A Study of Scattering of Scatterers Using Equivalence Principle Algorithm

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Abstract— For theoretical models used in microwave remote sensing of earth terrain, it is quite common to represent the medium as a random discrete medium where the medium can be modeled as a layer of scatterers of basic shapes embedded in the host medium and bounded by upper and lower rough surfaces. By solving the wave propagation and the scattering in the medium through Radiative Transfer (RT) theory, the scattering returns from the medium can be obtained. Traditionally, analytical expression of scattering from these scatterers of basic shapes is derived and computed with simplification based on basic assumptions. However, it is generally found from ground truth measurement that the actual shapes of those scatterers are much more complex and irregular. Although it is difficult to simulate the scattering from this wide range of scatterers of different sizes, orientation and shapes, it is possible to model the medium better by representing the scatterers with major types of scatterers of particular shapes, though the shapes may not be of basic geometrical shapes like spheres. In this study, it is proposed to use the Equivalence Principle Algorithm (EPA) method under the approach of computational electromagnetics to perform the calculation of scattering from these scatterers and incorporate this for the simulation of radar scattering returns from Radiative Transfer (RT) formulation. With this approach, the theoretical model developed can be extended to cover scattering from scatterers which are not easily represented by basic geometrical shapes. Comparison results of the simulation based on Mie scattering and EPA are presented. The model prediction is also compared with satellite SAR data with physical parameters input from ground truth measurement conducted in Antarctica.

1. INTRODUCTION

There have been theoretical models based on Radiative Transfer (RT) theory that incorporate Mie scatterers to simulate scattereres like ice, brine inclusions or air bubbles which are embedded inside the snow or sea ice layer [1–3]. Studies using this kind of models and comparison of theoretical results with ground truth measurement data conducted in Antarctica and satellite SAR data have also been done. While it is convenient to model the scatterers by representing them as spherical scatterers, most of the ice particles in snow or brine inclusions or air bubbles in sea ice are not actually spherical in shape. To better represent these scatterers, a new approach is proposed where the scattering from the scatterers are computed through the Equivalence Principle Algorithm (EPA) [4, 5]. The Equivalence Principle Algorithm (EPA) was developed based on equivalence theorem, also known as Huygens' equivalence principle, which states that the fields inside or outside a closed surface can be determined by tangential components of the fields on the surface [4]. EPA is a numerical integral solution which uses the concept of equivalence surface and domain decomposition method (DDM). For complex scattering problems, EPA can also be applied by breaking the problems into sub domains for solving multi-object scattering problem and this facilitates the parallelization of solution, the reuse of solution, and also improves the conditioning of matrix system [4]. By creating an equivalence surface to enclose the scatterers inside, a new equivalence source can replace the original source which provides the same incident field inside the equivalence surface. Using the three equivalence principle operators containing the information of the inside scatterers, scattered currents are then calculated. The three equivalence principle operators, which are inside-out propagation, current solver and outside-in propagation, can form the scattering matrix which is used to solve the scattered field outside the equivalence surface that can be calculated from the scattered currents on equivalence surface. In this manner, the unknowns of the inside scatterers are transferred to the unknowns on the equivalence surface that encloses the elements in the final matrix equation [4]. Scatterers to be computed in EPA are in arbitrary 3D model, this allows EPA to operate and be independent of the shape restriction of scatterers. In this case, it is possible to extend the microwave remote sensing model for spherical scatterers embedded in the medium to include cases where the shape of scatterers is irregular or not spherical.

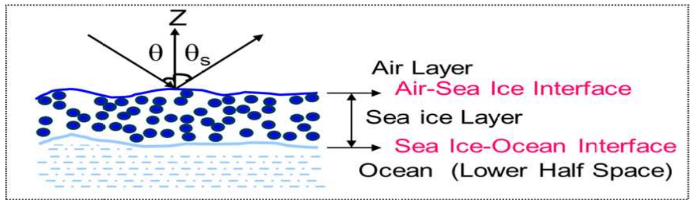

Figure 1: Model configuration.

2. MODEL DEVELOPMENT

As shown from Figure 1 above, scatterers inside a layer of sea ice are modeled as spherical objects and covered on top by air and bottom by half space of ocean. Mie scattering was previously used to calculate the phase matrix $\bar{\bar{P}}$ of the scatterers like ice, brine inclusions or air bubbles as given by:

$$\bar{\bar{P}}(\theta,\phi;\theta',\phi') = \langle|\psi|^2\rangle_n \cdot \bar{\bar{S}} = \begin{bmatrix} P_{vv} & P_{vh} \\ P_{hv} & P_{hh} \end{bmatrix} \tag{1}$$

where $\langle|\psi|^2\rangle_n$ is the dense medium array correction factor and $\bar{\bar{S}}$ is the Stokes matrix for a Mie scatterer with the close spacing amplitude correction [6, 7]. The dense medium array correction factor accounts for the coherency of scatterers in an electrically dense medium where the average distance between the scatterers is less than the wavelength [7, 8]. For electrically sparse medium, the dense medium array correction factor will approach to n_o where n_o is the number density of the spheres. The total backscattering from the medium will then be calculated using the Radiative Transfer (RT) equation where the method is described in [9].

