Power Absorption of Near Field of Elementary Radiators in Proximity of a Composite Layer

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Abstract—Near-field behavior of elementary electric and magnetic dipoles close to a plane layer (or layers) of engineered composite materials is analyzed using the rigorous analytical approach. Some results of computations are represented for composite media containing conductive inclusions. These composites provide shielding mainly due to absorption of electromagnetic energy. The effect of conductivity of inclusions and their geometry (through their aspect ratio) on the absorption and radiation efficiency of a radiator near composite layers is analyzed.

1. Introduction

The problems of studying electromagnetic interaction of different radiators with composite layered structures both in far- and near-field zones arise at the development of shielding enclosures for different electronic devices. In [1], the approach to engineering composites with the desired frequency response based on Maxwell Garnett (MG) formulation and a genetic algorithm is presented. An engineered infinitely large composite layer of finite thickness in [1] is considered for both normal and oblique incident plane waves. However, concepts of reflection and transmission coefficients, as well as of angles of incidence and polarization, are applicable only to the far-field region. In the near-field zone, it is better to consider field intensity attenuation due to such effects as excitation of evanescent waves, scattering, and different mechanisms of ohmic loss and energy transformation. In [2], the notions of absorption and radiation efficiencies in terms of power are introduced, and the corresponding power fluxes are calculated rigorously and explicitly via the spectra of the fields using the known solutions of boundary problems for parallel-plane, cylindrical, and spherical cases.

This paper considers the near-field behavior of elementary electric and magnetic dipoles close to a plane layer (or layers) of engineered composites, and the effect of conductivity of inclusions and their geometry (through the aspect ratio) on the absorption and radiation efficiency of a radiator near composite layers is studied.

2. Mathematical Model

2.1. Maxwell Garnett Formalism for Composites Containing Conductive Inclusions

The MG formulation is well-suited for modeling of linear electrodynamically isotropic multiphase mixtures of metallic or dielectric particles in a homogeneous dielectric base, where the parameters of the mixture do not change in time according to some law as a result of some external force—electrical, mechanical, etc.; inclusions are at the distances greater than their characteristic size; and the characteristic size of inclusions is small compared to the wavelength in the effective medium. The generalized MG mixing formula for multiphase mixtures with randomly oriented ellipsoidal inclusions is [1, 3],

$$\varepsilon_{eff} = \varepsilon_b + \frac{\frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{k=1}^3 \frac{\varepsilon_b}{\varepsilon_b + N_{ik}(\varepsilon_i - \varepsilon_b)}}{1 - \frac{1}{3} \sum_{i=1}^n f_i(\varepsilon_i - \varepsilon_b) \sum_{k=1}^3 \frac{N_{ik}}{\varepsilon_b + N_{ik}(\varepsilon_i - \varepsilon_b)}}$$
(1)

where $\varepsilon_b(j\omega) = \varepsilon_{\infty b} + \chi_b(j\omega)$ and $\varepsilon_i(j\omega) = \varepsilon_{\infty i} + \chi_i(j\omega)$ are the relative permittivity of the base and of the *i*-th type of inclusions, respectively. In (1), f_i is the volume fraction occupied by the inclusions of the *i*-th type; N_{ik} are the depolarization factors [4] of the *i*-th type of inclusions, where indices k = 1, 2, 3 corresponds to x, y, and z coordinates. If the inclusions are thin cylinders, their two depolarization factors are close to 1/2, and the third can be calculated as in [5], $N \approx (a)^{-2} \ln(a)$, where a = l/d is a cylinder's aspect ratio (length/diameter). Since the MG formula is linear, the resultant effective permittivity of the mixture can be also represented through effective high-frequency permittivity and susceptibility function,

$$\varepsilon_{eff}(j\omega) = \varepsilon_{\infty eff} + \chi_{eff}(j\omega). \tag{2}$$

If inclusions are conducting (metallic), their frequency characteristic in terms of relative permittivity is $\varepsilon_i(j\omega) = \varepsilon' - j\varepsilon'' = \varepsilon' - j\sigma/\omega\varepsilon_0.$ (3) The base material might be quite transparent over the frequency range where high shielding effectiveness is desirable. However, if there are conducting inclusions, the shielding effectiveness will be provided by absorption of electromagnetic energy due to conductivity loss and to the dimensional resonance in the particles. Presence of conductive particles will also increase reflection from the composite layer. In this paper, non-conductive composite materials (with dilute phase of conducting inclusions) are modeled. Non-conductive composites mainly absorb (rather than reflect) the energy of unwanted radiation. The effect of conductivity of inclusions and their geometry on the absorption and radiation efficiency of a radiator near composite layers is studied using the method described below.

