Resonant Micro-strip Lines Analog to Electromagnetically Induced Transparency

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Abstract— Electromagnetically induced transparency (EIT) observed in three-level atomic vapor media is a coherent quantum process whose optical response to a laser beam is modified by a second beam with a well-determined detuning. In references, some classical analogs to EIT have been presented. We propose a microstrip transmission line structure to mimic the optical behavior of a three-level Lambda type EIT phenomenon. This research may enable us to understand how the quantum coherence works in real atomic vapor experiments.

1. INTRODUCTION

Quantum coherent control effects realized via atomic phase coherence, such as three-level Lambda-type electromagnetically induced transparency, have captured interest of many researchers over the last decades [1]. EIT is such a quantum optical phenomenon, in which a resonant laser beam (probe field) incident into a medium (e.g., an rubidium atomic vapor) will get absorbed (i.e., the medium is opaque), while if there is a second resonant laser beam (control field) simultaneously incident inside the same medium, neither would be absorbed, and the medium would become transparent. There are a number of phenomena that are relevant to EIT, including multi-photon population trapping [2], inversionless light amplification [3], phase coherence control [4, 5] and cancellation of spontaneous emission [6].

Here we shall address the intriguing optical behavior of a three-level EIT atomic vapor. Consider a Lambda-configuration three-level atomic system with two lower levels $|1\rangle$, $|2\rangle$ and one upper level $|3\rangle$ (see Figure 1 for the schematic diagram of the three-level EIT system). This atomic system interacts with the electric fields of the probe and control light waves, which drive the $|1\rangle$–$|3\rangle$ and $|2\rangle$–$|3\rangle$ transitions, respectively. Such a three-level system can be found in metallic alkali atoms (e.g., Na, K, and Rb) [7]. The density matrix elements of the three-level atomic system is given by [7]

$$\begin{aligned}\frac{d\rho_{31}}{dt} &= -\left(\Gamma_3 + i\Delta_p\right)\rho_{31} + \frac{i\Omega_c}{2}\rho_{21} + \frac{i\Omega_p}{2}\rho_{11},\\ \frac{d\rho_{21}}{dt} &= -\left[\gamma_2 + i\left(\Delta_p - \Delta_c\right)\right]\rho_{21} + \frac{i\Omega_c^*}{2}\rho_{31}.\end{aligned} \tag{1}$$

Here, Γ_3 and γ_2 stand for the spontaneous emission decay rate and the collisional dephasing rate, respectively. Ω_c and Ω_p denote the Rabi frequencies of the control field and the probe field. They are defined by $\Omega_c = \wp_{32}E_c/\hbar$ and $\Omega_p = \wp_{31}E_p/\hbar$ with E_c and E_p the slowly-varying amplitudes (envelopes) of the control field and the probe field, respectively. The frequency detunings of the two applied optical fields are defined as $\Delta_p = \omega_{31} - \omega_p$, $\Delta_c = \omega_{32} - \omega_c$ with ω_p and ω_c the mode frequencies of the probe field and the control field, respectively. In general, the population (ρ_{11}) in the ground level $|1\rangle$ is almost unity (i.e., $\rho_{11} \to 1$ and $\rho_{23} = \rho_{33} = 0$) [7].

We shall now concentrate our attention on the influence of the external control field on the three-level atomic population, which is analyzed based on the concepts of *quantum interference* and *dark states*. The three-level EIT occurs when the condition, $\Delta_p = \Delta_c$, of two-photon resonance is fulfilled. One can see from the second equation in (1) that the upper level $|3\rangle$ would be empty, i.e., the steady solution $\rho_{31} \to 0$. Thus, the two lower levels$|1\rangle$, $|2\rangle$ of the three-level system would form a dark state, i.e., $\Omega_c\rho_{21} + \Omega_p\rho_{11} = 0$ (destructive quantum interference). This, therefore, means that the probability amplitudes of levels $|1\rangle$ and $|2\rangle$ are $N\Omega_c$ and $-N\Omega_p$, respectively, and the dark

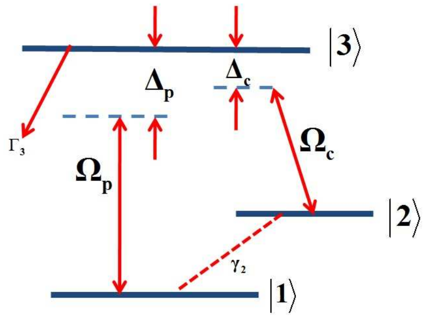

Figure 1: The schematic diagram of a three-level EIT atomic system. The control and probe laser beams drive the $|2\rangle$–$|3\rangle$ and $|1\rangle$–$|3\rangle$ transitions, respectively.

