The Imbedding Method in the Theory of Horn Array Antennas—Hypershort Impulses and the Near Fields

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Abstract—The problem of hyper short impulses distortion with horn array antennas radiation considers from spectrum analysis point of view. As main reason for misphasing of Fourier-components of field the collective effect resonances of horns overirradiaton were considered. The imbedding equations for transparent coefficients (field directional diagram) and reflection coefficients of linear HAA as functions of radiated field frequency have been build. Some results of numerical experiment are given and a part of near fields (inhomogeneous modes) was discussed.

1. Introduction

The usage of nanotechnologies in radiolocation has met some problems with distortion of hyper short (HS) impulses being radiated by horn array antennas (HAA). Qualitative explanation of this effect is connected with arising of reactive fields formed near antenna's system. But quantitative description based on the traditional methods meets serious difficulties. For correct description of radiation of ultra wide band (UWB) impulse process it's necessary to examine the internal problem of electrodynamics of HAA. Let's take into consideration that models usually used to describe narrow band signals radiation can't be considered adequate for UWB impulses.

Using the spectral method distortion of UWB signal during the radiation can be explained by misphasing and changing of its Fourier-component's amplitudes, arising in horn band. The latter can be considered as a transitional layer, matching waveguides with free space. If field in feeding waveguides $-E_{in}$ and in free space $-E_{out}$ is written in the mixed representation

$$E(\vec{q}, z; \omega) = \int d\vec{\rho} \cdot E(\vec{\rho}, z; \omega) \exp\{-i\vec{q}\vec{\rho}\}, \qquad \vec{\rho} = (x, y), \tag{1}$$

than the main characteristic of HAA—the transparence coefficient $T(\vec{q}, \vec{q}'; \omega)$ can be determent as a kernel of integral equation

$$E_{out}(\vec{q}, z; \omega)\big|_{z=H} = \int d\vec{q}' \; T(\vec{q}, \vec{q}'; \omega) \cdot E_{in}(\vec{q}', z; \omega)\Big|_{z=0} \tag{2}$$

Here H is thickness of the transition layer or horns height.

It's clear that when describing UWB impulse radiation in terms of spectral theory the demand to the measurement accuracy $T(\vec{q}, \vec{q}'; \omega)$ is much bigger than in the case of narrow band signal. In particular, the wide spectrum of the signal forces to take into consideration the group effects—i. e., overirradiation of horns in grating. This is usually neglected in narrowband field. Periodic property of grating space structure in combination with wide space signal spectrum leads to the fact that the definite group of frequencies inevitably lays in the field of Wood anomalies, where the important role is played by near fields—inhomogeneous modes of space spectrum of the radiation field.

Thus, the basic problem at the spectral approach to the solution of a problem on radiation of UWB-impulses by HAA consists in a choose of method allowing to solve the internal problem of HAA electrodynamics maximum correctly and to describe amplitude, phase and spatial vector of radiation of a monochromatic signal as function of its frequency. As such an approach it is proposed to use the imbedding method.

2. Imbedding Equations for Linear HAA

The imbedding method is used as base for getting the equation for transparent coefficient of HAA. The kernel of this method is in the following. A great number of solutions of similar problems is examined, these problems differ only with the value of one parameter—the imbedding parameter. In the considered case such a parameter is the height of the horn h—transparent layer thickness. The "utmost" solutions are: the field radiated by the system of the feeding waveguides ($h = 0$) and the field of researched HAA ($h = H$). Farther the solutions evolution equation is built in this functional space. Thus there can be established the connection between the solutions of the problems with corresponding different values of the parameter. The solution with one value of the imbedding parameter is relatively simple and is taken as known ($h = 0$). Than the solution of the researched problem ($h = H$) can be received as the solution of the Cauchy problem for imbedding equation (first order differential equation) with the initial condition as a solution of the problem at $h = 0$. Let's take into consideration that problems of waveguides radiation ($h = 0$) are rather simpler than problem of HAA radiation ($h = H$).

Thus, the transition from electrodynamics characteristics of waveguides' cut ($h = 0$) to the corresponding characteristics of horns can be seen in describing intermediate systems—the elements of the truncated horns family received one from another by increasing the height of the walls as it's shown in Fig. 1

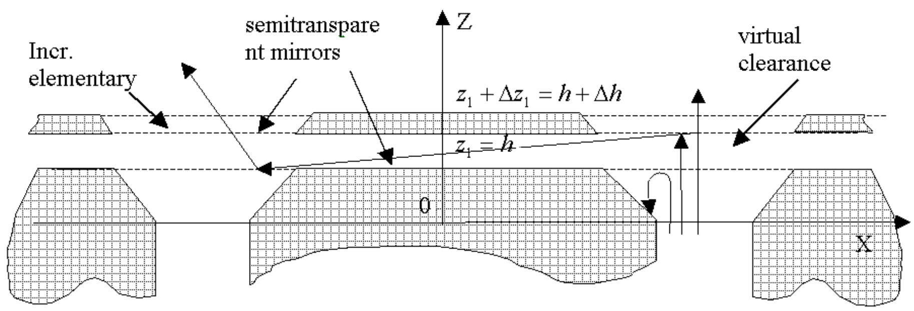

Figure 1: The evolution of horn layer under increasing the imbedding parameter.

