Wave Packet Propagation of Guided Optical Modes in a Thin Left-handed Film near a Frequency of Zero Power Flux

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Abstract— Dispersion of the propagation constants of waveguide modes of a thin left-handed film is considered. It is shown that by certain frequency the group velocity of mode can be vanished. An analysis of spatio-temporal transformation of intramode packet with spectrum near this frequency is performed. It is shown that a propagation velocity of the packet is proportional to a square root of a width of the packet spectrum and for packets with spectrum about 10 MHz can be by four orders less than the group velocity of the light in a volume left-handed material.

Modern technology of metamaterials allows creating thin left-handed waveguides on the volume substrate. These waveguides have both negative values of permittivity and permeability in optical range of wavelengths [1]. The waveguides have unique properties which considerably differ from usual waveguides. The features of dispersive characteristics of such waveguides and the features of the propagation of optical pulses in them are considered below.

It is shown in [2, 3] that the propagation constants β of waveguide modes of TE-type of thin films with thickness h including on the basis of left-handed metamaterials are the real roots of the dispersion relation which can be represented in the form

$$\tanh \chi_f h + \frac{\chi_s \mu_f \chi_f^{-1} \mu_s^{-1} + \chi_c \mu_f \chi_f^{-1} \mu_c^{-1}}{1 + \chi_s \chi_c \mu_f^2 \chi_f^{-2} \mu_s^{-1} \mu_c^{-1}} = f(\beta, \omega) = 0, \tag{1}$$

where $\chi_* = \left(\beta^2 - \varepsilon_* \mu_* (\omega/c)^2\right)^{1/2}$; μ_* and ε_* are the relative permeability and the relative permittivity, respectively; c is the velocity of light in a vacuum; sign "$*$" at subscript is used instead of letters "c" (coating medium), "f" (film) and "s" (substrate).

We assume that the dispersion dependences of permittivity ε_f and permeability μ_f of a volume medium of left-handed metamaterial on the frequency of electromagnetic field ω are described by the well-known relations [4]

$$\varepsilon_f = 1 - \omega_p^2/\omega^2, \quad \mu_f = 1 - F\omega^2 \left(\omega^2 - \omega_m^2\right)^{-1}, \tag{2}$$

where ω_p is a plasma frequency, ω_m is a frequency of magnetic resonance, F is a filling factor of metamaterial. The dispersion of material parameters of the usual right-handed material is negligible in comparison with the dispersion of the left-handed material and isn't taken into consideration below.

The dispersion dependences of propagation constants $\beta(\omega)$ of waveguide modes of TE-type for the left-handed film with thickness $h = 330$ nm in the case of air coating media ($\varepsilon_c = 1$ and $\mu_c = 1$) and non-magnetic substrate ($\mu_s = 1$) with relative permittivity equaled $\varepsilon_s = 2$ are represented in Fig. 1. The dependences have been calculated for parameters $\omega_p = 3.46 \cdot 10^{15}$ rad/s, $\omega_m = 1.63 \cdot 10^{15}$ rad/s and $F = 0,5$ which are in accord with data of papers [1, 5]. In this case the frequency range in which both conditions $\varepsilon_f \leq 0$ and $\mu_f \leq 0$ are satisfied is within the limits from $1.63 \cdot 10^{15}$ rad/s (the wavelength equals $\lambda = 1.16\,\mu$m) to $2.31 \cdot 10^{15}$ rad/s ($\lambda = 0.82\,\mu$m).

The dispersion dependences $\beta(\omega)$ for modes TE_0 and TE_1 are not only in the range of fast modes ($k_s < \beta < k_f$) that typically for usual films of right-handed materials but also in the range of slow modes ($\beta > \max(k_f; k_s)$) which is absent in the usual films [6]. At the same time the dependence $\beta(\omega)$ for mode TE_1 is divided into two branches by point with infinite derivative $d\beta/d\omega = \infty$ in which the group velocity of mode is vanished $v_g = d\omega/d\beta = 0$. The phase and group velocities of mode which correspond to a lower branch of the dispersion dependence have the same directions, and which correspond to a upper branch have opposite directions. The dispersion dependences $\beta(\omega)$ of remaining modes are in the fast range. However, in contrast to usual film waveguides of right-handed materials the dispersion dependences for modes of them are monotone the dependences

