Restoration of Antenna Patterns Using Iterative Method

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Abstract— A new approach to reconstructing antenna far-field patterns from the missing part of the pattern is presented in this paper. The antenna far-field pattern can be reconstructed by utilizing the iterative Hilbert transform, which is based on the relationship between the real and imaginary part of the Hilbert transform. A moving average filter is used to reduce the errors in the restored signal as well as the computation load. Under the constraint of the causality of the current source in space, we could successfully reconstruct the data. Several examples dealing with line source antennas and antenna arrays are simulated to illustrate the applicability of this approach.

1. INTRODUCTION

The Hilbert transform is a commonly used technique for relating the real and imaginary parts of a spectral response under causal conditions. It results from application of Cauchy's Residue Theorem in complex variable theory, and can be expressed in both continuous and discrete forms. The conventional reconstruction algorithm involves applying the Hilbert transform to the logarithmic function of the Fourier transform to obtain the unknown component [1]. Similarly, given the phase of a spectrum, it might be desirable to compute the magnitude function [2]. So far, Hilbert transforms have been widely applied in various areas of signal reconstruction, such as image reconstruction, minimum phase signal reconstruction and phase response reconstruction [3–5]. However, as far as the authors known, there have been no studies carried out on signal reconstruction in the missing block of an antenna far-field pattern signal.

In the area of microwaves, it is desired to obtain knowledge of the transient response of a system. It is common to measure both the magnitude and phase of spectral responses using a network analyzer [6–8]. It is important to appreciate that measured patterns are usually not perfectly symmetric and nulls are often partially filled. The real and imaginary parts of a spectral response are not independent of each other. The real part includes all the information of the imaginary part, and the imaginary part also includes all the information of the real part. In other words, both of them constitute the Hilbert transform pair, whose properties can be applied to any causal signal. Since we are dealing with a linear time-invariant (LTI) system, changing the input of the system through a time shift does not change the transfer function of the original system, and the amplitude spectrum is also unchanged. An antenna problem is equivalent to a shift in the spatial coordinates.

In this paper, we take the approach of developing iterative algorithms to reconstruct the loss or distortion of data from the phase of the antenna pattern spectrum, based on the above properties, i.e., the signal's correlation of the real and imaginary parts in the Hilbert transform. The remainder of this paper is organized as follows: the computational method of the iterative Hilbert transform is in Section 2. Section 3 introduces failed signal reconstructing data from the far-field power radiation pattern in the angular domain. Three design examples are given in Section 4, followed by the conclusion in Section 5.Text including equations must be typed single spaced in any of the following font types: Times, Times Roman, Times New Roman or Symbol. Use 10 pt for the text, 12 pt for the headings and 11 pt for subheadings, and 18 pt for the title. The title should be typed in capital letters and centred. The text should be set in two columns that are 8.8 cm wide and justified, and separated by a margin of 0.4 cm.

2. PRINCIPLE OF ITERATIVE HILBERT TRANSFORM

The signal that we consider to be real has a rational frequency response. Since we are interested in conditions under which the signal can be uniquely specified by the phase of its Fourier transform, the signal is assumed to be causal. For such a signal $x(t)$, where $t > 0$, the frequency response in the Fourier transform domain, $X(\omega)$, can be obtained by using the following Fourier transform:

$$X(\omega) = \int_0^t x(t)e^{-j\omega t}dt = \mathrm{Re}(X(\omega)) + j\mathrm{Im}(X(\omega)) \tag{1}$$

Once the real part $\mathrm{Re}(X(\omega))$ and imaginary part $\mathrm{Im}(X(\omega))$ are determined, the relations are verified in the frequency domain using the expressions:

$$\begin{aligned} \mathrm{Re}(X(\omega)) &= \frac{1}{2\pi}\left[j\mathrm{Im}(X(\omega)) * \frac{2}{j\omega}\right] = H[\mathrm{Im}(X(\omega))] \\ \mathrm{Im}[X(\omega)] &= \frac{1}{2\pi}\left[\mathrm{Re}(X(\omega)) * \frac{2}{j\omega}\right] = -H[\mathrm{Re}(X(\omega))] \end{aligned} \tag{2}$$

where * represents the convolution integral, and $H[.]$ is the Hilbert transform. We can note that one can reconstruct the real part of the spectrum from the imaginary part, and vice versa [1].

