\rho = b + t + 0, \phi) = E_z^{(II)}(\rho = b + t - 0, \phi) = e_z^c. \tag{9}$$

These continuity conditions are transformed into spectral domain and they are used to relate the unknowns $A^{(i)}$, $B^{(i)}$ to the aperture fields. The fields are then related to the aperture fields as follows

$$E^{(II)}(\rho,\phi) = \int_{\phi'} \{\overline{\overline{T}}^{(II)}(b,\phi|b+t,\phi') \cdot \mathbf{e^c}(\phi') + \overline{\overline{T}}^{(II)}(b,\phi|b,\phi') \cdot \mathbf{e^b}(\phi')\}d\phi' \tag{10}$$

$$H^{(II)}(\rho,\phi) = \int_{\phi'} \{\overline{\overline{Y}}^{(II)}(b,\phi|b+t,\phi') \cdot \mathbf{e^c}(\phi') + \overline{\overline{Y}}^{(II)}(b,\phi|b,\phi') \cdot \mathbf{e^b}(\phi')\}d\phi' \tag{11}$$

where, $\overline{\overline{T}}'_s$, $\overline{\overline{Y}}'_s$ are the dyadic Green's functions. Then, the integral equations on the aperture fields are obtained by using the continuities of magnetic fields at the aperture surfaces,

$$H_\phi^{(II)}(\rho = b + 0, \phi) = H_\phi^{(III)}(\rho = b - 0, \phi) \qquad (\frac{\phi W}{2} < \phi < \pi) \tag{12}$$

$$H_z^{(II)}(\rho = b + 0, \phi) = H_z^{(III)}(\rho = b - 0, \phi) \tag{13}$$

$$H_\phi^{(I)}(\rho = b + t + 0, \phi) = H_\phi^{(II)}(\rho = b + t - 0, \phi) \qquad (\frac{\phi W}{2} < \phi < \pi) \tag{14}$$

$$H_z^{(I)}(\rho = b + t + 0, \phi) = H_z^{(II)}(\rho = b + t - 0, \phi). \tag{15}$$

Applying the Galerkins procedure to these integral equations, we get the determinant equation for the phase constant β. In the Galerkins, the unknown aperture fields are expressed in terms of the appropriate basis functions $\xi_{\phi i}(\phi)$ and $\xi_{zi}(\phi)$ as,

$$e_\phi^b(\phi) = \sum_{i=1}^{N} b_{\phi i}\xi_{\phi i}(\phi), \qquad e_z^b(\phi) = \sum_{i=1}^{N} b_{zi}\xi_{zi}(\phi) \tag{16}$$

$$e_\phi^c(\phi) = \sum_{i=1}^{N} c_{\phi i}\xi_{\phi i}(\phi), \qquad e_z^c(\phi) = \sum_{i=1}^{N} c_{zi}\xi_{zi}(\phi) \tag{17}$$

where $b_{\phi i}$, b_{zi}, $c_{\phi i}$, and c_{zi} are the unknown expansion coefficients. The basis functions $\xi_{\phi i}(\phi)$, $\xi_{zi}(\phi)$, which incorporate the singularities of fields properly near the conductor edge [8–10], are used in the following computations. For the case with the conductors of zero thickness, the aperture region (II) will be eliminated in the procedure and the aperture field e^b equals to e^c.

The definition of the characteristic impedance is somewhat ambiguous for the hybrid mode propagation along microstrip line. We adopt the voltage-current definitions

$$Z_{VI} = \frac{V_o}{I_o} \tag{18}$$

where V_o is the voltage between the center strip and the ground conductor, and I_o is the total current flowing in the z-direction on the strip conductor. The voltage V_o is evaluated by integrating the radial component of electric field $E_\rho^{(III)}$ between the ground ($\rho = a$) and the signal ($\rho = b$) conductors,

$$V(\phi) = \int_a^b E_\rho(\rho, \phi)d\rho \tag{19}$$

where ϕ may be any in $0 < \phi < \phi_W/2$. Therefore $V(\phi)$ is integrated with ϕ over $0 < \phi < \phi_W/2$ to get

$$V_o = \frac{2}{\phi_W}\int_0^{\frac{\phi_W}{2}} V(\phi)d\phi. \tag{20}$$

The current I_o can be evaluated by the line integral C of the magnetic field around the strip conductor [7]

