Critical Study of DCIM, and Development of Efficient Simulation Tool for 3D Printed Structures in Multilayer Media

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Abstract—Since the discrete complex image method (DCIM) has been widely used in conjunction with the Method of Moments (MoM) to efficiently analyze printed structures, some lingering issues related to the implementation of DCIM and their brief clarifications are first reviewed. Then, an efficient and rigorous electromagnetic simulation algorithm, based on the combination of MoM and DCIM, is proposed and developed for the solution of mixed-potential integral equation (MPIE) for printed structures with multiple vertical strips in multilayer media. The algorithm is possibly the most efficient approach to handle multiple vertical conductors, even spanning more than one layer, in printed circuits.

1. Introduction

Spatial-domain method of moments (MoM) is a widely used technique for the solution of mixed-potential integral equation (MPIE) for printed geometries in multilayer planar media [1], thanks to the introduction of an efficient closed-form approximation method [2] and its improved versions of the spatial-domain Green's functions [3, 4]. This approach, known as discrete complex image method (DCIM), basically approximates the spectral-domain Green's functions in terms of complex exponentials, and then casts the integral representations of the spatial-domain Green's functions into closed-form expressions via Sommerfeld identity [5]. Although DCIM is quite robust and works well to get the closed-form Green's functions, it has some limitations in the form of a limited range of validity depending upon the implementation of the method.

Some issues originating from the implementation of DCIM are discussed and possible clarifications are provided in Section 2. In Section 3, application of the closed-form Green's functions in conjunction with the spatial-domain MoM is reviewed, with the emphasis given to efficient handling of multiple vertical conductors. Finally, conclusions are provided in Section 4.

2. Discussions on Closed-form Green's Functions

It is well known that spectral-domain Green's functions can be written analytically in planar multilayer media, and their spatial-domain counterparts can be obtained from the inverse Hankel transform of the spectral-domain Green's functions [4, 6], as

$$G = \frac{1}{4\pi} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \widetilde{G}(k_\rho) \tag{1}$$

where $k_\rho^2 = k_x^2 + k_y^2$, ρ is the variable in cylindrical coordinate system, G and $\widetilde{G}$ are Green's functions in the spatial and spectral domain, respectively, $H_0^{(2)}$ is the Hankel function of the second kind and SIP is the Sommerfeld integration path. Since the integrand usually exhibits oscillatory nature and slow convergence, rendering the transformation computationally very expensive, spectral-domain Green's functions can be approximated by complex exponentials, via the generalized pencil-of-function (GPOF) method [6], to obtain closed-form expressions from the inverse Hankel transform. Since the crucial step in this approach is the approximation of the spectral-domain Green's functions, which is detailed in [3, 4], discussions on the accuracy of the method for large distances have concentrated mainly on the approximation procedure, because the resulting closed-form Green's functions are, in general, accurate enough for distances as far as $k_0\rho = 20 - 30$ ($\rho/\lambda = 3 - 4$), beyond which they may deteriorate significantly.

In the literature, there were basically three attributable sources of problems in the implementation of DCIM: (i) not extracting the quasi-static terms, (ii) introducing a wrong branch point in the process of approximation, and (iii) not extracting the surface wave poles (SWP). In the original implementation of DCIM, as introduced in [2], there were only one level of approximation, and it was necessary to extract the quasi-static terms to make the remaining portion of the spectral-domain Green's functions converge to zero for large k_ρ values. However, with the introduction of two-level and multi-level approximation algorithms [3, 7], the necessity of finding the quasi-static terms and their extraction before the approximation has been eliminated. The issue of introducing wrong branch point originates from the following observations: spectral-domain Green's functions, when the source is in a bounded layer, have no branch point at $k_\rho = k_s$, although they have k_{zs} term in the denominator, where k_s is the wave number of the source layer; and the approximating exponentials with k_{zs} factor in the denominator seem to have branch point at $k_\rho = k_s$. However, one should note that the exponential approximation is always performed over the deformed path of SIP and the function to be approximated over this path is single valued with the right choice of the branch. Therefore, the resulting exponentials divided by k_{zs} is a singlevalued function with this right choice of the branch. The last problem concerning the SWPs is inherent to the approach unless the SWP contributions are totally extracted from the functions to be approximated. The detailed discussions on these issues and some clarifications can be found in [4].

3. MoM-DCIM Application for Multiple Vertical Strips

In the analysis of printed geometries with multiple vertical strips, a method based on MoMDCIM is employed, as proposed in [7], and it is extended to efficiently handle multiple vertical strips. The algorithm and its efficient handling of multiple vertical strips can be described by examining one of the inner-product terms in the MoM matrix entries, as follows:

$$\left\langle \frac{\partial}{\partial x}T_x(x,y),\, G_z^q * \frac{\partial}{\partial z}B_z(y,z)\right\rangle = \iint dxdy \frac{\partial}{\partial x}T_x(x,y)\cdot \int dy' B_z(y') \int dz' \frac{\partial}{\partial z'}B_z(z')G_z^q(x-x',y-y',z,z') \quad (2)$$

where $T_x(x,y)$ and $B_z(x,y)$ are the testing and basis functions used in the evaluation of corresponding MoM matrix entry. Writing the spatial-domain Green's function G_z^q in terms of its spectral-domain representation $\widetilde{G}_z^q$, followed by the change of the order of integrations, (2) can be cast into the following form