To restore the missing data one need to follow the steps shown in Fig. 1 where ˆ represents estimated data.

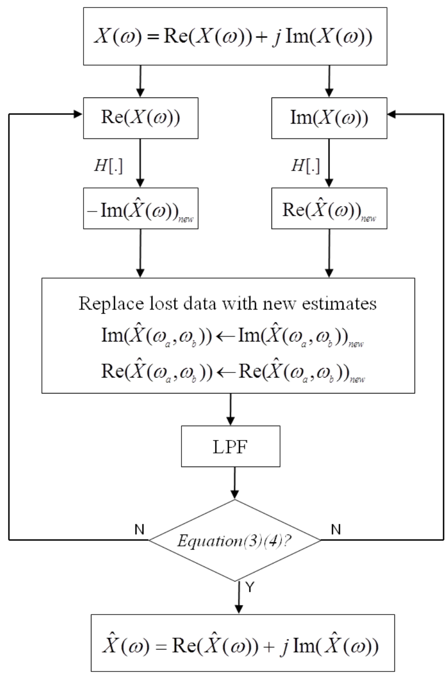


Figure 1: Flow chart of the iterative Hilbert transform.

After a number of iterations, the lost data will be reconstructed. However, there are some errors between the actual pattern and the restored pattern. The restored pattern not only has a high-frequency component but also requires a high computational load. To avoid this situation, a moving average filter is employed. The moving average filter is a linear filter, which is widely used for image enhancement [1, 9] and noise reduction in image processing. By using the MA filter at each recursive step, the fluctuating influence can be filtered out and the characteristics of these systems remain unchanged. In the meantime, the errors rapidly converge to a certain criteria, resulting in the computational load being reduced. The recursive step will not be stopped until (3) and (4) are satisfied, where $\varepsilon$is a small criterion. The average step size $k = 3$ is chosen in the following numerical simulations.

$$|R_c(\omega) - R_p(\omega)| < \varepsilon \tag{3}$$

$$|I_c(\omega) - I_p(\omega)| < \varepsilon \tag{4}$$

where R_c is the real part of the restored data at the current iteration step and R_p is the real part of the restored data at the previous step. The same applies to the imaginary part I_c and I_p.

3. SIGNAL RECONSTRUCTION IN A FAR FIELD PATTERN

The antenna radiation problem consists of solving for the fields that are created by an impressed current distribution J. In order to simplify the solution to solve the two Maxwell's curl equations, we use the potential function. As a result, by having a knowledge of the vector potential A one can solve the radiation problem. It is best if the radiated electric field of an electromagnetic system is proportional to the Fourier transform of the spatial current distribution on the electromagnetic structure. If we assume a current distribution only existing along the z-axis, centered on the origin and confine the observation point to a fixed φ in the y-z plane ($\varphi = 90°$), then the vector potential can be expressed as [10]:

$$\vec{A}_z = \hat{z}\frac{e^{-j\beta r}}{4\pi r}\int_{z1}^{z2} J_z(z)e^{j\beta z\cos\theta}dz = \hat{z}\frac{e^{-j\beta r}}{4\pi r}\int_{z1}^{z2} J_z(z)e^{j\beta zu}dz \tag{5}$$

where $\beta = 2\pi/\lambda$,$u = \cos\theta$, and r is the spatial far-field variable and the integral is over the extent of the current source. Then the far-field can be expressed as:

$$\vec{E} = -j\omega A_\theta\hat{\theta} = j\omega\sin\theta\vec{A}_z\hat{\theta} \ = j\omega\sin\theta\frac{e^{-j\beta r}}{4\pi r}\int_{z1}^{z2} J_z(z)e^{j\beta zu}dz\hat{\theta} \tag{6}$$

Equation (5) indicates that the vector potential A is proportional to the Fourier transform of the spatial current distribution with the variable changed from θ to u. For our proposed method, the vector potential A provides all the necessary information in the frequency domain as the difference between the vector potential A and far-field pattern is just a constant $j\omega\mu\sin\theta$.