To make the problem simpler let's use the method of periodical prolongation of the structure, i.e., let's add the researched HAA, consisting of N horns, with identical systems to the left and to the right to make it periodical structure. Under such a representation of horn grating its space spectrum of radiation becomes discrete. From the mathematical point of view it means that we change integral equations to matrix equations.

Farther it is necessary to express the transparence coefficient of "increased" HAA $T(h + \Delta h)$ in terms of $T(h)$, the reflection coefficient $r(h, \Delta h)$ and transparence coefficient $t(h, \Delta h)$ of the elementary layer.

According to the ideology of the works [1–2] the field being necessary for calculating $T(h+\Delta h)$ is considered in endless thin (virtual) clearance that divides the truncated horn of height h from the increased elementary layer. The clearance borders can be considered semitransparent mirrors with transparent coefficients $r(h, \Delta h)$ and $R(h)$. Here $R(h)$ it is a reflection coefficient and of truncated HAA of height h. Taking into consideration multiple reflections of field from the layer's borders the next equation [3] takes place

$$T(h + \Delta h) = [R(h + \Delta h) - r(h + \Delta h, \Delta h)] \cdot t^{-1}(h + \Delta h, \Delta h) \cdot R^{-1}(h) \cdot T(h), \tag{3}$$

written in finite difference.

Imbedding equation (3) is not closed, there is an unknown function $R(h)$ in it. The equation for reflection coefficient for truncated HAA can be received by variation of co-relations of integral equations method also known as MMM [4]. This method gave good results in the description of reflection from ideally conducting surfaces.

The distinctive part of the problem for HAA is the presence of waveguides—special insertions in ideally conducting surfaces. On these parts of the surface the Dirichlet condition doesn't take place that leads to essential complication of the method equation. Generalizing of method equations can be received knowing that the field in the spaces where Dirichlet condition doesn't take place can be represented as the superposition of waveguide's modes.

The equation for $R(h)$, evident view of which has being shown in [5] is a following matrix Riccati equation

$$\frac{1}{2i}\frac{d\hat{R}}{dz} = \hat{R}(\hat{I} - \hat{\hat{D}})\hat{V}^{-1} - \hat{R}\left[\hat{\hat{D}}(\hat{I} - \hat{H}\hat{V}) + (\hat{I} - \hat{\hat{D}})\hat{H}\hat{V}\right]\hat{V}^{-1}\hat{R} + (\hat{I} - \hat{H}\hat{V})\hat{V}^{-1}\hat{R} \tag{4}$$

Here $\hat{\hat{D}} = \hat{W}^{-`1}\hat{\hat{C}}^{-1}\hat{F}$, $\hat{H} = \hat{F}\hat{K}^{-1}\hat{W}^{-1}$, $\hat{K} = \hat{V}\hat{C} - \frac{1}{\Lambda}\hat{\mu}$. Matrix $\hat{\hat{C}}$ has the following components $\widetilde{C}_{kl} = \frac{1}{\Lambda}\int\limits_0^\Lambda e^{-i\frac{2\pi}{\Lambda}(k-l)x+i\nu_l h(x)}dx$, and matrix $\widetilde{C} - C_{kl} = \frac{1}{\Lambda}\int\limits_0^\Lambda e^{-i\frac{2\pi}{\Lambda}(k-l)x-i\nu_l h(x)}dx$, $h(x)$ is the form of a horn's profile, $A_{kn} = \int\limits_0^\Lambda dx \int\limits_{-\infty}^{\infty} dx' H_0^{(1)}(x, x', h(x), h(x')) \cdot e^{-iq_k x + iq_n x'}$, $\mu_{kn} = \frac{4}{b} \cdot \sum\limits_{p=1}^{\infty} \widetilde{\chi}_{kp}\frac{1}{\widetilde{\nu}_p}\chi_{kp}$, $\widetilde{\nu}_p = \sqrt{k_0^2 - \widetilde{q}_p^2}$, $\widetilde{q}_p = \frac{\pi}{b}p$, b is waveguide width, $\chi_{pn} = \int\limits_{\frac{\Lambda-b}{2}}^{\frac{\Lambda+b}{2}} \varphi_p(x - \frac{\Lambda - b}{2}) \cdot e^{iq_n x} dx$, $\varphi_p(\cdot)$ is p-th waveguide's mode, $\hat{\tilde{\chi}} = \hat{\chi}^{*T}$, $W_{mn} = e^{iv_n z_1} \cdot \delta_{nm}$, δ_{nm} is Kronecker's symbol, $\hat{I}$ is identity matrix. $F_{kn} = \frac{2}{\Lambda} x \cdot \sin c \left[\frac{2}{\Lambda}(k - n)x\right]$, matrix $\hat{V}$ is diagonal with elements $v_{pk} = \frac{2}{v_k} \cdot \delta_{kp}$.

