Conditions for Negative Refraction and Negative Refractive Index in Lossy Media

Liming Ji and Vasundara V. Varadan
Microwave and Optics Laboratory of Imaging and Characterization, Department of Electrical Engineering
University of Arkansas, 700 W. Research Center Blvd., Fayetteville, Arkansas 72701, USA

Abstract— Negative refractive index (NRI) has been reported to be the requirement for negative refraction of electromagnetic waves in isotropic media. Such discussions have assumed that the material is lossless. Metamaterials have been used to create effective media with NRI by exciting plasmon-like resonances. In reality, there are no lossless metamaterials. In this paper, we derive the conditions for negative refraction of electromagnetic waves in isotropic and lossy media. The angles for phase refraction and energy refraction are different in lossy media. NRI is required for the negative refraction of phase but is not a requirement for achieving negative refraction of energy in lossy media. Negative permittivity or negative permeability is required for negative energy refraction at TM polarization and TE polarization, respectively. NRI is not necessary for negative energy refraction in lossy media.

1. INTRODUCTION

Negative refraction of electromagnetic waves has been studied since it was first envisaged by Veselago [1]. It was suggested that negative refraction could be employed to implement the superlensing effect, which has the potential to overcome the diffraction limit of conventional lenses [2–6]. For obliquely incident EM waves at the interface of two lossless materials, enforcing boundary conditions and using Snell's law can show that the angles of phase refraction and energy refraction are the same. Thus, if the second medium is a lossless NRI material, then NRI results in negative phase refraction and negative energy refraction [7, 8]. Experimental results have shown that man-made media with negative indices exhibit very high loss [9–11]. We may expect a different relation between NRI and negative refraction in isotropic media with loss added as a new unknown factor.

In this paper, a detailed theoretical analysis for negative refraction of waves in isotropic, lossy media was performed. This analysis can apply to media of any geometry. We demonstrate that NRI is not required for obtaining negative energy refraction in lossy isotropic media. The study follows the classic treatment of obtaining the transmitted wave for an obliquely incident TE or TM wave at a material interface where the second medium is assumed to be lossy [12]. Expressions were obtained for the angle of phase refraction and the angle of energy refraction of transmitted waves. As is well known, the two angles are different in lossy media [13]. Then, we discuss the conditions for NRI and negative refraction. We also show that TE polarization requires a negative permeability for negative energy refraction and TM polarization requires a negative permittivity for negative energy refraction. Negative phase refraction in lossy media needs NRI but negative energy refraction in lossy media does not require NRI. Three scenarios of wave refraction in a dispersive region are possible based on our theoretical study: (1) negative energy refraction occurs without negative refractive index provided either the permittivity or permeability is negative; (2) negative phase refraction (NRI) can occur without negative energy refraction; (3) both negative energy refraction and negative phase refraction can simultaneously occur in some frequency regions.

2. NEGATIVE REFRACTION FOR TM POLARIZATION IN LOSSY MEDIA

We consider the schematic as shown in Fig. 1. A plane wave is obliquely incident on a semi-infinite lossy medium (region 2) from a lossless medium (region 1). The incident angle is θ_1. ϕ and ψ are the refraction angles in region 2 for energy transmission and phase transmission, respectively. The refraction of phase and energy are treated as two different angles for a more general derivation. Starting from the positive z direction at 0°, the refraction angle, in our convention, increases to positive values when rotating clockwise and decreases to negative values when rotating counterclockwise. Negative phase refraction occurs when $90^\circ < \psi < 180^\circ$ and negative energy refraction occurs when $-90^\circ < \phi < 0^\circ$. The wave propagation numbers in regions 1 and 2 are:

$$\text{Region 1: } \gamma_1 = j\beta_1 \tag{1}$$

$$\text{Region 2: } \gamma_2 = \alpha_2 + j\beta_2 = j\sqrt{\mu\varepsilon} \tag{2}$$

where μ and ε are the complex permeability and permittivity in region 2. Since region 1 is lossless, its propagation coefficient does not have a real part. If region 2 exhibits NRI, then $\beta_2 < 0$.

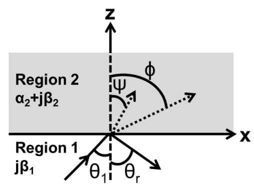

Figure 1: The schematic of a plane wave travelling through the interface of two semi-infinite media. Region 1 is lossless and region 2 is lossy. The incident angle is θ_1. The refraction angles for phase and energy, measured clockwise from the positive z-axis, are represented by ψ and ϕ, respectively. ψ can be in the first (0–90°) and fourth quadrants (−90°–0°). ϕ can be in the first and second (90–180°) quadrants.