4. NUMERICAL EXAMPLES

For the first example, as we stated in the previous section, the vector potential A can be obtained by dividing $\vec{E}$ by $\sin\theta$ and the constant $j\omega\mu$ from the far-field pattern. Therefore, we only need

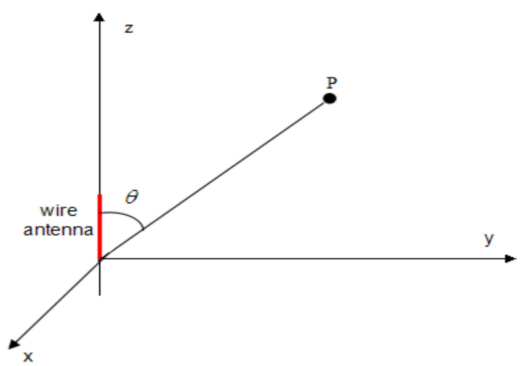
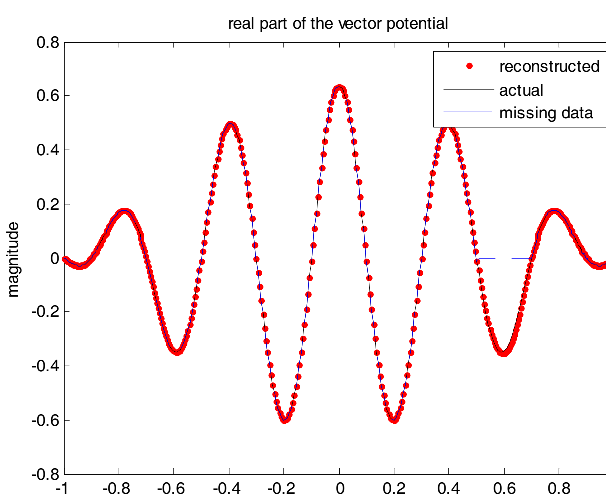
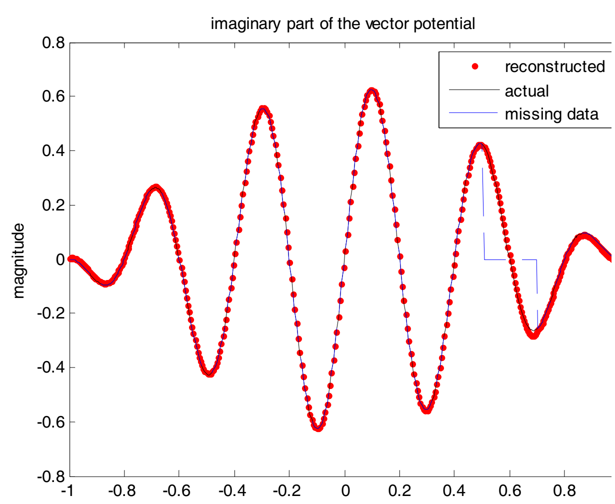


Figure 2: A line source along the z-axis.



(a)



(b)

Figure 3: (a) The real part of the original, reconstructed and missing data $A$with a filter. (b) The imaginary part of the original, reconstructed and missing data $A$with a filter.

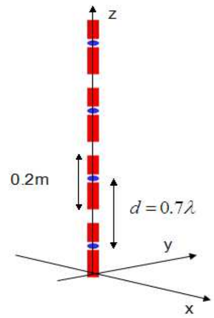


Figure 4: Straight wire antenna array.