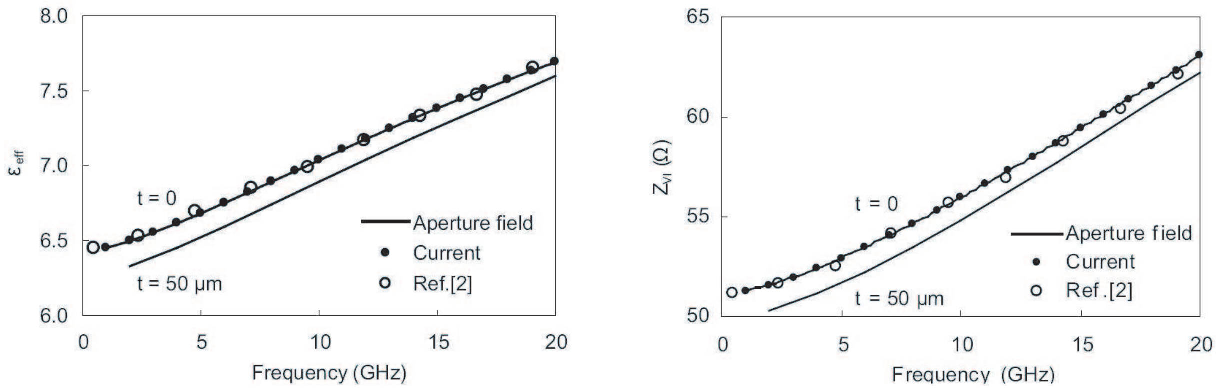
$$I_o = \oint_c \mathbf{H} \cdot dl. \tag{21}$$

3. Numerical Procedure and Results

The conventional methods have treated the propagation characteristics of a microstrip line on a cylindrical substrate assuming the conductor thickness to be zero [2]. The present method, when the aperture field is adopted as the source quantity in the formulation, can afford to present the characteristics of the case with finite as well as zero thickness. Also, the present formulation procedure could employ the current on the strip instead of the aperture field as the source quantity, although this procedure could be applied only to the case with zero thickness. Fig. 2 shows the frequency dependency of the effective dielectric constant ε_{eff} and the characteristic impedance Z_{VI} of a microstrip line on a cylindrical substrate with larger R [2]. The effective dielectric constant ε_{eff} is obtained in terms of the phase constant β as

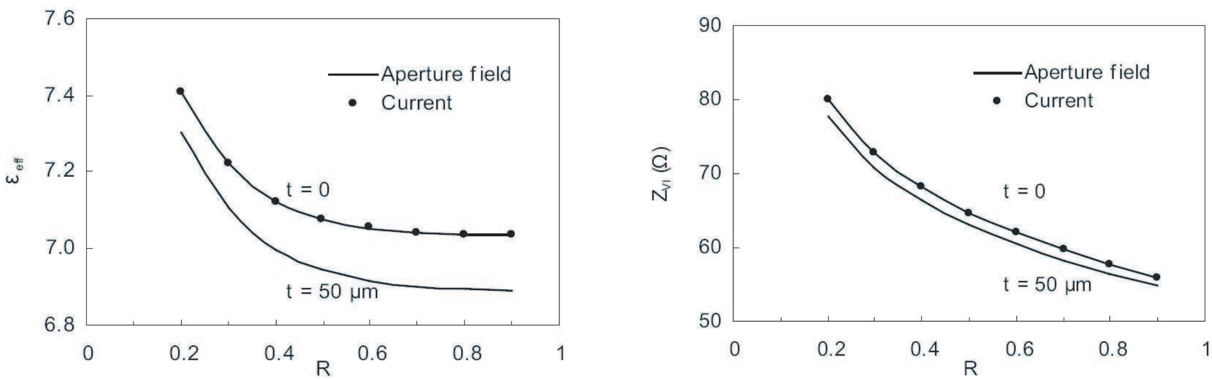
$$\varepsilon_{eff} = \{\beta/\omega\sqrt{\varepsilon_0\mu_0}\}^2 \tag{22}$$