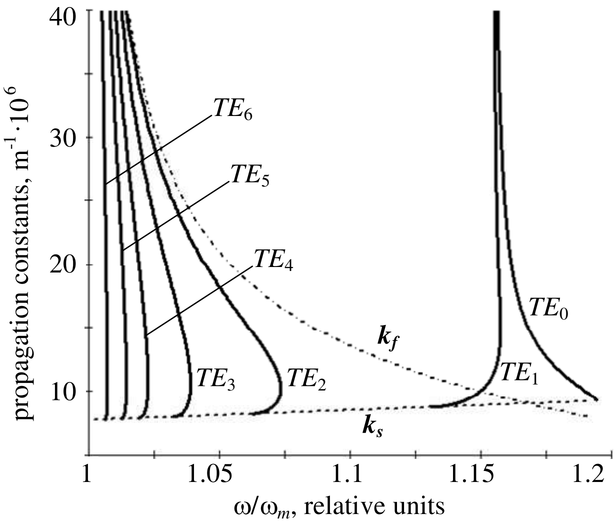

Figure 1: The dispersion dependences of the propagation constants of TE-modes of the left-handed film with thickness $h = 330\,\text{nm}$ in the case of the air coating medium and the non-magnetic substrate with relative permittivity $\varepsilon_s = 2$. Here k_f and k_s are dispersion dependences of wave numbers for volume materials of the film and the substrate, respectively.

$\beta(\omega)$ for fast modes of waveguides with left-handed film are divided into two branches by point that corresponds to the case $\upsilon_g = 0$ (similarly to the dependences for TE_1). At the same time by increase of mode number of the left-handed film the dispersion dependences which correspond to this mode number approach to the point of singularity of permeability μ_f and there are practically merged.

A cause of appearance on the dispersion dependences of the propagation constants of waveguide modes of the left-handed film two branches which correspond to the opposite directions of group velocity of mode is that the power flux of the light field in the film is opposite in its direction to power fluxes in right-handed coating medium and substrate [3]. Depending on the frequency ω of the light field of mode a total flux of power can has a direction either coinciding with power fluxes into the coating medium and the substrate or coinciding with the power fluxes in the film. By certain frequency ω_0 the power flux in film is equalized by fluxes in the coating medium and the substrate, the total flux of power of the mode and its group velocity are vanished. The waveguide modes of the usual right-handed films don't have the similar effect [6].

The frequency ω_0 in which the group velocity of the mode is vanished ($\upsilon_g = d\omega/d\beta = 0$) and the propagation constant β_0 corresponding to the group velocity can be determined as roots of the dispersion relation (1) that defines function $\omega\,(\beta)$ implicitly and equations $d\omega/d\beta = -(\partial f/\partial\beta)/(\partial f/\partial\omega) = 0$ [7]. In general case the frequency ω_0 belongs not only to the definition domain of the dispersion curve $\beta(\omega)$ of the mode (for this mode $\upsilon_g = 0$) but gets in area of definition curves $\beta(\omega)$ of other modes (see Fig. 1). Dispersion properties of these modes near the frequency ω_0 can be described within the framework of usual dispersion theory and aren't considered below.

From Fig. 1 follows that for case under consideration two waveguide modes can exist on the frequency ω in the neighborhood of point ω_0 ($\omega < \omega_0$). One of these modes corresponds to the lower branch of dependence $\beta(\omega)$ and other mode corresponds to the upper branch. The dependences $\beta(\omega)$ in the neighborhood of point ω_0 can be represented in terms of series by degrees of the square root of the frequency deviation $\Delta\omega = \omega_0 - \omega$. The dispersion dependences of the propagation constant β^+ (modes with the same directions of phase and group velocities) and the propagation constant β^- (modes with the opposite directions) to a first approximation by value $\Delta\omega^{1/2}$ can be described by following expressions:

$$\beta^{\pm} = \beta_0 \mp \sqrt{\Delta\omega/a} = \beta_0\,(1 \mp \delta\beta)\,, \tag{3}$$

where $a = -0.5\left(d^2\omega/d\beta^2\right)_{\omega=\omega_0}$, $\delta\beta = \beta_0^{-1}\sqrt{\Delta\omega/a}$.

The dependence of y-component of the electric intensity vector of the light field of waveguide mode of TE-type by the frequency ω (the mode propagates in plane of the film along z-axis) on the coordinate x along the normal to the film can be presented with accuracy at multiplier in the form:

$$E_c = (-1)^{\eta}\left(1-\mu_f^2\chi_c^2\chi_f^{-2}\right)^{-1/2}\exp\left[\chi_c(h-x)\right],\quad x>h \tag{4}$$

$$E_f = \left(1-\mu_f^2\chi_s^2\chi_f^{-2}\right)^{-1/2}\left[\cosh(\chi_f x)+\mu_f\chi_s\chi_f^{-1}\sinh(\chi_f x)\right],\quad 0\le x\le h \tag{5}$$

$$E_s = \left(1-\mu_f^2\chi_s^2\chi_f^{-2}\right)^{-1/2}\exp(\chi_s x),\quad x<0 \tag{6}$$

where η is a mode number equaled to a quantity of zeros of the function $E_f(x)$.