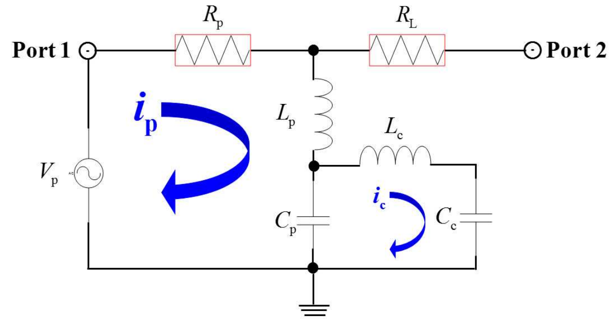

Figure 2: The circuit analog of the three-level quantum coherent effects. The circuit loop containing L_p, C_p corresponds to the $|1\rangle$–$|3\rangle$ transition pathway in the atomic system. The circuit loop on the bottom-right area is the analog of the $|2\rangle$–$|3\rangle$ atomic transition pathway.

state of the atomic system is given by $|\text{Dark}\rangle = N(\Omega_c|1\rangle - \Omega_p|2\rangle)$ with the normalized coefficient $N = 1/\sqrt{\Omega_c^*\Omega_c + \Omega_p^*\Omega_p}$. The minus sign in the dark state $|\text{Dark}\rangle$ represents an effect of destructive quantum interference between the two transition pathways $|1\rangle \to |3\rangle$ and $|2\rangle \to |3\rangle$. This leads to the effect of *electromagnetically induced transparency* to the probe field, since there is no effective interaction between the dark state and the applied fields (control and probe fields) [1], and hence no net transition from the ground level $|1\rangle$ to the upper level $|3\rangle$ occurs (i.e., $\rho_{31} \to 0$). Therefore, the multilevel atomic coherence (various quantum interference effects) can be controllably manipulated by adjusting the intensities (or the Rabi frequencies) of the control field.

EIT is a quantum coherent effect. In many classical systems, however, such an effect can be mimicked. Since a simple oscillating system (e.g., a RLC circuit and a spring oscillator) can simulate a two-level atomic system, and a coupled oscillating system (e.g., coupled circuits and double-spring oscillators) can exhibit some resonance behavior of three-level atomic system. In the literature, schemes based on optical and photonic scenarios, such as planar metamaterial analogue of EIT for plasmonic sensing, plasmonic analog of EIT, circuit analog of EIT as well as EIT-like effect in micromechanical resonators [8], have been suggested or experimentally demonstrated. Here we shall suggest a scenario of employing microstrip lines to mimic the EIT effect.

2. CIRCUIT ANALOG OF A THREE-LEVEL ATOMIC SYSTEM

Before we design the microstrip line analog of EIT, we shall address a circuit analog of the above quantum interference effect [8]. The dynamical behavior of the two-loop circuit (shown in Figure 2) is governed by

$$\begin{aligned}\frac{d^2 i_p}{dt^2} &= -\frac{R_p}{L_p}\frac{di_p}{dt} - \frac{1}{L_pC_p}i_p + \frac{1}{L_pC_p}i_c + \frac{1}{L_p}\frac{dV_p}{dt},\\ \frac{d^2 i_c}{dt^2} &= -\frac{1}{L_c}\left(\frac{1}{C_p}+\frac{1}{C_c}\right)i_c + \frac{1}{L_cC_p}i_p.\end{aligned} \tag{2}$$

It can be easily found that the mathematical structure of the circuit Equation (2) is analogous to that of the density matrix Equation (1) of the three-level atomic system: specifically, we can have the relations $i_p \to \rho_{31}$, $i_c \to \rho_{21}$, and $1/C_c \to \Omega_c$. The term of dV_p/dt, i.e., the voltage signal source, corresponds to the Rabi frequency, Ω_p, of the probe field. For the circuit analog of the two- or three-level atomic system as shown in Figure 2, one can conclude that the curve of the parameter $20\log|S_{21}|$ in the circuit analog of the two-level system (i.e., the simple $R_pL_pC_p$ circuit) has only one cuspidate point (a downward peak) indicating two-level resonant absorption, while in the circuit analog of three-level systems, there would be two downward cuspidate points [8]. These downward peaks can be interpreted as the "Autler-Townes effects". Since the circuit composed of two resonators (L_pC_p, L_cC_c) via capacitor coupling has one Autler-Townes doublet, there is only one EIT transparency window between the two downward cuspidate points in Autler-Townes doublet (corresponding to the circuit analog of the three-level system). This classical EIT-like RLC circuit can be viewed as the basis for designing the EIT-like microstrip lines.