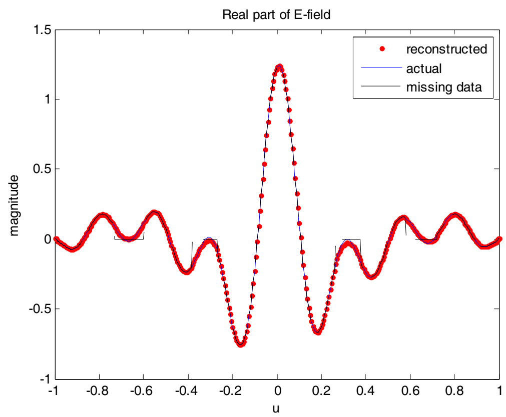


Figure 5: Real component of the actual, reconstructed and missing data.

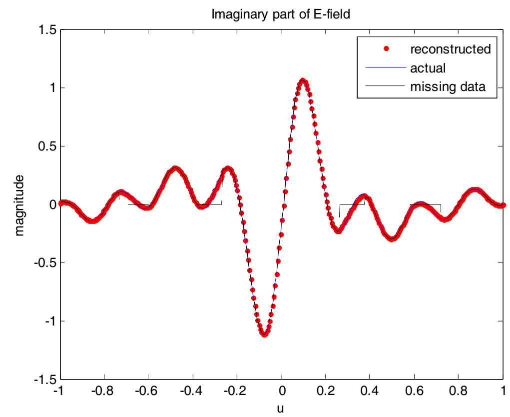


Figure 6: Imaginary component of the actual, reconstructed and missing data.

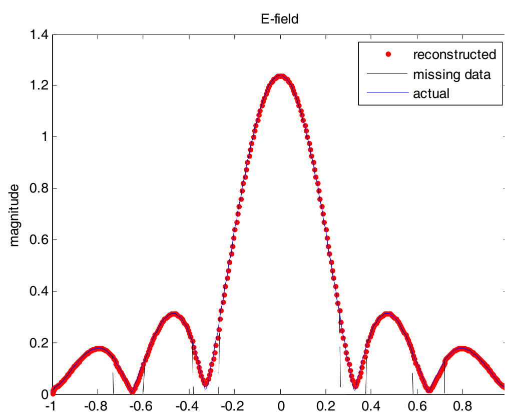


Figure 7: Far-field pattern of the actual, restored and missing data.

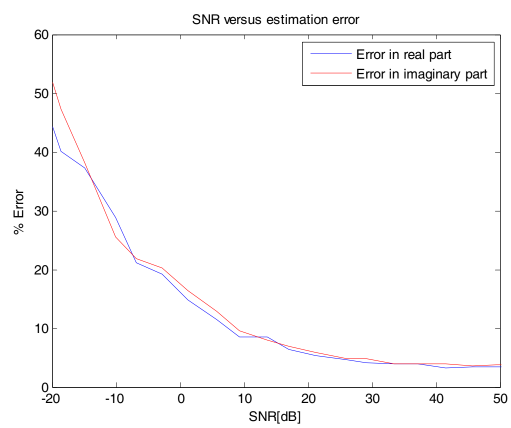


Figure 8: SNR and the estimation error.

to restore the pattern using the proposed method for vector potential. Here, we consider a short dipole as a straight wire antenna, shown in Fig. 2. The wire excited with $1V$ is oriented along the z-axis. The length of the wire antenna is $l = \lambda$. The far-field pattern has the variable changed with $u = \cos\theta$. We use an MA filter for a fast convergence. In addition, we can also see the actual data (solid line), restored data (dotted line), and missing data (dashed line). The missing data in the figures are reconstructed through recursive steps. The error is defined by the two norm of the difference between the actual data. The error is 10.8% and 12.8%, respectively. We can see that the restored signal fluctuates by more than 10%. In order to filter out these fluctuations, we use the moving average filter to reconstruct the missing data at each iterative step. The results are shown in Figs. 3(a) and 3(b) with errors of 4.7% and 5%, respectively. The step size of the MA filter was chosen as 3. For the second example, consider also the pattern from a straight wire antenna array. The structure of the antenna array is plotted in Fig. 4. There are 4 elements equally spaced along the z-direction. All of the elements are above the x-o-$y$plane. The length of each element is