The results of zero thickness conductors are calculated by both the aperture field and the current bases, and both results are in excellent agreement and they agree well with the conventional ones [2] over the frequencies. The figure includes the results of the case with finite thickness of the strip conductor (50 μm) showing the effects of the conductor thickness on ε_{eff} and Z_{VI}.



(a) ε_{eff}. (b) Z_{VI}.

Figure 2: Frequency dependency of propagation characteristics. $\varepsilon_r = 9.6$, $h = 1\,mm$, $W = 1\,mm$, $R = 0.9$.



(a) ε_{eff}. (b) Z_{VI}.

Figure 3: Curvature dependency of propagation characteristics. $\varepsilon_r = 9.6$, $h = 1\,mm$, $W = 1\,mm$, $f = 10\,GHz$.

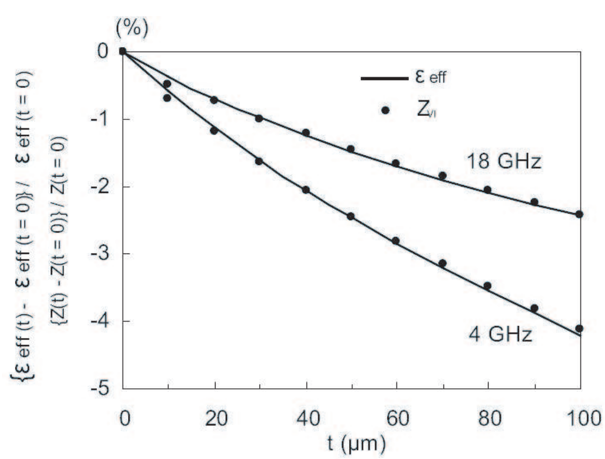


Figure 4: Thickness effect on propagation characteristics. $\varepsilon_r = 9.6$, $h = 1\,mm$, $W = 1\,mm$, $R = 0.9$.

The present methods is equally applicable to the a cylindrical microstrip line with larger and smaller curvature rate R. Fig. 3 shows the curvature dependency of ε_{eff} and Z_{VI}. The value of ε_{eff} increases rapidly when curvature rate R is 0.5 or less. That is, the concentration of the electromagnetic field in the dielectric substrate becomes stronger as the curvature ratio becomes smaller. Therefore, the effect of the thickness of the conductor becomes smaller for the smaller R. Fig. 4 shows the conductor thickness effect where the relative changes of ε_{eff} and Z_{VI} are presented with the thickness variation of conductor. Both ε_{eff} and Z_{VI} are decrease monotonously up to 100 μm thickness conductor. It should be noted that the effect of the conductor thickness becomes smaller for higher frequency ($f = 18$ GHz), as opposed to a cylindrical coplanar waveguides (CCPWs), where the effect becomes larger for higher frequency. This is why the electromagnetic field concentrates more in the dielectric substrate between the strip and the ground conductors and the effect of conductor thickness becomes smaller for higher frequency.

4. Conclusion

Novel analytical method based on extended spectral domain approach (ESDA) is presented for a cylindrical microstrip line. The method is able to treat the effect of the finite thickness of a strip conductor by utilizing the aperture electric fields as source quantities. The numerical procedure incorporates the effects of the edge singularities properly and can afford the efficient and accurate calculation method for the characteristic impedances in addition to the phase constants of a cylindrical microstrip line. The calculated results for zero-thickness conductor by both procedures, based on current or aperture field, are in good agreement and also they agree well with the published data. The results obtained by the present method show the curvature dependency of the propagation characteristics and reveal the effect of conductor thickness, which is different from that of a cylindrical coplanar waveguides (CCPWs).

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