Dynamical Evolution of the Laser Linewidth at Switch-on

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Abstract— We investigate the dynamical evolution of the spectrum of a homogeneously broadened (semiconductor) laser switched from far below-threshold to above threshold. Four dynamical regimes are identified with spectral features which remain hidden in a standard detection based on the total laser output.

1. INTRODUCTION

Lasers have been studied for over fifty years and much work has been devoted to understanding and characterizing their main features: their narrow linewidth and threshold properties, thus also their efficiency. Although the first realization of a diode laser dates back to 1962 [1], semiconductor lasers have taken a leading role in the 1980's thanks to considerable technological advances and to their potential for telecommunications. These efforts have been well-rewarded and nowadays semiconductor lasers are ubiquitous both for their direct applications (e.g., telecommunications) and for their *support* role (e.g., pump lasers). Curiously, some of the questions we address in this paper do not seem to have received a large amount of attention, or have found only partial answers which can be tracked to separate papers published over the years.

Here, we present an overview of the questions pertaining the *average, macroscopic* evolution of the laser field intensity starting from below threshold until the equilibrium distribution of emission is reached above threshold, where equilibrium is to be intended as dynamically stationary both for the emitted power and for the spectral distribution. We achieve this goal by analyzing the growth of an ensemble of modes for a multimode laser, described by a standard rate-equation model [2], which are used to monitor at the same time the dynamics of the emitted power and its spectral distribution. Indeed, the analysis is carried out by considering a very large number of cavity modes, chosen to give a good sampling of the spontaneous linewidth (113 modes in the numerical simulations), and following their temporal evolution (and extinction for most of them) towards the above-threshold emission. In this paper, we present a physical discussion of the various aspects of the dynamics, leaving the laborious technical aspects of the analysis to papers already published or in progress.

2. FROM BELOW TO ABOVE THRESHOLD

We consider the transition from the average equilibrium distribution of radiation below threshold, which reflects the average spontaneous emission seeded into the lasing modes, to the equilibrium, above threshold emission — i.e., after all transients in the total emission and in the individual mode emission (thus, in the spectral distribution) have died out. As already mentioned, we use a dense and extended longitudinal cavity mode distribution to obtain a detailed picture of those components which eventually disappear when stable lasing occurs.

The transition is obtained by applying a sudden change (Heaviside function) to the pump rate in a standard rate-equation model [2]. Although some of the results we obtain may be somewhat model-dependent, it is well-established that rate equations provide a very robust underlying dynamical structure, thus reliable predictions. In the following, we will use interchangeably the concept of modal or spectral dynamics, since the former represents a discrete sampling of the latter and is the tool we use to efficiently follow the changes in the optical spectrum. However, here we will focus on the *spectral* aspects, while in the more technical papers we use the modal approach to quantitatively analyze the details of the dynamics.

To better place the problem into context, we are going to discuss the spectral evolution of the laser emission (curve a in Fig. 1(a)) compared to the standard picture of the transient, as provided by the output of a detector placed in front of the laser. Four different regimes of operation can be identified, as summarized in Fig. 1(a):

I. below transparency, the evolution of the spectrum (i.e., represented by the growth of the individual modes) is characterized by a slight broadening, rather than narrowing, due to the reduced amount of absorption in the wings (Fig. 1(b), curve a). Thus, the spectral line slightly deforms given that the wing modes experience a somewhat larger differential growth (Fig. 6 in [3]).

II. The differential gain (Fig. 6 in [3]) now privileges the central modes and the actual mode selection process *begins* to take place, however, in this region, the dynamics of the carrier density (i.e., population inversion in a generic laser) are decoupled from those of the modal intensity (i.e., the electromagnetic — e.m. — field emission) [4–8] and its evolution can be studied independently. The spectral emission (curve b in Fig. 1(b)) is slightly narrower than that of the spontaneous emission below threshold, but differs entirely from that of the true lasing action. What may appear counterintuitive, but can be numerically and analytically [3] proven to hold, is that this region extends to e.m. intensity values which are not negligible (Fig. 1(a)) and may amount to up to 10–20% of the peak value.