3. IMPLEMENT USING TWO RESONANT MICROSTRIP LINES WITH RADIAL STUBS

The applied substrate is made up of GaAs whose relative dielectric constant is 12.9 and loss tangent is 0.0002 (See Figure 3). The microstrip line has width of 0.140473 mm, different length (in mm) for attenuation element (equivalent to a resistor, while its length is equal to half wavelength) and oscillator element (equivalent to an inductor, while its length is less than a quarter wavelength) for achieving 50 Ω characteristic impedance, 7.054 effective dielectric constant and 0.003 mm skin depth from the line surface when working at 2 GHz frequency.

The analog to the three-level system is set up by two resonant microstrip lines OSC1(See Fig-

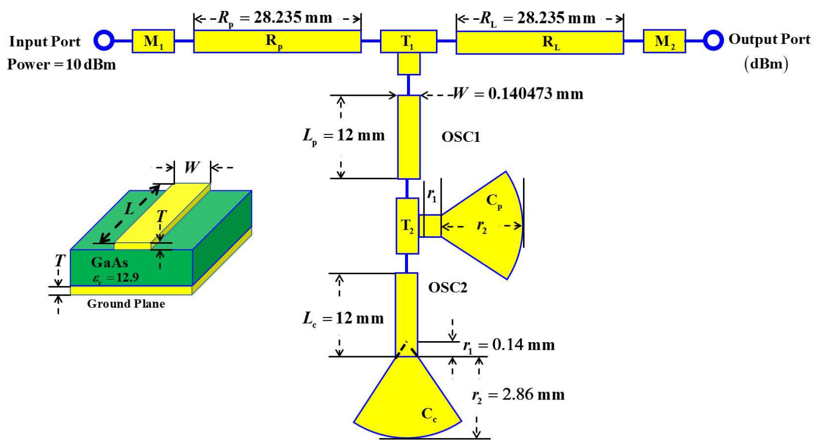

Figure 3: Scheme of EIT-like two resonant microstrip line structure. The upper one resonant microstrip line serves as the oscillator OSC1 and the lower one as the oscillator OSC2.