In a first approximation by parameter $\delta\beta$ the equations which describe the dispersion of space dependences of the electric intensity $E_*^{\pm}$ and x-component of the magnetic intensity vector $H_*^{\pm} = -\beta E_*^{\pm}/(\mu_*\mu_0\omega)$ that corresponds to the electric intensity can be obtained as:

$$E_*^{\pm} = \left(1\mp\xi_*\beta_0^2\delta\beta\right)E_{*0},\quad H_*^{\pm} = \left[1\mp\left(1+\xi_*\beta_0^2\right)\delta\beta\right]H_{*0}, \tag{7}$$

where the sign "$*$" denotes the same that in (1) (see the explication to (1));

$$\xi_c(x) = \frac{h-x}{\chi_{c0}}+\frac{\mu_{f0}^2\left(\chi_{f0}^2-\chi_{c0}^2\right)}{\chi_{f0}^2\left(\chi_{f0}^2-\mu_{f0}^2\chi_{c0}^2\right)},\quad x>h; \tag{8}$$

$$\begin{aligned}\xi_f(x) &= \mu_{f0}\frac{\chi_{f0}^2-\chi_{s0}^2}{\chi_{s0}\chi_{f0}^3}\frac{\sinh(\chi_{f0}x)}{\cosh(\chi_{f0}x)+\mu_{f0}\chi_{s0}\chi_{f0}^{-1}\sinh(\chi_{f0}x)}\\ &\quad+\frac{x}{\chi_{f0}^2}\frac{d}{dx}\ln\left[\cosh(\chi_{f0}x)+\mu_{f0}\chi_{s0}\chi_{f0}^{-1}\sinh(\chi_{f0}x)\right]\\ &\quad+\frac{\mu_{f0}^2\left(\chi_{f0}^2-\chi_{s0}^2\right)}{\chi_{f0}^2\left(\chi_{f0}^2-\mu_{f0}^2\chi_{s0}^2\right)},\quad 0\le x\le h;\end{aligned} \tag{9}$$

$$\xi_s(x) = \frac{x}{\chi_{s0}}+\frac{\mu_{f0}^2\left(\chi_{f0}^2-\chi_{s0}^2\right)}{\chi_{f0}^2\left(\chi_{f0}^2-\mu_{f0}^2\chi_{s0}^2\right)},\quad x<0; \tag{10}$$

the number zero "0" at subscripts of all magnitudes signifies that they have to be calculated by the frequency ω_0; $\mu^{(0)}$ is the fundamental magnetic constant.

Let's consider the propagation of limited packet of the guided modes of TE-type. We assume that the propagation constants of modes from the packet belong one dispersion curve and the frequencies of them are within interval $\Delta\Omega = \omega_0 - \omega_{\min} << \omega_0$ (see Fig. 1). Two different pairs of intramode packets correspond to such interval. The modes of these packets have the opposite directions of phase velocities. At that the modes of one of the packet of every pair (with the propagation constants β^+) have the same directions of phase and group velocities. The modes of other packet of pair (with the propagation constants β^-) have the opposite directions of these velocities. We assume below that the spectral density of the modes in these packets is the same and normalized to unit. Analytical computation of spatio-temporal dependences of the power fluxes $S_\nu^{\pm}(z,t)$ of the packets of guided modes of TE-type of the first ($\nu=1$, the phase velocity along the positive direction of z-axis) and the second ($\nu=2$, the phase velocity along the negative direction of z-axis) pairs shows that in this case the relations are correct $S_1^+(z,t) = S_2^-(z,t) \equiv S(z,t)$ and $S_2^+(z,t) = S_1^-(z,t) \equiv -S(-z,t)$,

$$S(z,t) = \frac{4\beta_0^2}{\mu_0\omega_0}\int\limits_{-\infty}^{\infty}\frac{\xi(x)}{\mu(x)}\left(E_0^+(x)\right)^2 dx Re\left[\psi(z,t)\left(\Delta\psi(z,t)\right)^*\right], \tag{11}$$

$$\psi = \frac{i}{t}\left[Ex(z,t)-\frac{z}{2\sqrt{at}}Erf_{\Delta}(z,t)\right], \tag{12}$$

$$\Delta\psi(z,t) = \frac{i}{t\sqrt{at}}\left[Ex(z,t)\left(\sqrt{\Delta\Omega t}+\frac{z}{2\sqrt{at}}\right)-\frac{z}{2\sqrt{at}}+\left(\frac{i}{2}-\frac{z^2}{4at}\right)Erf_{\Delta}(z,t)\right], \tag{13}$$

where

$$Ex\,(z,t) = \exp\left[i\left(\sqrt{\frac{\Delta\Omega}{a}}z - \Delta\Omega t\right)\right] - 1, \tag{14}$$

$$Erf_{\Delta}\,(z,t) = \frac{1+i}{\sqrt{2}}\sqrt{\pi}\exp\left(\frac{iz^2}{4at}\right)\left[erf\left(\frac{1+i}{\sqrt{2}}\frac{z}{2\sqrt{at}}\right) - erf\left(\frac{1+i}{\sqrt{2}}\left(\frac{z}{2\sqrt{at}} - \sqrt{\Delta\Omega t}\right)\right)\right], \tag{15}$$

$\mu\,(x)$ is a dependence of permeability on the coordinate x, $erf\,(\zeta)$ is an integral of errors [7].