III. This region is characterized by a very strong growth of the field intensity and by the interdependent oscillations which take place between the total e.m. field emission and the carrier density (relaxation oscillations, cf. curves a and b in Fig. 1(a)). What is hidden by the traditional outlook based on the description of the total intensity growth, is the fact that the spectral width in this regime is still very large. Further, the intermodal dynamics, i.e., the spectral features, display an unexpected behaviour [3].

IV. When looking at the total emitted power, e.g., the signal coming from a detector, one concludes that the laser has reached steady state emission at the end of Region III and that this fourth region, which occupies a very considerable fraction of the transient evolution from below to above threshold (over 90% of the time interval) has no place in a dynamical description. This mistaken impression comes from the observation of the total intensity, rather than the modal intensities (or spectral features). Curve c in Fig. 1(a) shows the evolution of the central mode, i.e., the one which eventually dominates the emission. During the relaxation oscillations this mode carriers a fraction of the modal power which is well below its equilibrium contribution (e.g., compare peak emission in Fig. 1(a), curves a and c or spectral contributions at line

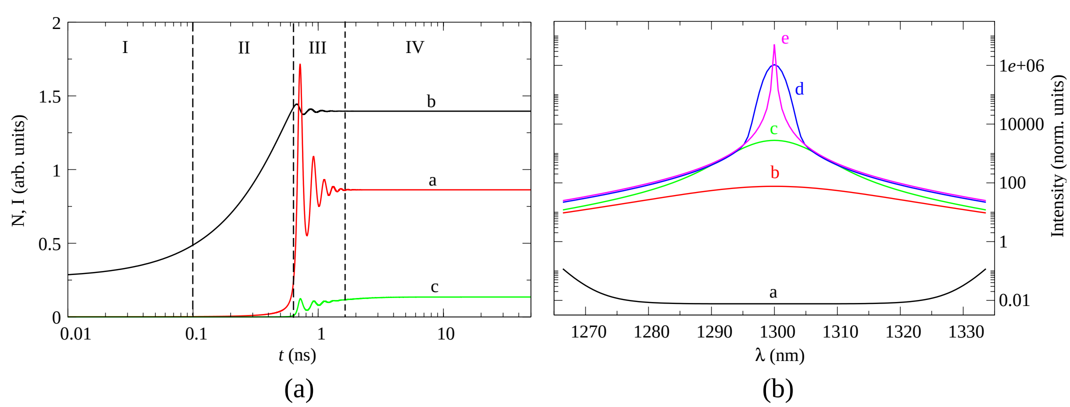

Figure 1: (a) Schematic representation of the four regimes which occur in the transition between the below-threshold and the above-threshold stationary spectral distributions when switching on a laser. Total intensity a, as measured by a detector placed in front of the laser, computed on the basis of a standard rate equation model [2]. A pump step is applied at time $t = 0$ to bring the laser from below to above threshold (parameters as in [9]). Curve b represents the carrier density (or population inversion), while curve c displays the temporal evolution of the central lasing mode. Notice the use of a logarithmic scale for time to better display the various regimes. For graphical purposes, both intensities a and c are divided by 10^5, while the carrier number b is divided by 10^8. (b) Evolution of the spectral distribution of the laser emission (representing the lineshape). Curve a is the initial, below threshold equilibrium distribution (lateral increase due to reduced off-resonance absorption in the modeling choice [2]). Spectrum b corresponds to the crossing of the stationary threshold (i.e., shortly before the end of Region II), while c is the spectrum at the intensity peak (curve a in (a)). Spectrum d corresponds to the end of Region III, i.e., at the end of the relaxation oscillations, while the stationary (narrow-line) spectrum e is attained only at the end of Region IV. For details cf. [3, 9].

center in Fig. 1(b), curves c and e). The evolution of the spectrum throughout Region IV is very slow but leads to a substantial change in lineshape (cf. curves d and e in Fig. 1(b)). This dynamical (apparent) *peculiarity* is thoroughly discussed in [9].

This description highlights the underlying complexity of the evolution which takes place between the initial, below threshold, equilibrium spectral distribution and the corresponding equilibrium state above threshold. The analysis is done using semiconductor laser parameters, but it qualitatively holds for any other kind of device with homogeneous emission linewidth.

Below, we offer some details about the physics characterizing each region. For Region I it suffices to realize that the, at first sight counterintuitive, broadening and deformation of the emission line is due to the (standard) modeling choices which give a larger differential increase of photons in the spectral wings of the intracavity laser field because of the reduced absorption below transparency.