$l = 0.2$ m, and diameter 2 mm. The distance between each array is $d = 0.6\,m$. The driven elements are fed at the center with $1V$ excitation at 350 MHz. The real and imaginary parts (solid line) generated the 512 point data shown in Fig. 5 and Fig. 6. Each part includes four blocks of missing data (dashed line), with the side of the block having 30 points of missing data and the middle of the block 20 points of missing data, for a total of 100 points. As is the case with Example 2, once there are more than 100 points of missing data, the reconstructed data will have larger distortion. This iterative algorithm with an MA filter is stopped after 110 iterations. The reconstructed data (dotted line) of the real and imaginary parts are also shown in Fig. 5 and Fig. 6, respectively. The errors associated with the estimated data and the actual data are 8.59%, 8.53%, 7.66% and 10.75% in Fig. 5. The errors associated with the estimated data and the actual data are 9.02%, 7.87%, 9.97% and 9.19% in Fig. 6. From Fig. 7 we can see that the far-field pattern is well reconstructed.

5. CONCLUSION

In this paper, we presented an iterative Hilbert transform method for reconstructing missing data from an antenna pattern, on the premises that the current source exists on z-axis, has the form of a Fourier transform. In all the numerical examples, this method effectively restored the lost fragments of the antenna pattern based on the signal's relationship to the real part and imaginary part. The MA filter representation is also utilized for better estimation of the data and more efficient computation. In addition, the rates of convergence depend on the amount of data missing. In practice, we have found that the missing data can be approximately 20% of the whole data in patterns and also can be distributed in multiple blocks of the signal at the same time. Finally, the iterative algorithms, as presented, can be used in more applications where causality plays an important role

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REFERENCES

1. Oppenheim, A. V. and R. W. Schafer, "Digital Signal Processing," *3nd. Englewood Cliffs*, Prentice-Hall, NJ, 1999.
2. Hayes, H. M., J. S. Lim, and A. V. Oppenheim, "Signal reconstruction from phase or magnitude," *IEEE Trans. on Acoustics, Speech and Signal Processing*, Vol. 28, No. 6, Dec. 1980.
3. Gabor, D., "Theory of communication," *Institute of Electrics Engineering*, Vol. 93, No. 3, 429–457, Nov. 1946.
4. Quatieri, T. F. and A. V. Oppenheim, "Iterative techniques for minimum phase signal reconstruction from phase or magnitude," *IEEE trans. on Acoustics, Speech and Signal Processing*, Vol. 29, No. 6, 1187–1193, Dec. 1981.
5. Carballar, A. and M. A. Muriel, "Phase Reconstruction from Reflectivity in Fiber Bragg Gratings" *IEEE Light wave Technology*, vol. 15, no. 8, pp. 1314-1322, Aug. 1997.
6. Guillemin, E. A., *Synthesis of Passive Networks*, Krieger, New York, 1977.
7. Kazemipour, A., X. Begaud, and D. Allal, "E-field measurement, accuracy and uncertainties," *ECEE*, 189–191, June 2007.
8. James, G. C., "Phase retrieval and near field antenna measurements," *Processing URSI Symp, Sydney, Australia*, 510–512, 1992.
9. Martens L. J. and A. N. Venetsanopoulos, "A comparison of adaptive order statistic filters using different images," *IEEE Systems Engineering*, 77–80, Aug 1989.
10. Stuzman, W. L. and G. A. Thiele, *Antenna Theory and Design*, John Wiley& Sons, 1981.
11. Kolundzija, B., et al., *WIPL-D: Electromagnetic Modeling of Composite Metallic and Dielectric Structures-Software and User's Manual*, Artech House, Norwood, MA, 2000.