It should be noted that the product of two functions, $\psi\,(z,t)\,(\Delta\psi\,(z,t))^*$, in relation (11) describes dispersive distortions of the mode packet both at the expense of the dispersion of the propagation constants of waveguide modes (see (3)) and at the expense of the dispersion of the spatial distribution shape of the fields of the waveguide modes (see (7)). Analysis and numerical simulation have showed that the function $\Delta\psi$ is degenerated into the function ψ in the case of neglect of the dispersion of shape. However such neglect is unjustified as contributions of both types of the dispersion into spatio-temporal dynamics of the mode packet have the same order.

From Fig. 1 follows that for above considered left-handed film a small neighborhood of frequency $\omega_0 \approx 1.75\cdot 10^{15}$ rad/s ($\beta_0 \approx 1.02\cdot 10^7\,\mathrm{m}^{-1}$) that is in the definition area of the dispersion dependence of the propagation constant of waveguide mode TE_2 with parameter $a \approx 1.66\,\mathrm{s/m^2}$ is outside of the definition area of the dispersion dependences of other modes. For this mode the time dependence of the power flux $s\,(z,t) = S\,(z,t)/\max\left[S\,(0,t)\right]$ of the wave packet with width $\Delta\Omega = 100\,\mathrm{GHz}$ at different sections $z = const$ normalized to maximum of the flux by $z = 0$ is represented in Fig. 2. A considerable spreading of the mode packet with time is conditioned by both the strong material dispersion of left-handed film (see (2) and Fig. 1) and the waveguide dispersion of considered planar structure. In spite of zero group velocity by frequency ω_0 the maximum of the mode packet during 1.4 ns is shifted along the waveguide axis by 1 mm that corresponds to the propagation velocity of the mode packet approximately $6.7\cdot 10^5$ m/s. It is almost by three orders less than the group velocity in the volume left-handed material equaled $1.1\cdot 10^7$ m/s. From analysis of relations (11)–(15) follows that in considered case the propagation velocity of the mode packet can be approximately evaluated by means of following relation

$$\upsilon = \sqrt{2a\Delta\Omega}. \tag{16}$$

As follows from last formula the propagation velocity of group packet decreases proportionally to the square root of the spectrum width of the packet $\Delta\Omega$ and by $\Delta\Omega = 10\,\mathrm{MHz}$ for the considered mode TE_2 has very small value $8.1\cdot 10^3$ m/s.

Thus the dispersion properties of planar fibers based on the thin left-handed film have qualitative differences from the properties of usual structures based on the right-handed films. The effect of

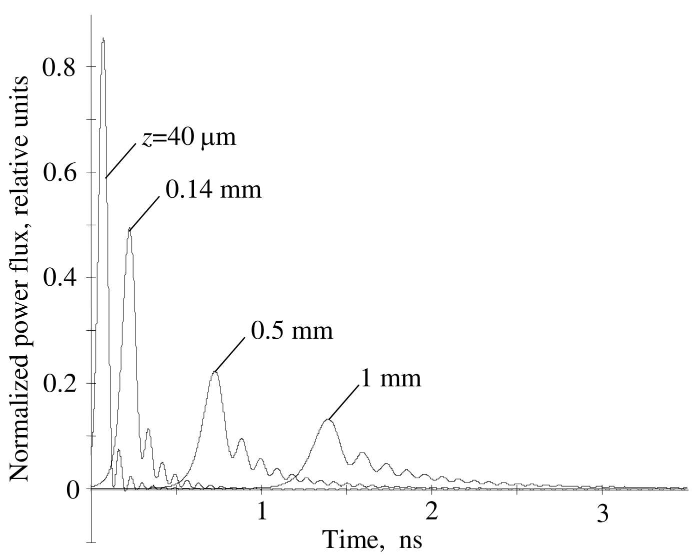

Figure 2: Normalized time dependence of the power flux of the intramode packet with width $\Delta\Omega = 100\,\mathrm{GHz}$ at different sections $z = const$ for mode TE_2 with parameter $a = 1.6554\,\mathrm{s/m^2}$.

considerable reduction of the propagation velocity of the light in the left-handed films can be used in the integrated spatio-temporal modulators of the light.

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