Compensation of Disturbed Load Currents Using Active Power Filter and Generalized Non-active Power Theory

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Abstract— The principal and analysis of usage of the generalized non-active power theory for the parallel compensation of periodic and non-periodic load current disturbances is presented in the paper. The effectiveness of the generalized non-active power theory to mitigate non-periodic current disturbances is demonstrated by simulation and laboratory experiments.

1. INTRODUCTION

Today, the compensation of subharmonic, non-periodic, and stochastic disturbances in the electric power systems has emerged as a very serious problem to be solved.

The original Fryze's theory of the non-active current may be extended using a general averaging time interval T_C that can be chosen either one-half fundamental cycle, one full fundamental cycle T, that is the same as that in Fryze's theory, or multiple fundamental cycles, depending on the character of the load current, compensation objectives and the energy storage capacity of a power electronics-based compensator.

If the current $\mathbf{i}(t)$ is the current of an unbalanced load, which may contain, in general, harmonic and also non-periodic or stochastic components, and the active current $\mathbf{i}_a(t)$ is the demanded source current, then $\mathbf{i}_n(t) = \mathbf{i}(t) - \mathbf{i}_a(t)$ is the non-active current that should be injected by a parallel compensator connected to the PCC (Point of Common Coupling).

By choosing $T_C \to 0$, the GNP (Generalized Non-active Power) theory gives the same results as those obtained by using the well-known IRP (Instantaneous Reactive Power) theory. If $T_C \to \infty$, the non-active component in the non-periodic load current is completely eliminated, so the source current is only active.

A principal and analysis of usage of this GNP theory for the parallel compensation of periodic and non-periodic disturbances is presented in the paper. A non-linear unbalanced load generating periodic as well as non-periodic currents connected to an unsymmetrical non-sinusoidal voltage source is considered.

2. GENERALIZED NON-ACTIVE POWER THEORY

For the three-phase system, the voltage and current vectors may be expressed as follows

$$\mathbf{v} = [v_a,\, v_b,\, v_c]^T \quad \mathbf{i} = [i_a,\, i_b,\, i_c]^T \tag{1}$$

The active current is defined in the GNP theory [1] by

$$\mathbf{i}_a(t) = \frac{P(t)}{V_P^2(t)}\mathbf{v}_P(t) = Y(t)\mathbf{v}_P(t) \tag{2}$$

$$P(t) = \frac{1}{T_C}\int_{t-T_C}^{t} \mathbf{v}^T(\tau)\mathbf{i}(\tau)\mathbf{d}\tau \tag{3}$$

$$V_P^2(t) = \frac{1}{T_C}\int_{t-T_C}^{t} \mathbf{v}_P^T(\tau)\mathbf{v}_P(\tau)\mathbf{d}\tau \tag{4}$$

where $v_P(t)$ is the reference voltage vector.

The non-active current is the remaining part of the current vector

$$\mathbf{i}_n(t) = \mathbf{i}(t) - \mathbf{i}_a(t) \tag{5}$$

If the current $\mathbf{i}(t)$ is the current of an unbalanced load and the current $\mathbf{i}_a(t)$ is the demanded source current, then $\mathbf{i}_n(t)$ is the current that should be injected by a parallel compensator connected to the PCC.

3. ANALYSIS OF GNP THEORY

It was shown [1–3] that the precision of the calculation of the currents $\mathbf{i}_a$, $\mathbf{i}_n$ by using (2), (5) in unbalanced non-sinusoidal systems with non-periodic load currents is sufficient if T_C is chosen to be 5–10 times that of the fundamental period. Let us demonstrate some basic attributes of the GNP theory on an example of the one phase voltage and current

$$v = \sqrt{2}\,V \sin\omega t \tag{6}$$

$$\begin{aligned} i &= \sqrt{2}\,I\,(1+\Delta\sin\Delta\omega t)\sin\omega t + \sqrt{2}\,I_h \sin\omega_h t \\ &= \left(\sqrt{2}\,I + \sqrt{2}\,\Delta I \sin\Delta\omega t\right)\sin\omega t + \sqrt{2}\,I_h\sin\omega_h t = \sqrt{2}\,I\sin\omega t + i_{np} + i_h \end{aligned} \tag{7}$$

The current contains three components: the fundamental, non-periodic (specifically subharmonic here), and harmonic one.