An incident wave from region 1 reaches the boundary at $z = 0$. The wave is incident only in the x-z plane. It will be partially reflected back into region 1 and refracted into region 2. The fields in both regions can be written as follows:

Region 1:

$$\mathbf{E_i} = (\hat{\mathbf{x}}E_{0ix} - \hat{\mathbf{z}}E_{0iz})e^{-j\beta_{ix}x - j\beta_{iz}z} \tag{3}$$
$$\mathbf{H_i} = \hat{\mathbf{y}}H_{0i}e^{-j\beta_{ix}x - j\beta_{iz}z} \tag{4}$$
$$\mathbf{E_r} = (\hat{\mathbf{x}}E_{0rx} + \hat{\mathbf{z}}E_{0rz})e^{-j\beta_{rx}x - j\beta_{rz}z} \tag{5}$$
$$\mathbf{H_r} = \hat{\mathbf{y}}H_{0r}e^{-j\beta_{rx}x - j\beta_{rz}z} \tag{6}$$

Region 2:

$$\mathbf{E_t} = (\hat{\mathbf{x}}E_{0tx} - \hat{\mathbf{z}}E_{0tz})e^{-(\alpha_{tx}+j\beta_{tx})x - (\alpha_{tz}+j\beta_{tz})z} \tag{7}$$
$$\mathbf{H_t} = \hat{\mathbf{y}}H_{0t}e^{-(\alpha_{tx}+j\beta_{tx})x - (\alpha_{tz}+j\beta_{tz})z} \tag{8}$$

The subscripts "i", "r" and "t" represent incident, reflected and transmitted waves, respectively. The propagation coefficients in x and z directions in each region are related to each other:

Region 1:

$$\beta_{ix} = \beta_1 \sin\theta_1 \tag{9}$$
$$\beta_{iz} = \beta_1 \cos\theta_1 \tag{10}$$

where we have assumed that θ_1 is a real value in the range $0° < \theta_1 < 90°$.

Region 2:

$$(\alpha_{tx} + j\beta_{tx})^2 + (\alpha_{tz} + j\beta_{tz})^2 = (\alpha_2 + j\beta_2)^2 \tag{11}$$

In order to find the wave propagation coefficients in region 2, we apply boundary conditions at $z = 0$. The tangential field components on the two sides of the boundary are continuous.

$$E_{0ix}e^{-j\beta_{ix}x} + E_{0rx}e^{-j\beta_{rx}x} = E_{0tx}e^{-(\alpha_{tx}+j\beta_{tx})x} \tag{12}$$
$$H_{0i}e^{-j\beta_{ix}x} + H_{0r}e^{-j\beta_{rx}x} = H_{0t}e^{-(\alpha_{tx}+j\beta_{tx})x} \tag{13}$$

Equations (12) and (13) are valid for all values of x only when the exponential terms are the same (generalized Snell's Law):

$$j\beta_{ix} = j\beta_{rx} = \alpha_{tx} + j\beta_{tx} \tag{14}$$

Equating real and imaginary parts in Eq. (14), $\alpha_{tx} = 0$ for region 2. Hence, we can rewrite Eq. (7) and Eq. (8):

$$\mathbf{E_t} = (\hat{\mathbf{x}}E_{0tx} - \hat{\mathbf{z}}E_{0tz})e^{-\alpha_{tz}z - (j\beta_{ix}x + j\beta_{tz}z)} \tag{15}$$
$$\mathbf{H_t} = \hat{\mathbf{y}}H_{0t}e^{-\alpha_{tz}z - (j\beta_{ix}x + j\beta_{tz}z)} \tag{16}$$

The coefficients β_{tz} and α_{tz} are obtained from Eq. (11):

$$(\alpha_{tz} + j\beta_{tz})^2 = \beta_1^2 \sin^2\theta_1 + (\alpha_2 + j\beta_2)^2 \tag{17}$$

We can have the following equation by equating the imaginary parts on the two sides of Eq. (17),

$$\alpha_{tz}\beta_{tz} = \alpha_2\beta_2 \tag{18}$$

Equation (18) is valid only when region 1 is lossless and region 2 is lossy. $\alpha_{tz} > 0$ is enforced because a passive medium cannot generate energy. We find the planes of constant phase defined by

$$\beta_{ix}x + \beta_{tz}z = C \tag{19}$$

where C is a constant. The direction of phase transmission is perpendicular to the plane of constant phase. Hence, the phase refraction angle is

$$\tan\psi_{TM} = \frac{\beta_{ix}}{\beta_{tz}} = \frac{\beta_1 \sin\theta_1}{\beta_{tz}} = \frac{\alpha_{tz}\beta_1\sin\theta_1}{\alpha_2\beta_2} \tag{20}$$