As the initial condition for it serves the reflection coefficient of system of feeding waveguides, which could be found by using the mode-matching method.

3. A Physical Picture of Distortions of a UWB Signal at Radiation by HAA. Wood Resonance and Near Fields

Periodic expansion of HAA used in the stated approach allows not only to simplify a problem in mathematical aspect, but also to make more clear interpretation of destruction mechanism of the signal's form. It is known, that at interaction of a field with periodic structure there only components of a discrete spectrum are interconnected. In case of linear HAA it is possible to present a set of the wave vectors forming this spectrum, as $\vec{k}_n = (\nu_n, q_0 + nk)$, here $k = 2\pi/\Lambda$—is a vector of the inverse lattice, Λ is distance between the nearest radiators, $n \in Z$, $\nu_n = \sqrt{k^2 - (q_0 + nk)^2}$ and q_0 is a corresponding projection of allocated components of field angular spectrum. In case of the scattering problem, usually it is a projection of an external field's wave vector.

If frequency of a field $\omega = ck$ is such that one of its space components gets in area of Wood resonance $\nu_n \cong 0$, then anomalies are observed in distribution of a field on modes.

At radiation of the narrowband signal, carrying (central) frequency is chosen so that the condition $\lambda_0 > \Lambda \, (k_0 < \frac{2\pi}{\Lambda})$ is satisfied. In this case in a space spectrum of radiation only one mode is homogeneous (lateral petals in the directional diagram are absent). Thus all field modes, both homogeneous, and inhomogeneous, are far from Wood's resonance (Fig. 2 (a)). Therefore the problem of distortion of the form of the narrowband signal usually does not arise.

For a UWB signal the range of wave numbers change is great. It grasps a lot of resonant points (Fig. 2 (b)).

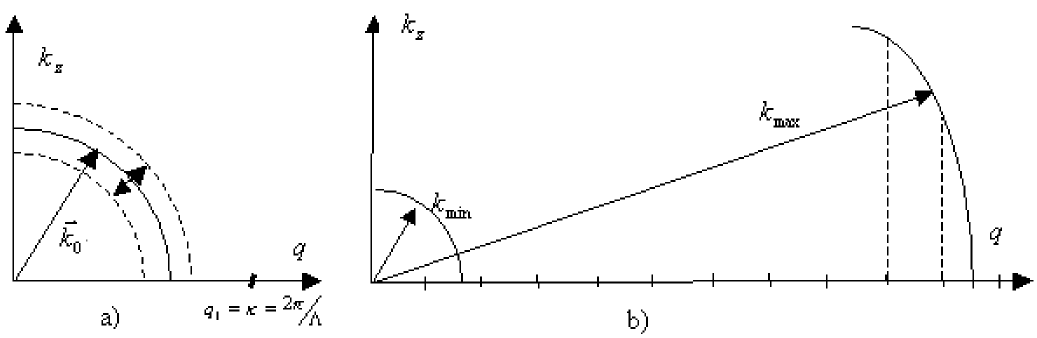

Figure 2: The range of wave numbers change for narrowband -(a) and UWB -(b) signal.

As follows from the formula (1), the transparency coefficient (the directional diagram) HAA is substantially determined by the feature of matrix reflection coefficient $R(h)$. Let's present its elements as

$$R_{n,m}(\omega) = |R_{n,m}(\omega)| \cdot \exp\{i\Phi_{n,m}(\omega)\}$$

The magnitude $\tau_{n,m}(\omega) = -\frac{d}{d\omega}\Phi_{n,m}(\omega)$ defines a group delay for n-th mode of a scattering field. The index m defines an external field wave vector $\nu_m = \sqrt{k^2 - (q_0 + k \cdot m)^2}$. If $\tau_{n,m}(\omega)$ varies with change of frequency then the output form of a signal most likely is distort. In other words, any deviation of frequency dependence $\tau_{n,m}(\omega)$ from the linear law must be analyzed.