3. REGION II

In the single-mode picture of any laser representation, one considers this region as the latency regime [10] of a Class B laser [11] where the carriers (or population inversion) accumulate at their own rate before attaining the threshold value. This simplified representation, although effective, does not provide information about the line reshaping. Direct numerical simulations, well supported by an analytical solution of the carrier density growth [3], show that the linewidth at the instant where the static threshold is crossed (i.e., when the carrier density dynamically crosses its asymptotic value for the first time) is still approximately half the initial linewidth (curve b in Fig. 1(b)). Thus, even though some linewidth reduction takes place due to the frequency-selective growth in the presence of the cavity feedback, its effect is largely insufficient to ensure the final spectral purity which is attained at steady state operation. This is not surprising, given that stimulated emission has not yet really taken hold. As already mentioned, the decoupled regime, where the stimulated emission follows the carrier density without appreaciably influencing it, extends to e.m. field intensity values well in excess of what one would reasonably consider a negligible perturbation (cf. total intensity value a at the transition between Regions II and III, compared to its asymptotic value in Region IV).

4. REGION III

From the point of view of a global description (i.e., the output measured by the detector) this is the crucial region, since most of the field intensity growth and its relaxation to the asymptotic state take place here. The quadrature oscillations between the total intensity (curve a, Fig. 1(a)) and the carrier density b, damped within a few cycles are characteristic of the turn-on of any Class B laser.

However, the spectral evolution shows a much more complicated picture. First, a nonnegligible ensemble of cavity modes participates in the peak. This is a direct consequence of the residual linewidth, but its existence is entirely hidden in the internal dynamics, which is not normally visible. A hint of the complexity of the problem can be had by comparing the evolution of the total intracavity field b to that of the central mode c. In the oscillatory regime, the central mode's peak does not even reach its asymptotic value (at $t = 20\,\mathrm{ns}$), while the total intensity has a peak which is double as large. This implies that other modes play a dominant role in the dynamics, with peak intensities which overcome their stationary values by more than a factor 2 (to compensate for the central mode(s) for which we observe a power defect).

This is indeed the case, as shown in [3]. Lateral modes, symmetrically placed a few units from line center ($\approx 4\,\mathrm{nm}$ far away) carry the largest peaks (Fig. 11 in [3]). The dynamics are therefore dominated by this set of modes which deform the transient spectral distribution of the e.m. radiation. This larger differential gain for the side modes is accompanied by a larger relaxation oscillation frequency and stronger damping (Fig. 12 in [3]).

Looking at the spectral distribution provides a complementary picture of the this discussion since a substantial amount of line reshaping takes place in this region. Curves b–d (Fig. 1(b)) show three different spectral distributions, at the beginning b, at the e.m. intensity peak c and at the end d of Region III, respectively. The emergence of the peak is clearly due to a differential growth of the central modes, without a strong domination of the centre one. Indeed, in the very short time interval between the start of Region III and the total intensity peak (a in Fig. 1(a)) the spectrum evolves from b to c (Fig. 1(b)) showing a wideband evolution of the intensity distribution. Interestingly, we notice that the spectral components located between 5 and 10 nm from linecenter (i.e., between 8 and 17 modes away from the central one) reach their steady state value already at the total intensity peak occurrence! During the relaxation oscillation phase the group of modes closer to line center (i.e., ± 7 modes) substantially grow (a few order of magnitudes!), however without any strong mode selection. The line shape which pops out of the "background" — bearing the approximate features of a very wide gaussian — vaguely resembles a gaussian (in semilogarithmic scale) with spectral width in the nanometer range. Notice that during the relaxation oscillations the far wing-modes keep growing until they reach their steady state contribution.

5. REGION IV

The observation of the total laser output (curve a, Fig. 1(a)) states that the transition is completed at the end of Region III and that the laser has reached its equilibrium above-threshold state. Comparison with the computed dynamics of the carrier density (or population inversion, in a generic laser) confirms this assertion b. Observation of the side modes [3, 9], instead, contradicts this statement since the modal dynamics is not yet complete. Curve c in Fig. 1(a) clearly shows that the central mode is steadily growing in Region IV (reaching its steady state at the end of the figure). For quantitative comparison, the vertical scale in the figure has been chosen to be the same for the two intensities and thus does not allow for a clear view of the central mode's growth. Notice also that, in order to better display the various regimes in a single figure, the time axis is plotted in logarithmic scale. Thus, the temporal dynamics appears compressed in its latter parts, while instead it occupies approximately 90% of the transient.