The direction of energy transmission in region 2 can be obtained from the Poynting vector. After deriving the transmitted electric field from Maxwell's equations, we have the Poynting vector in region 2:

$$\mathbf{P_{TM}} = \frac{1}{2}\left\{\hat{\mathbf{z}}\left[\frac{\beta_{tz}\varepsilon' + \alpha_{tz}\varepsilon''}{\omega(\varepsilon'^2 + \varepsilon''^2)}\right] + \hat{\mathbf{x}}\frac{\beta_{ix}\varepsilon'}{\omega(\varepsilon'^2 + \varepsilon''^2)}\right\}|H_{0t}|^2 e^{-2\alpha_{tz}z} \tag{21}$$

Combined with Eq. (20), the angle of energy transmission in region 2 is:

$$\tan\phi_{TM} = \frac{\varepsilon'\beta_1\sin\theta_1}{\beta_{tz}\varepsilon' + \alpha_{tz}\varepsilon''} = \frac{\varepsilon'\alpha_{tz}\beta_1\sin\theta_1}{\varepsilon'\alpha_2\beta_2 + \alpha_{tz}^2\varepsilon''} \tag{22}$$

The angles for phase refraction and energy refraction for TE polarization can be derived in a similar way:

$$\tan\psi_{TE} = \frac{\beta_1\sin\theta_1}{\beta_{tz}} = \frac{\alpha_{tz}\beta_1\sin\theta_1}{\alpha_2\beta_2} \tag{23}$$

$$\tan\phi_{TE} = \frac{\beta_1\sin\theta_1\mu'}{\beta_{tz}\mu' + \alpha_{tz}\mu''} = \frac{\mu'\alpha_{tz}\beta_1\sin\theta_1}{\mu'\alpha_2\beta_2 + \alpha_{tz}^2\mu''} \tag{24}$$

Instead of depending on the permittivity as for TM polarization, the angle of energy refraction for TE polarization is dependent on the permeability of region 2.

3. DISCUSSION

We analyze the conditions for negative refraction at TE polarization. The conditions for negative refraction at TM polarization can be obtained by simply replacing permeability with permittivity. The following parameters are always non-negative: α_{tz}, β_1, $\sin\theta_1$, α_2 and μ''. We require $\beta_2 < 0$ in region 2 for ψ_{TE} >90°, which indicates negative phase refraction,. The real part of the refractive index has the same sign as β_2. Hence, NRI is necessary and is the only requirement to implement negative phase refraction. However, NRI is not necessary to implement negative energy refraction in region 2. Negative energy refraction in region 2 requires a negative numerator simultaneously with a positive denominator in Eq. (24). The conditions for negative phase and energy refraction at TE polarization in region 2 are:

$$\text{Negative Phase Refraction (TE): } \beta_2 < 0 \tag{25}$$

$$\text{Negative Energy Refraction (TE): } \mu' < 0 \tag{26}$$

$$\beta_2 < -\frac{\alpha_{tz}^2\mu''}{\mu'\alpha_2} \tag{27}$$

Similar conditions are found for TM polarization according to Eq. (20) and Eq. (24). The conditions for negative phase refraction and negative energy refraction in region 2 at TM polarization are:

$$\text{Negative Phase Refraction (TM): } \beta_2 < 0 \tag{28}$$

$$\text{Negative Energy Refraction (TM): } \varepsilon' < 0 \tag{29}$$

Table 1: The required conditions for negative phase refraction and negative energy refraction of waves travelling from a lossless medium into a lossy medium.

	Polarization	ε'	μ'	β_2 $(\omega n_2')$
Negative Phase Refraction	TE	$\mu'\varepsilon'' + \varepsilon'\mu'' < 0$		$\beta_2 < 0$
	TM			
Negative Energy Refraction	TE		$\mu' < 0$	$\beta_2 < -\frac{\alpha_{tz}^2 \mu''}{\mu' \alpha_2}$
	TM	$\varepsilon' < 0$		$\beta_2 < -\frac{\alpha_{tz}^2 \varepsilon''}{\varepsilon' \alpha_2}$

$$\beta_2 < -\frac{\alpha_{tz}^2 \varepsilon''}{\varepsilon' \alpha_2} \tag{30}$$

All the required conditions for negative refraction are summarized in Table 1.

When negative energy refraction occurs in each polarization, β_2 (same sign as the refractive index in region 2) should be less than a positive value. Hence, it can be negative or positive to realize negative energy refraction.