2.2. Power Fluxes and Radiation and Absorption Efficiency in a Parallel-plane Structure

The near-field behavior of elementary radiators in proximity of a composite planar layer is studied using the unified rigorous analytical approach developed in [2,9]. Herein, this approach is specified for the parallel-plane geometry. Power radiation efficiency and absorption efficiency are calculated, using formulas similar to those introduced in [2],

$$\eta_{rad} = 10 \log_{10} [(P_{rad} - P_{loss})/P_{rad}] \text{ and } \eta_{abs} = 10 \log_{10} [P_{loss}/P_{rad}].$$
(4)

The radiated power P_{rad} and the power loss P_{los} are defined for a parallel-plane dielectric layer (see Figure 1):

$$P_{rad} = P_{Z1} + P_{Z2}; \quad P_{loss} = P_{Z1} - P_{Z3}.$$
(5)

The z-component of the Poynting vector in the parallel-plane geometry is

$$p_z = 0.5Re(E_x H_y^* - E_y H_x^*), \tag{6}$$



Figure 1: Parallel-plate geometry with a dielectric layer.

where $E_{x,y}$ and $H_{x,y}$ are the corresponding phasors for the tangential components of electric and magnetic field, and the asterisk stands for complex conjugating. The power through any cross-section S in the plane z is $P_z = \iint p_z dS$.

As is done in [10], the spectral densities $U^{e,m}$ and $I^{e,m}$ of scalar electric (e) and magnetic (m) potentials are introduced, and the expansion in terms of the complete system of eigenfunctions (Fourier representation) is applied. The scalar potentials $U^{e,m}$ and $I^{e,m}$ play part of the generalized voltages and currents, respectively, and they are used instead of the unknown field components. The potentials are obtained from the rigorous solution of the boundary problem, taking into account physical effects of diffraction, absorption, refraction, and numerous reflections. The tangential components of the electromagnetic field contain spatial spectra of the scalar potentials,

$$\vec{E}_{\tau} = \iint_{\chi_1\chi_2} (U^e \vec{t} + U^m \vec{f}) d\chi_1 d\chi_2; \quad \vec{H}_{\tau} = \iint_{\chi_1\chi_2} (I^e \vec{t} + I^m \vec{f}) d\chi_1 d\chi_2.$$
(7)

The complete system of vector eigenfunctions is

$$\vec{t} = (-j\chi_1 \vec{x}_0 - j\chi_2 \vec{y}_0)e^{-j\chi_1 x - j\chi_2 y}; \quad \vec{f} = (-j\chi_2 \vec{x}_0 + j\chi_1 \vec{y}_0)e^{-j\chi_1 x - j\chi_2 y}$$
(8)

Vectors \vec{x}_0 and \vec{y}_0 are the Cartesian unit vectors. Then, the power flux is



Figure 2: Complex permittivity of the composite: base material is Teflon ($\varepsilon' = 2.2$); aspect ratio for inclusions a = 1500; volumetric fraction of inclusions is 0.15%; conductivity of inclusions σ is a parameter.



Figure 3: Complex permittivity of the composite: base material is Teflon ($\varepsilon' = 2.2$); volumetric fraction of carbon inclusions is $0.7/a < p_c$; conductivity is $\sigma = 40000 \text{ S/m}^2$; aspect ratio *a* is a parameter.

$$P_{z} = 2\pi^{2} Re \iint_{\chi_{1}\chi_{2}} \chi^{2} (U^{e} I^{e*} + U^{m} I^{m*}) d\chi_{1} d\chi_{2}, \qquad (9)$$

where $\chi^2 = \chi_1^2 + \chi_2^2$, and $\chi_{1,2}$ are the spatial frequencies along x- and y-coordinates in Fourier representation for the field components. Substitution of the Fourier representation for the field components (7) into Maxwell's equations yields the 2-nd order differential equations for $U^{e,m}$ and $I^{e,m}$. In the cross-sections z_1 and z_2 , where the reflected waves exist, and inside the dielectric layers, the solutions for $U^{e,m}$ and $I^{e,m}$ are

$$U^{e,m} = U^{e,m}_{inc} \cdot e^{-\gamma z} + U^{e,m}_{refl} \cdot e^{+\gamma z}, \quad I^{e,m} = (U^{e,m}_{inc} \cdot e^{-\gamma z} - U^{e,m}_{refl} \cdot e^{+\gamma z})/Z^{e,m}, \tag{10}$$

where $\gamma^2 = \chi^2 - k_0^2$ is the square of the propagation constant, and $k_0 = \omega \sqrt{\mu_0 \varepsilon_0}$ is the wave number in free space. The characteristic impedance of the medium is $Z^{e,m}$. The scalar potentials $U_{in}^{e,m}$ and $U_r^{e,m}$ correspond to the incident and reflected waves, respectively. They are obtained as the coefficients of two linearly independent solutions for the boundary problem formulated for the one-dimensional Helmholtz equation (in z-direction). In the cross-section z_3 , there are no reflected waves, and the values $U_{refl}^{e,m}$ and $U_{refl}^{e,m}$ are zero. To calculate the power flux through the cross-section z_1 in a lossless medium, two cases should be considered: $|\chi| < k_0$, and $|\chi| > k_0$.

Figure 4: Shielding effectiveness (SE) in terms of plane wave formulation for an infinite layer of a composite material: (a) corresponding to Figure 2; (b) corresponding to Figure 3.