The deficit in power carried at line center, which amounts to saying that the line narrowing has not yet been completed at the beginning of Region IV, implies that the energy distribution is still far from its equilibrium value. Whence the existence of the residual dynamics which connects the *final* spectral distribution at the end of Region III to the asymptotic, equilibrium one typical of the laser lineshape. It is important to notice that, since the total intensity (curve a in Fig. 1(a)) shows a deficit, other modes must carry an excess of power at the beginning of the relaxation period, Region IV. This is indeed the case [9]. Inspection shows a close connection between the faster relaxation (cf. Figs. 11 and 12 of [3]), due to stronger growth and faster damping of selected side modes, and the amount of energy they carry at the end of the relaxation oscillations for the total intensity a. Thus, in Region IV. some modes decay away, while others (the central ones) grow towards their asymptotic value. The spectral picture, Fig. 1(b), confirms this analysis since it shows that strong line narrowing takes place in Region IV (compare curves d, at the beginning, and e at the end of the time interval), characterized by a substantial growth of the central mode and by a very strong decay of most of the modes which have contributed to the fast settling of the relaxation oscillations.

The slow growth, compared to the rest of the very fast dynamics, may appear at first to be quite puzzling, since it results from the laser response to the step-like increase of the pump rate to bring the laser from below to above threshold. One wonders why the laser should at first react fast and why the total e.m. energy settles to its equilibrium state in a time scale which is approximately one order of magnitude faster than the corresponding spectral content. After all, one would intuitively expect that, since the laser emission is dominated by one (or at most a few) mode(s), the temporal response of the ensemble should not differ too drastically from that of the individual mode(s) characterizing the laser performance above threshold.

An in-depth analysis shows that, within the framework of rate-equation models — giving in this context a rather accurate picture of reality —, the actual modal relaxation/growth rates structurally depend on the distance from line center through a combination of losses and gain (Section IV in [9] and Section III in [3]). This can be easily recognized by looking at the (approximate) structure of the dynamical equations for the modal intensities [3]:

$$\frac{dI_j}{dt} \approx [\alpha_j \mathcal{N}(t) - \mathcal{K}_j] I_j + B_j \mathcal{F}(\mathcal{N}(t)), \tag{1}$$

$$\mathcal{K}_j = \alpha_j N_0 + \frac{1}{\tau_p}; \quad B_j = \beta_j B; \quad \alpha_j = \Gamma g_p \left[1 - 2 \left(\frac{\lambda_j - \lambda_p}{\Delta\lambda_G} \right)^2 \right], \tag{2}$$

where I_j represents the intensity of the j-th mode, $\mathcal{N}$ the carrier density, α_j is the modal gain, $\mathcal{K}_j$ is the effective relaxation mode constant, B_j is the modal spontaneous emission coupling coefficient, $\mathcal{F}(\mathcal{N}(t))$ the (average) spontaneous emission factor, Γ the cavity confinement, g_p the gain, λ_j and λ_p represent the modal and peak wavelengths, respectively, $\Delta\lambda_G$ the linewidth, τ_p the cavity lifetime and β_j the fraction of spontaneous emission coupled into the j-th mode. Since spontaneous emission is added to each mode but varies only very slowly, the equilibrium condition — corresponding to $\frac{dI_j}{dt} = 0$ — implies that the bracket for the central mode (always a negative number) is the least negative (Table I in [9]) — i.e., the smallest number if absolute value —. Thus the corresponding time scale is the slowest. Since the slowest mode dominates the evolution and acts as a Master Mode [12], the slow convergence of the spectrum is thus explained.

6. CONCLUSION

In conclusion, with the help of a detailed numerical analysis we have shown that the transient from the initial to the final equilibrium states in a laser is characterized by several regimes with a dynamics far more complex than the one monitored by the simple output power. Surprising spectral features are identified in the various transient regimes.

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