We listed the three possibilities for negative refraction in Fig. 2 based on the above derivations. Positive energy refraction and positive phase refraction (normal refraction) occur if their refraction vectors fall in the first quadrant. Negative energy refraction will occur if the energy refraction vector falls in the second quadrant ($-90° < \Phi < 0°$). Negative phase refraction will occur if the phase refraction vector falls in the fourth quadrant ($90° < \psi < 180°$). The energy refraction vector cannot fall into the third and the fourth quadrants to exhibit backward energy refraction. Similarly, the phase refraction vector cannot fall into the second and the third quadrants because that will violate Snell's law.

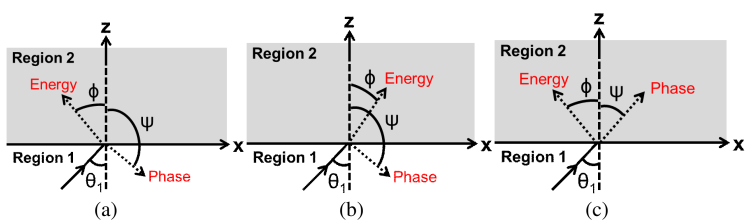

Figure 2: Schematics showing three possibilities for negative refraction. All angles are measured from the positive z-axis. If $90° < \psi < 180°$(fourth quadrant), phase refraction is negative; if $-90° < \Phi < 0°$ (second quadrant), then energy refraction is negative. (a) Negative energy refraction and negative phase refraction; (b) Positive energy refraction and negative phase refraction; (c) Negative energy refraction and positive phase refraction.

4. CONCLUSION

We have derived the conditions for negative refraction of phase and energy in lossy media in this paper. The incident and refraction angles determined by Snell's law for lossless media do not apply in lossy media. The refraction angles of phase and energy are different in lossy media. Negative phase refraction is independent of polarization but negative energy refraction depends on the polarization of incident waves. Negative permittivity is required to realize negative energy refraction at TM polarization. Similarly, negative permeability is required to realize negative energy refraction at TE polarization. Based on our derivation, NRI is required for the negative refraction of phase in lossy media. However, it is not required for the negative refraction of energy. Hence, there are three possibilities for refraction: (a) simultaneous negative energy refraction and negative phase refraction, (b) simultaneous positive energy refraction and negative phase refraction and (c) simultaneous negative energy refraction and positive phase refraction.

REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Sov. Phys. Uspe.*, Vol. 10, No. 4, 509–514, July 1968.

2. Pendry, J. B., "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, October 2000.
3. Zhang, X. and Z. Liu, "Superlens to overcome the diffraction limit," *Nature Mater.*, Vol. 7, 435–441, June 2008.
4. Aydin, K., I. Bulu and E. Ozbay, "Subwavelength resolution with a negative-index metamaterial superlens," *Appl. Phys. Lett.*, Vol. 90, No. 254102, June 2007.
5. Fang, N. and X. Zhang, "Imaging properties of a metamaterial superlens," *Appl. Phys. Lett.*, Vol. 82, No. 2, 161–163, January 2003.
6. Iyer, A. K. and G. V. Eleftheriades, "Free-space imaging beyond the diffraction limit using a veselago-pendry transmission-line metamaterial superlens," *IEEE Trans. Antennas Propag.*, Vol. 57, No. 6, 1720–1727, June 2009.
7. Smith, D. R., J. B. Pendry, and M. C. K. Wiltshire, "Metamaterials and negative refractive index," *Science*, Vol. 305, 788–792, Aug. 2004.
8. Ziolkowski, R. W. and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phys. Rev. E*, Vol. 64, No. 056625, 1–15, October 2001.
9. Varadan, V. V. and A. R. Tellakula, "Effective properties of split-ring resonator metamaterials using measured scattering parameters: Effect of gap orientation," *J. Appl. Phys.*, Vol. 100, No. 034910, 1–8, August 2006.
10. Varadan, V. V. and L. Ji, "Temperature dependence of resonances in metamaterials," *IEEE Trans. Microw. Theory Tech.*, Vol. 58, No. 10, 2673–2681, October 2010.
11. Shelby, R. A., D. R. Smith, S. Schultz, "Experimental veripcation of a negative index of refraction," *Science*, Vol. 292, No. 6, 77–79, April 2001.
12. Varadan, V. V. and L. Ji, "Does a negative refractive index always result in negative refraction? — Effect of loss," *IEEE MTT-S International Microwave Symposium Digest*, 61–64, June 2009.
13. Holmes, J. J. and C. A. Balanis, "Refraction of a uniform plane wave incident on a plane boundary between two lossy media," *IEEE Trans. Antennas Propag.*, Vol. 26, No. 5, 738–741, September 1978.