On Fig. 3 diagrams of dependences $|R_{n,m}(k')|$ and $\Phi(k')$ are presented. They are calculated with the help of imbedding method represented for a case of normal falling $(q_0 + k \cdot m = 0)$ of an external field on the periodic surface modeling linear HAA.

Here wave parameter k' is a dimensionless wave vector $k' = k\Lambda/2\pi = \omega'$. Let's notice, that deviations from linear dependence near the values of parameter $k' = n$, $n \in Z$ corresponding to points of Wood's resonance, are observed.

Let's note also, that far from resonant points, the kind of dependence $\Phi = \Phi(\omega')$ can be counted linear, but in the different areas of a frequency spectrum separated by resonant values of parameter, the corner of

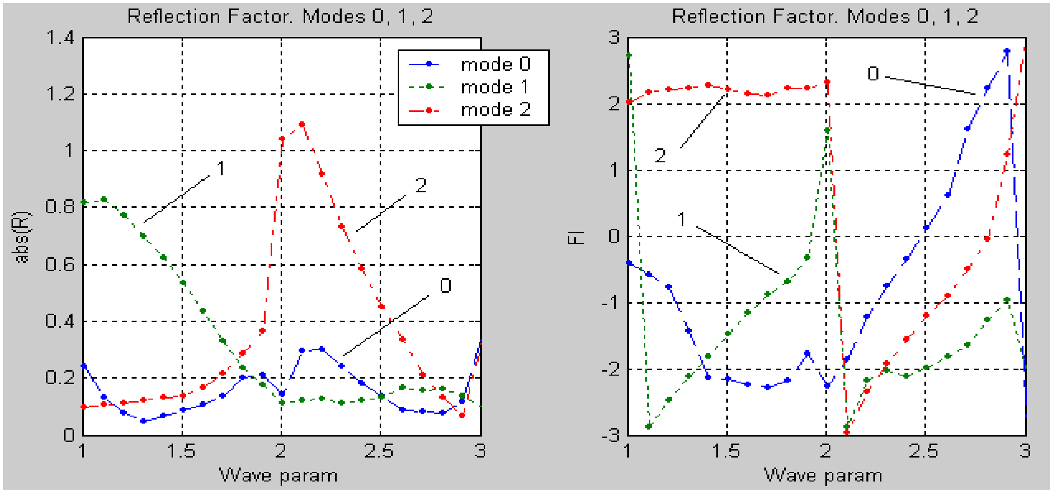

Figure 3: The diagrams of dependences $|R_{n,m}(k')|$ and $\Phi(k')$.

an inclination of curves essentially differs. As the spectrum of a UWB signal spans the big number of such areas even without taking into account Wood's anomalies dependence $\Theta = \Theta(\omega')$ can be approximated only by wiselinear, but not linear dependence. It also is necessary to take into account at the analysis of the reasons of the distortion of the form of radiated signal.

Complete results of the carried out numerical experiment will be submitted in the report.

4. Conclusion

The problem of ultra short impulses radiated by HAA is observed. From the spectrum analysis point of view impulse distortion depends on its Fourier components misphasing. To describe this effect the matrix transparence coefficient $\hat{T}(\omega)$ of horns layer is introduced as transitional layer that matches waveguides with free space. To calculate $\hat{T}(\omega)$ the imbedding equations were built. They allow considering horns overirradiation effects and borders effects that bound with its finite dimensions. Group delay variation that leads to signal disintegration can be represented as resonant interactions (Wood anomalies).

Reactive fields formed near antenna's system can be represented as superposition of inhomogeneous modes. The importance of near fields (inhomogeneous modes) grows sharply near the points of Wood resonant.

This quality summary were confirmed by diagrams of $R_{nm}(\omega)$ dependence that were calculated using imbedding equations describing external field interactions with periodical surface that models linear HAA.

REFERENCES

1. Barabanenkov, Y. N., V. L. Kuznetsov, and M. Y. Barabanenkov, "Progress in electromagnetic research," *PIER*, Vol. 24, 39–75, 1999.
2. Barabanenkov, Y. N. and V. L. Kuznetsov, *Radiotechnika I Elektronika*, Vol. 44, No. 6, 659, 1999 (in Russian).
3. Bakhrakh, L. D., V. L. Kuznetsov, and I. I. Vizgina, Antennas, No. 8–9, 42, 2004 (in Russian).
4. Masel, R. I., R. P. Merril, and W. H. Miller, Http:// Phys. Rev. B., Vol. 12, No. 12, 5545, 1975.
5. Bakhrakh, L. D., V. L. Kuznetsov, and I. I. Vizgina, *Nauchnii Westnik Mstuca, Series Mathematic and Physic*, No. 91, 48–62, 2005.