$$\left\langle \frac{\partial}{\partial x}T_x(x,y),\, G_z^q * \frac{\partial}{\partial z}B_z(y,z)\right\rangle = \iint dudv F_z^q(u,v,z=cons) \int dy B_z(y-v)\frac{\partial}{\partial x}T_x(x'+u,y) \quad (3)$$

where $x - x' = u$, $y - y' = v$ and

$$F_z^q \cong \frac{1}{4\pi}\int_{SIP} dk_\rho k_\rho H_0^{(2)}\big(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|\big)\cdot GPOF\left\{\int dz' \frac{\partial}{\partial z'}B_z(z')\widetilde{G}_z^q(k_\rho, z=cons, z')\right\} \quad (4)$$

Note that the auxiliary function $F_z(u,v)$ is obtained analytically in terms of complex exponentials and it is an explicit function of $u = x-x'$ and $v = y-y'$, and the inner integral of (3) can easily be obtained analytically for most basis and testing functions. Therefore, the same inner-product terms corresponding to other vertical strips can be obtained simply by evaluating $F_z(u,v)$ for different values of u and v, as long as the basis functions used to represent the current densities along them have identical z'-dependencies. Consequently, having more than one vertical conductors in a printed circuit would not require significant amount of additional computation.

The formulation described above is applied to a microstrip line lying along x-direction with four vertical y-spanning strips to assess and demonstrate the computational efficiency of the method. Here are the parameters of the microstrip line: the dielectric constant of the medium is 4.0; the length and width of the line is 18.0 cm and 0.1 cm, respectively; the thickness of the substrate is 0.4 cm; the frequency of operation is 2 GHz; and 71 horizontal basis functions along x-direction are employed. As the thickness of the substrate is uniform, which is usually the case for most of antenna and microwave applications, two basis functions are used over every vertical strip, and naturally they have the same z and z' dependencies, satisfying the only criterion for the efficiency of the method for multiple vertical strips. To validate the method, the current distribution along the microstrip line is first obtained, and compared to that from a commercially available EM simulation software, *em* by Sonnet, as shown in Fig. 1. An excellent agreement is observed; slight differences in the amplitude can be attributed to the inherent models of the approaches: *em* by Sonnet solves the problem in shielded environment while the method proposed here solves it in open environment, which inevitable causes some slight differences on the resonant frequencies of the structure.

Once the validation is complete, the computational efficiency of the proposed method is assessed in terms of the CPU time obtained from a 1.5 GHz Centrino CPU. The microstrip line is first analyzed with one vertical strip (at $x = 7.0$ cm), and then the number of vertical strips is increased to four by one-by-one. As the ultimate measure for the efficient handling of multiple vertical metallization, in addition to the first one, matrix fill time for additional vertical strips are listed in Table 1. For the matrix fill times in case I, the necessary auxiliary functions are calculated only once and used repeatedly, but for case II, the auxiliary functions are re-calculated for every entry corresponding to each basis and testing functions introduced with the addition of new vertical strips. It is observed that efficient use of auxiliary functions has significantly reduced the computational complexity of the whole method. This can be stated with adding new vertical strips to the microstrip line with one vertical strip costs about 4.0 seconds whereas it requires 70.0 seconds in case of not using auxiliary functions repeatedly. Note that CPU times are obtained by using only the symmetry of the MoM matrix and it has not been used any acceleration technique for the evaluation of MoM matrix entries.

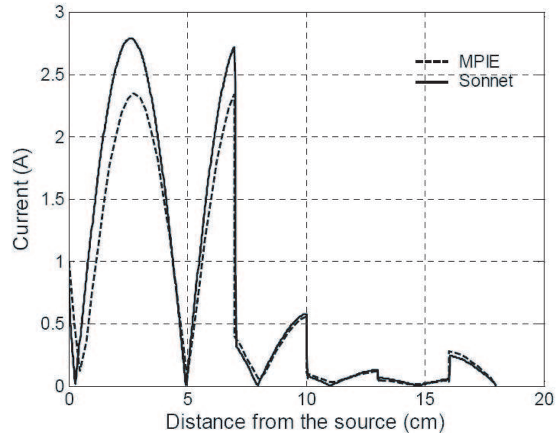

Figure 1: Magnitude of the current along the microstrip line with 4 vertical strips.

Table 1: MoM matrix fill times for each additional vertical strip.

Number of vertical strips	**MoM matrix fill-time (sec)**	
	CASE I	**CASE II**
1	11.8	69.8
2	4.0	68.6
3	4.1	72.2
4	4.2	75.8

4. Conclusions

Issues related to the implementation of DCIM have been first clarified, as it is used in conjunction with the MoM in the algorithm proposed in this paper. The algorithm, based on the DCIM-MoM technique, is assessed in terms of its accuracy and the efficiency in the analysis of printed geometries with multiple vertical conductors. It has been shown mathematically and numerically that, as long as the vertical dependencies of the basis or testing functions are chosen to be the same, the inclusion of additional vertical conductors is extremely efficient. Therefore, this approach seems to be a good candidate to use in conjunction with an optimization algorithm in a CAD tool.

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