Figure 5: FDTD modeled power decrease through the composite layer. Field is radiated by the electric dipole placed at h = 5 mm below the layer (see Figure 1).

Case 1. When $|\chi| < k_0$, the propagation constant $\gamma = j\beta$ is imaginary in a lossless case, and the impedances $Z^e = \gamma/(j\omega\varepsilon_0)$ and $Z^m = j\omega\mu_0/\gamma$ are real, so the power flux for propagating waves is

$$P_{\rm z\,prop} = 2\pi^2 \iint_{\chi_1\chi_2} \chi^2 \Big[\Big(|U_{inc}^e|^2 - |U_{refl}^e|^2 \Big) / Z^e + \Big(|U_{inc}^m|^2 - |U_{refl}^m|^2 \Big) / Z^m \Big] d\chi_1\chi_2. \tag{11}$$

Case 2. When $|\chi| > k_0$, the propagation constant $\gamma = \beta$ is real, and the characteristic impedance $Z^{e,m} = jX^{e,m}$ is imaginary. The power flux for evanescent waves in this case is

$$P_{\rm z\,evan} = 4\pi^2 \iint_{\chi_1\chi_2} \chi^2 \Big[{\rm Im}(U^e_{inc}U^{e*}_{refl}) / X^e + {\rm Im}(U^m_{inc}U^{m*}_{refl}) / X^m \Big] d\chi_1 d\chi_2.$$
(12)

The exact expressions for the coefficients $U_{inc}^{e,m}$ and $U_{refl}^{e,m}$ are found from the solution of a boundary problem with the known volume densities for the source. Obviously, the power flux through the surface that crosses a medium without loss is independent of the z-coordinate, because the coefficients $U_{inc}^{e,m}$ and $U_{refl}^{e,m}$ are independent of the propagation z-coordinate. The total power flux (11), (12) is comprised of two terms: one is determined by the propagating waves ves with $\gamma = j\beta$, while the second is determined by evanescent waves with $\gamma = \beta$. Only for the regions where there are no reflected fields, $(U_{refl}^{e,m} \text{ and } I_{refl}^{e,m}$ are zero) the power flux is determined only by propagating waves. In general case, the propagation constant is complex. For multilayered structures, the cascading of transfer matrices can be used even for near fields, as is done in [9].

3. Computations

The frequency dependences for permittivity of the Teflon-based composites containing conductive fibers modeled using (1) are shown in Figures 2 and 3. The corresponding frequency dependences of shielding effectiveness (SE = $-20 \log_{10}(E_{tr}/E_{inc})$) defined in a plane-wave formulation for infinite plane panels made of these composites are presented in Figure 4. S.E. increases with the increase of conductivity and aspect ratio of inclusions. Figure 5 shows the rate of power decrease through the absorbing layer $\eta_{trans} = -10 \log_{10}(P_z/P_{ref})$. The results in Figure 5 are modeled using FDTD codes. The source is an elementary electric dipole parallel to the layer. The 20-mm thick layer is a Teflon-based ($\varepsilon_b = 2.2$) composite with conducting inclusions (a = 100; $\sigma = 40000 \text{ S/m}$; concentration is 0.7/a, below the percolation threshold). The reference plane for calculating P_{ref} is z = -1 mm.

Figures 6 and 7 show the dependences of the absorption coefficient (4) versus distance of the electric dipole from the composite layer for different frequencies, conductivities of inclusions, and their aspect ratio. The electric dipole is parallel to the layer surface. When the point of observation is in the far-field region, the absorption in composites increases with the increase of conductivity and aspect ratio of inclusions. In contrast to the far-field region, in the near-field zone the higher conductivity and higher aspect ratio do not necessarily lead to greater absorption. Absorption depends on the source type, distance between the source and the layer, and the effective constitutive parameters of the composite [2]. Trends of the curves in Figures 6 and 7 at varying a and σ are different for different frequencies. This can be explained by variations in frequency dependences of the effective parameters of composites.

Figure 6: Absorption coefficient versus distance h between the electric dipole and the composite layer (d = 1 mm); frequency is 0.1 GHz, 0.5 GHz, 3 GHz, and 9 GHz. Conductivity σ of inclusions is a parameter.

Figure 7: Absorption coefficient versus distance h between the elementary electric dipole and the composite layer (thickness d = 1 mm); frequency is 0.1 GHz, 1 GHz, 3 GHz, and 9 GHz. Aspect ratio a of inclusions is a parameter.

4. Conclusions

In this paper, the analytical formulas for absorption and radiation coefficients for radiators near a composite dielectric layer are obtained by rigorous boundary problem solution. The complex frequency-dependent permittivity of a composite dielectric containing conductive inclusions is modeled using Maxwell Garnett effective medium formulation. The results of computations for near-field of an elementary electric dipole close to a plane composite layer show that the behavior of absorption of near fields in the composite layer with respect to the conductivity and aspect ratio of inclusions is different from the far-field behavior. Near-field absorption in a layer depends on the distance of the radiator from the composite layer and the particular effective permittivity of the composite layer at the particular frequency.

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