By using the concept of the GNP theory (2)–(4), the respective active current can be expressed as follows

$$i_a = Yv = \sqrt{2}\,I\sin\omega t + i_2 + i_{anp} + i_{ah} \tag{8}$$

where

$$i_2 = \frac{\sqrt{2}}{T_C}\left[-\frac{I}{2\omega}\sin 2\omega\tau\right]_{t-T_C}^{t}\sin\omega t \tag{9}$$

$$i_{anp} = \sqrt{2}\Delta I\left[G_D\sin(\Delta\omega t - \phi_D) + \frac{G_D^-}{2}\sin[(2\omega-\Delta\omega)t-\phi_D^-] - \frac{G_D^+}{2}\sin[(2\omega+\Delta\omega)t-\phi_D^+]\right]\sin\omega t \tag{10}$$

$$\begin{aligned} &G_D = \sin\phi_D/\phi_D, \quad \phi_D = \Delta\omega T_C/2 \quad G_D^+ = \sin\phi_D^+/\phi_D^+, \quad \phi_D^+ = (2\omega+\Delta\omega)T_C/2 \\ &G_D^- = \sin\phi_D^-/\phi_D^-, \quad \phi_D^- = (2\omega-\Delta\omega)T_C/2 \end{aligned} \tag{11}$$

$$i_{ah} = \frac{\sqrt{2}I_h}{2}\left[\begin{array}{l} G_h^+\sin\left(\omega_h t - \phi_h^+\right) + G_h^-\sin\left(\omega_h t - \phi_h^-\right) - G_h^+\sin\left((\omega_h+2\omega)t - \phi_h^+\right) \\ -G_h^-\sin\left((\omega_h-2\omega)t-\phi_h^-\right) \end{array}\right] \tag{12}$$

$$G_h^+ = \sin\phi_h^+/\phi_h^+, \quad \phi_h^+ = (\omega_h+\omega)T_C/2 \quad G_h^- = \sin\phi_h^-/\phi_h^-, \quad \phi_h^- = (\omega_h-\omega)T_C/2 \tag{13}$$

The components i_2, i_{anp}, i_{ah} are more or less excluded from the active current component i_a depending on the value of the general averaging time interval T_C and the frequencies $\Delta\omega$, ω_h.

If $T_C = kT/2$, $k = 1, 2, 3, \ldots$, the current component i_2 is equal to zero. By changing the value of T_C we can evaluate how the components i_{anp}, i_{ah} are more or less excluded from the active current component i_a.

By using Formulas (12)–(13), we can analyze how the current component with the frequency ω_h, which is present in the load current i_L, will appear in the active current i_{La} when the GNP theory is applied. Let us suppose here that the frequency ω_h is not only an integer multiple of the fundamental frequency ω (harmonic component), but generally any other one, that is subharmonic $\langle 2\pi 1,\, 2\pi 49\,\mathrm{Hz}\rangle$, harmonic ($2\pi h50\,\mathrm{Hz}$, $h = 1, 2, 3, \ldots$) or interharmonic higher than $2\pi 50\,\mathrm{Hz}$.

Figure 1 shows the impact of the value of T_C on the rate I_{La}/I_L for different frequencies, where I_{La}, I_L are the respective magnitudes of the current components presented in $i_{La}(t)$, $i_L(t)$ (for i_{La} given by the first two terms in (12)). We see that all harmonics presented in $i_L(t)$ are completely suppressed for T_C being one full fundamental cycle T or any multiple fundamental cycles. We see that components of interharmonic frequencies are substantially suppressed as well.