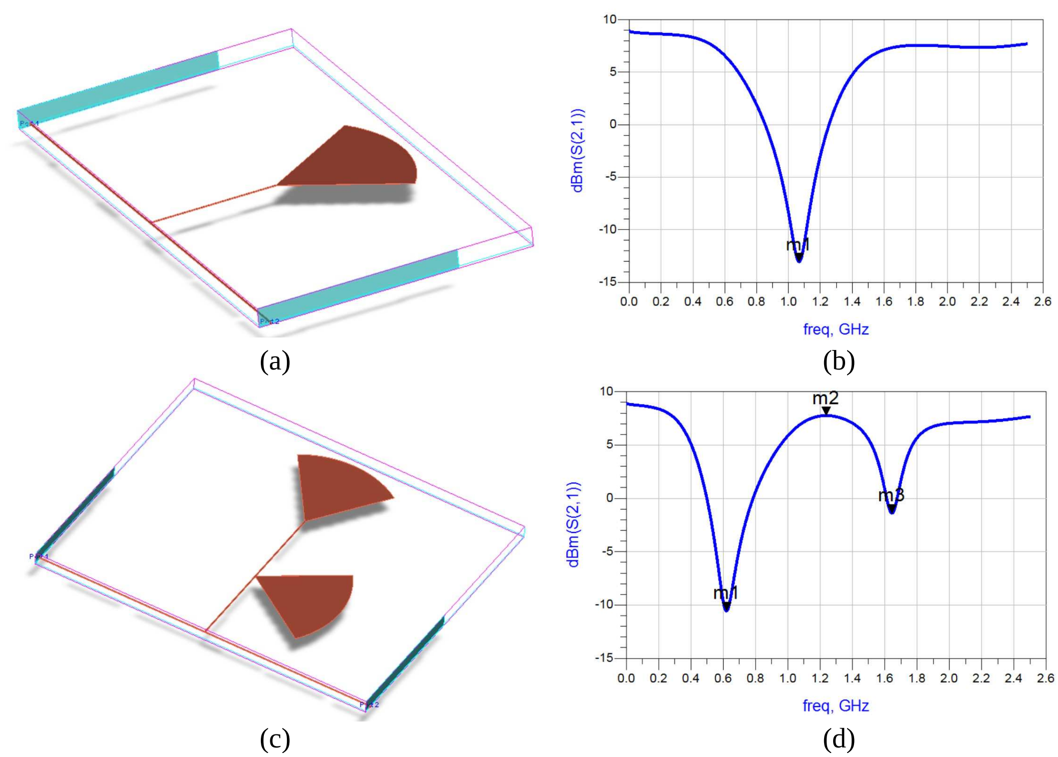

Figure 4: (a) OSC1 (an analog for a two-level atomic system). (b) A downward peak (1.07, −13) can be found in OSC1. (c) OSC1 and OSC2 are serially connected, performing as the analog for a three-level atomic system, in which OSC2 acts as a role of modifier of OSC1. (d) Two downward peaks $m_1 = (0.6, -10.5)$ and $m_3 = (1.7, -1.5)$ corresponding to the Autler-Townes doublet are found, and one upward peak (1.2, 7.8) corresponding to the transparency point can be acquired.

ure 4(a)) and OSC2 (See Figure 4(c), in which OSC2 has been serially connected with OSC1) using two fan-shape opened capacitive lines (i.e., two radial opened stubs). The first one resonator (i.e., OSC1) is configured by one straight microstrip line (i.e., L_p) and one series radial opened capacitance (i.e., C_p). The second resonator (i.e.,, OSC2) is configured by another straight microstrip line (i.e., L_c) and serially connected with another one radial opened capacitance (i.e., C_c). Combining the left-side-input port, the two resonant microstrip lines (located between the input loop and the output loop) and right-side-output port, the whole microstrip line circuit analog to EIT material is achieved. We point out that the left one straight microstrip line near the input port performs as a decaying factor (which corresponds to the spontaneous emission decay rate Γ_3 in the atomic system shown in Figure 1) and the right one (near the output port), of which the location is placed as the part-layout on microstrip line substrate for symmetry layout design with half-left amplitude, performs as attenuation factor in practice. At the input port, 10 dBm power (i.e., 10 milliWatt) is provided. Each quantity in this paper is all in dBm and acquired via s_{21} parameter for consistency and simplicity.

By adjusting the length to be the multiplication of $\lambda/2$ at the working frequency (i.e., 2 GHz), one can get an insertion loss performing as the spontaneous emission decay in the atomic system (i.e., $R_p \to \Gamma_3$). A section of micro-strip line with length scale smaller than a quarter wavelength (behaving as an inductor, L_p) is connected to a radial open stub with 3 mm axis length (behaving as a capacitor, C_p), and one can acquire a resonant microstrip line and the analog of the two-level atomic system (See Figure 4(a)). One can see a dip (downward peak, −13.048 dBm @ 1.067 GHz) corresponding to the largest absorption (See Figure 4(b)). For the three-level atomic system, two resonant microstrip lines are cascaded connected together (See Figure 4(c)). Thus, there are two downward peaks m_1 (−10.531 dBm @ 0.621 GHz) and m_3 (−1.514 dBm @ 1.657 GHz) corresponding to the Autler-Townes doublet and one upward peak m_2 (7.774 dBm @ 1.237 GHz) corresponding to the transparency point (See Figure 4(d)).

4. CONCLUSIONS

Two resonant microstrip lines can be designed for mimicking the EIT effect. One resistor line plays the role of the spontaneous emission decay of the excited state $|3\rangle$ in the three-level atomic system, two inductive lines and two capacitive radial stubs are utilized to realize two oscillators to act as the $|3\rangle$–$|1\rangle$ and $|3\rangle$–$|2\rangle$ transition frequencies, respectively. The higher quality factors of two resonators may make the transparency windows wider. Further relevant topics (such as quick intelligent floppy control on control field for EIT material or periodical layers of combination of dielectric and EIT material) will be investigated in the near future.

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