Now, let us continue with the load current component i_{np} induced by the subharmonic magnitude modulation. We see that, in addition to the amplitude modulation frequency $\Delta\omega$ in i_{np} (7), two more components with the frequencies $(2\omega-\Delta\omega)$, $(2\omega+\Delta\omega)$ have appeared in the spectrum of the magnitude of the current i_{anp} (10) after processing the current i (7) in (2).

By using Formulas (10)–(11), we can analyze how the current component with the magnitude modulation frequency $\Delta\omega$, which is present in the load current i_L, will appear in the active current i_{La} when the GNP theory is applied. If $\Delta\omega \ll 2\omega$, what is the usual case, then $G_D^+ \approx G_D^- \to 0$ (for $T_C = mT/2$, $m = 1, 2, 3, \ldots$) and only the first term in (10) with the amplitude G_D will be significant.

Figure 2 demonstrates how this dominating gain G_D depends on the subharmonic frequency Δf for different values $T_C = 0.02$, 0.1, $0.2\,\mathrm{s}$. We see that for the frequencies $\Delta f = k/T_C$, $k = 1, 2, 3,$

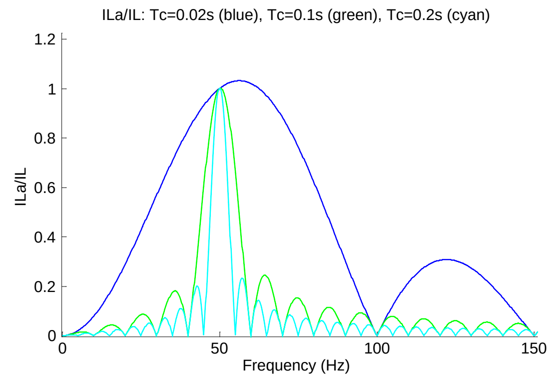


Figure 1: Impact of value of T_C on rate I_{La}/I_L for frequencies in range $\langle 0,\, 150\,\mathrm{Hz}\rangle$.

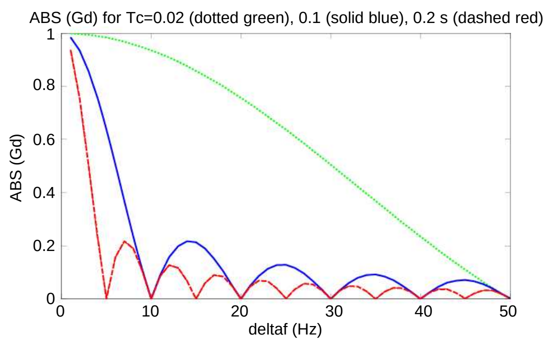


Figure 2: Dominating gain G_D in (14) versus subharmonic frequency Δf for different values $T_C = 0.02,\, 0.1,\, 0.2\,\mathrm{s}$.

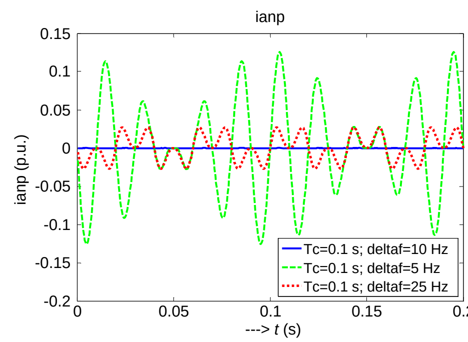


Figure 3: Time responses of current i_{anp} (14) for $T_C = 0.1\,\mathrm{s}$, $\sqrt{2}\,\Delta I = 0.2\,\mathrm{p.u.}$, and subharmonic frequencies $\Delta f = 5$, 10, and 25 Hz.

… the subharmonic components are nullified and for other frequencies the gain G_D is decreasing for Δf getting higher.

Finally, Figure 3 shows the time responses of the current i_{anp} (10) for $T_C = 0.1\,\mathrm{s}$, $\sqrt{2}\Delta I = 0.2\,\mathrm{p.u.}$, and the subharmonic frequencies $\Delta f = 5$, 10, and 25 Hz. We see that only for $\Delta f = 1/T_C = 1/0.1 = 10\,\mathrm{Hz}$ the signal $i_{anp} = 0$, while for other frequencies this signal is only partially suppressed.

Let us demonstrate the use of the GNP strategy presented above on an example of the three phase unbalanced non-sinusoidal system with the load current $\mathbf{i}_L$ that contains, in addition to the active current $\mathbf{i}_{La}$, also the reactive current $\mathbf{i}_{L1r}$ associated with the fundamental harmonic only, the fundamental harmonic unbalanced current $\mathbf{i}_{L1u}$, the harmonic current $\mathbf{i}_{Lh}$, and other non-periodic components specified later.

Table 1: Parameters of the load voltage and current used in simulation.

Variable	Magnitude (p.u.)	Phase
$\mathbf{v}_{L1}^{+} = \mathbf{v}_{LP}$	1	0
$\mathbf{i}_{L1}^{+}$	1	$-\pi/2$
$\mathbf{i}_{L1}^{-}$	0.5	0
$\mathbf{i}_{L5}$	0.2	0

Figures 4–5 show the results of the simulation of a model based on (1)–(5) with $\mathbf{v}_{LP} = \mathbf{v}_{L1}^{+}$ ($\mathbf{v}_{LP}$ is selected as the fundamental positive sequence component), $T_C = 5T = 0.1\,\mathrm{s}$. The voltage $\mathbf{v}_L$ contained, additionally to the fundamental positive sequence component $\mathbf{v}_{L1}^{+}$, also the negative sequence component $\mathbf{v}_{L1}^{-} = 0.2\,\mathrm{p.u.}$, the fifth harmonic (0.06 p.u.), and was modulated by the white noise signal with the power spectral density 0.0001875 as well. The load current with the parameters declared in Table 1 was modulated by the subharmonic signal with the magnitude $\Delta I_L = 0.2\,\mathrm{p.u.}$, and successively, with different frequencies 5, 10, 25 Hz. At the same time, the current contained

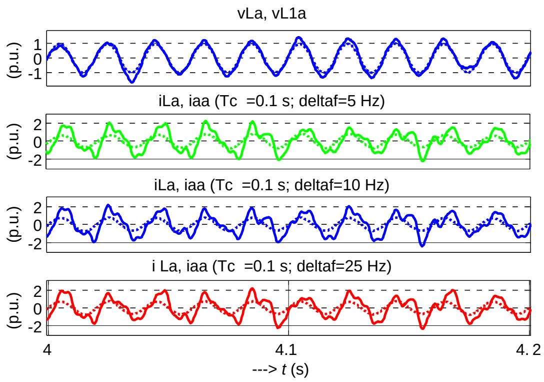


Figure 4: Time responses of voltages v_{La} (full), v_{L1a} (dotted-this was obtained by using PLL technique) and currents i_{La} (full), i_{aa} (dotted) in the phase "a" for three values 5, 10, 25 Hz of subharmonic frequency, $T_C = 5T = 0.1\,\mathrm{s}$. Load current i_{La} is simulated as highly distorted.

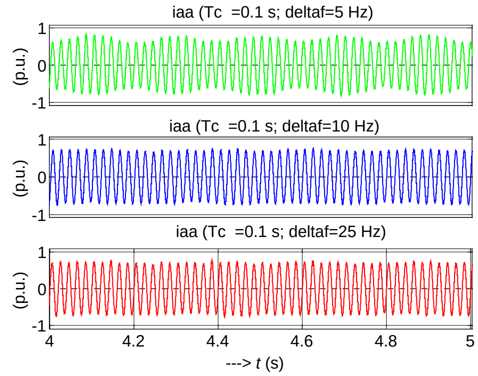


Figure 5: Comparison of subharmonic modulation presented in the active current component i_{aa} for three values 5, 10, 25 Hz of subharmonic frequency, $T_C = 5T = 0.1\,\mathrm{s}$.

the additional current interharmonics (104, 117, 134, and 147 Hz) specified by [4].

In Figure 4 we see the time responses of the voltages v_{La}, v_{L1a} (obtained by using a PLL technique) and the currents i_{La}, i_{aa} in the phase "a" for three values 5, 10, 25 Hz of the subharmonic frequency. We can recognize that the active current i_{aa} is sinusoidal in all three cases in contrast to the highly distorted original load current i_{La}.

But, depending on the value of the frequency of the signal of a subharmonic modulation, this subharmonic signal is more or less manifesting itself in the active current component i_{aa}. However, it can not be recognized in Figure 4 due to the scale selected there. Thus, Figure 5 shows this phenomenon within the time interval of 1 s. As it has been already said, the subharmonic component is fully nullified only for the frequencies $\Delta\omega = 2\pi k/T_C$, $k = 1, 2, 3, \ldots$ (for $T_C = 5T = 0.1\,\mathrm{s}$ it is just for the frequency $\Delta f = 10\,\mathrm{Hz}$). For subharmonic frequencies $\Delta\omega$ that do not match this equation, a general rule says that the lower the frequency is, the higher parasitic subharmonic modulation of the active current i_{aa} can be expected.

It is evident that if the non-active load current $\mathbf{i}_{Ln} = \mathbf{i}_L - \mathbf{i}_{La}$ (5) is completely compensated for by a parallel compensator, only the active load current i_{La} is exchanges with the source, i.e., $\mathbf{i}_S = \mathbf{i}_{La}$.

The choice of the value of T_C has a great influence on the quality of the compensation of non-periodic current components. The values of $T_C = (5\text{–}10)\,\mathrm{T}$ are high enough to compensate sufficiently for especially the interharmonics with frequencies above 100 Hz. The thing is that interharmonics can be more troublesome than harmonics and voltage fluctuations caused by a combination of interharmonics can cause voltage fluctuations in addition to waveform distortions, resulting in light flicker [5]. As for subharmonics, the lower their frequency is, the higher the value of T_C should be for a sufficient compensation. The full compensation of the subharmonic with the frequency $\Delta\omega$ needs the averaging time interval $T_C = 2\pi/\Delta\omega$ which, on the other hand, restricts the compensation dynamics in case of transient processes [1]. Another thing is that the energy storage requirement on a power electronics-based compensator is higher for lower T_C. That is why, a proper balance between a source current quality in steady states as well as in transients and energy storage requirements should be ensured for concrete conditions.

4. PARALLEL ACTIVE POWER FILTER COMPENSATION

The three phase power system was simulated in the Matlab/Simulink$^{\mathrm{TM}}$ environment. The parallel APF (Active Power Filter) was based on the PWM (Pulse Width Modulated) converter. The converter was modeled as a 4-level Capacitor Clamped Inverter (CCI) working with the sampling frequency 1600 Hz (due to use of IGCT switching elements). The converter reference current was $i^*_{CCI} = i_L - i_{La}$.

The nominal line-to-line voltage of the grid was $10\,\mathrm{kV_{RMS}}$, without any disturbances. The parameters of the 400 kVA load, modeled as the three phase system of current sources, are in Table 2.

In Figure 6 we see the waveforms of the grid voltage v_L, load current i_L, and grid current i_S

Current component	Value in phase	Current vector phase shift
i_{L1}^{+}	$I_N = 23.1\,\mathrm{A_{RMS}}$	$-\pi/2$
i_{L1}^{-}	$0.5I_N$	0
i_{L5}	$0.2I_N$	0
i_{L7}	$0.15I_N$	0

Table 2: Parameters of non-linear load.

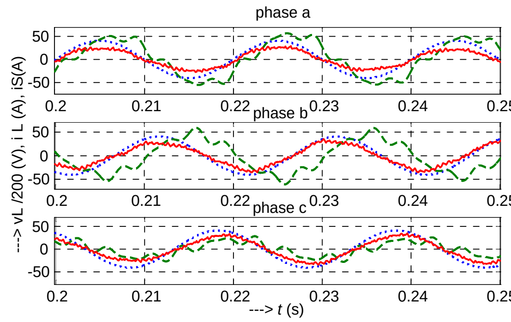
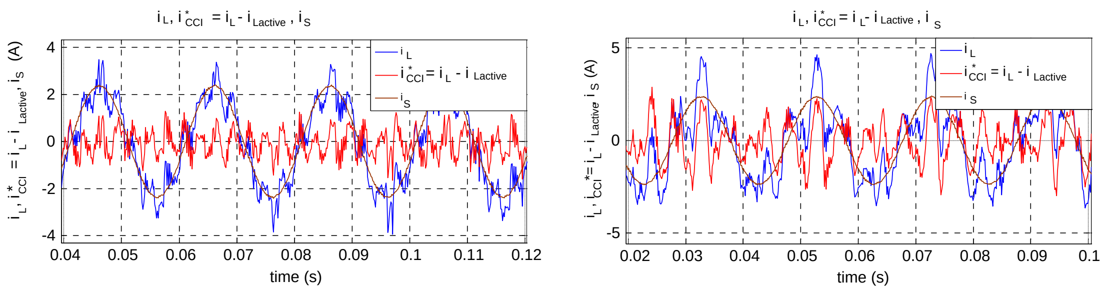


Figure 6: Waveforms of grid voltage $\mathbf{v}_L$ (dotted blue curve), load current $\mathbf{i}_L$ (dashed green curve), and grid current $\mathbf{i}_S$ (full red curve) after compensation in all three phases. The magnitude of load current $\mathbf{i}_L$ with components declared in Table 2 was modulated by subharmonic signal with magnitude ΔI_L=0.2I_N and frequency 5 Hz.



(a) (b)

Figure 7: Waveforms of load i_L and compensating reference phase currents $i_{CCI}^{*} = i_L i_{La}$ captured by the dSPACE™control system and processed in Matlab™, and the respective supposed grid currents i_S (right side: interharmonics of 104, 147 Hz are also included into load current), switching period 150 µs/frequency is 6.667 kHz.

after compensation in all three phases. The magnitude of the load current $\mathbf{i}_L$ with components declared in Table 2 was additionally modulated by the subharmonic signal with the magnitude $\Delta I_L = 0.2I_N$ and frequency 5 Hz. We see that there is a phase shift between the grid voltage and the load current which is not the case for the grid current. Also the magnitudes of the grid currents in all three phases are approximately the same, although the load current is, due to the presence of the negative component $\mathbf{i}_{L1}^{-}$, highly unbalanced. But, due to the choice $T_C = 5\,\mathrm{T}$, a part of the subharmonic component with the frequency 5 Hz is still present in the grid current in accordance with the results presented before. This is the reason why the magnitudes of the three phase grid currents are not exactly the same in the time window shown. The full compensation of this subharmonic component could be obtained by setting $T_C = 10\,\mathrm{T}$.

We have compared the discussed compensation strategy also through experimentations. Figure 7 shows load i_L and compensating reference phase current $i_{CCI}^{*} = i_L - i_{La}$ ($T_C = 5\,\mathrm{T}$) captured by the dSPACE™ control system and processed in Matlab™, and the respective supposed grid currents i_S for two different loads.

5. CONCLUSION

A principal of usage of the generalized non-active power theory for the parallel compensation of periodic and non-periodic (subharmonic, interharmonic) current disturbances is presented in the paper.

The impact of the value of T_C on the rate I_{La}/I_L for different frequencies has been studied in the frequency domain (I_{La} is a residuum of the load current magnitude I_L remaining in the active current after applying GNP strategy). It was found that especially the interharmonic current components above 100 Hz are sufficiently suppressed for $T_C = (5–10)\,\mathrm{T}$. As for subharmonics, the lower their frequency is, the higher the value of T_C should be used for a sufficient compensation. On the other hand, the compensation dynamics in case of transient processes is becoming worse for T_C increasing.

The effectiveness of the generalized non-active power theory to mitigate non-periodic current disturbances is demonstrated on the example of a reactive, unbalanced load current with several harmonic and interharmonic components, additionally modulated either by the subharmonic component or the white noise signal, by the simulations in the time domain and experimentally as well.

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