For the utilization of EPA, the phase matrix of a scatterer can now be replaced by the EPA calculated values where an equivalence surface is considered enclosing the scatterer. In EPA, the components of phase matrix are calculated using the equivalence principle operators given by:

$$\begin{bmatrix} J^{sca} \\ \frac{1}{\eta}M^{sca} \end{bmatrix} = \begin{bmatrix} -\hat{n}\times K^S_{HJ} \\ -\frac{1}{\eta}\hat{n}\times L^S_{EJ} \end{bmatrix} \cdot [L_{pp}]^{-1} \cdot \begin{bmatrix} -L^S_{EJ} & -\eta K^S_{EM} \end{bmatrix} \cdot \begin{bmatrix} J^{inc} \\ \frac{1}{\eta}M^{inc} \end{bmatrix} \tag{2}$$

where J^{inc} is equivalent incident electric current and M^{inc} is equivalent incident magnetic current while J^{sca} is equivalent scattered electric current and M^{sca} is equivalent scattered magnetic current. The first term of the equation $\begin{bmatrix} -\hat{n}\times K^S_{HJ} \\ -\frac{1}{\eta}\hat{n}\times L^S_{EJ} \end{bmatrix}$ is the inside-out propagation operator, second term $[L_{pp}]^{-1}$ is the current solver operator and third term $\begin{bmatrix} -L^S_{EJ} & -\eta K^S_{EM} \end{bmatrix}$ is the outside-in propagation operator [4]. After computing the J^{sca} and M^{sca}, the scattered fields can then be obtained and this scattered field can be used in the phase matrix in Equation (1). To verify the accuracy of the incorporation of EPA calculated scattering coefficient, several verification steps were carried out.

3. SIMULATION RESULTS AND ANALYSIS

Figure 2 and Figure 3 show the scattering coefficient of a single spherical object calculated from EPA and compared with that of Mie scattering and also FEKO software for two different sizes of sphere which were 1.2 mm and 0.5 mm (with relative permittivity $6.9 + i0.12$) at the frequencies of 5 GHz and 1 GHz, respectively. The results from the two figures show that all three methods give well agreed scattering coefficient values. Figure 4 shows the results of total backscattering coefficient from EPA and Mie scattering for a layer of medium with spherical scatterers (with relative permittivity $6.9 + i0.12$) of radius 13 mm plotted against 8 different incident angles at 5 GHz frequency. The simulated results of total backscattering coefficient from EPA agreed well with those of Mie scattering.

Figure 5 shows the backscattering coefficient comparison between EPA RT model, Mie scattering RT model and satellite SAR data for ice shelf sites A, B, C, I and P in Antarctica. The data from the RT model with Mie scattering was from paper [7] in 2012. For ground truth measurement of sites A, B, and C, data were collected in 2002, site I data were collected in 2003 and data for site P were collected in 2004 [7, 10]. From Figure 5, results computed from the EPA based

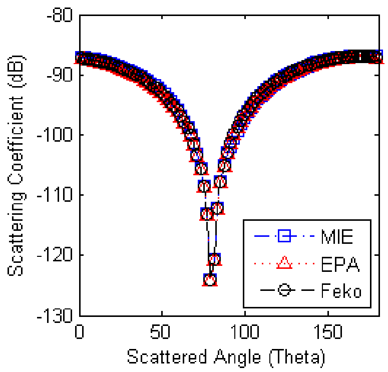

Figure 2: Scattering coefficient comparison among EPA, FEKO and MIE for a sphere with radius 1.2 mm at 5 GHz frequency.

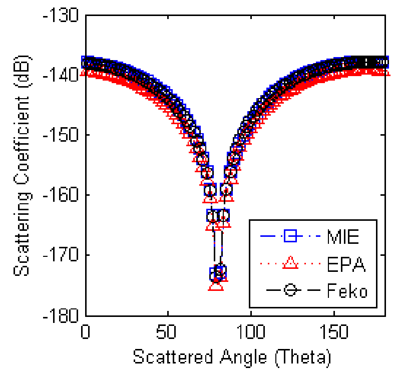

Figure 3: Scattering coefficient comparison among EPA, FEKO and MIE for a sphere with radius 0.5 mm at 1 GHz frequency.

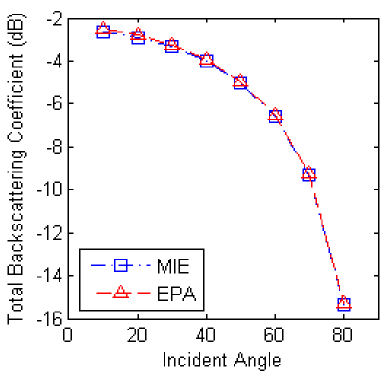

Figure 4: Total Backscattering coefficient [HH] comparison between EPA and MIE for a layer of medium with spherical scatterers of radius 13 mm at 5 GHz frequency for 8 different incident angles.

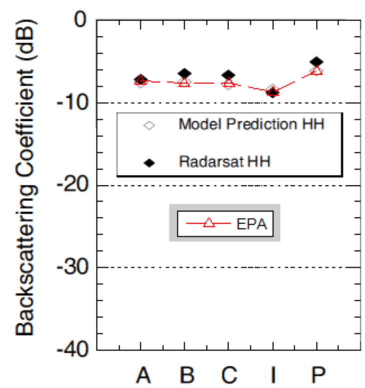

Figure 5: Backscattering coefficient comparison among EPA method, MIE model and satellite measured values for 5 ice shelf sites (Sites A, B, C, I and P) in Antarctica.

theoretical model match well with the RADARSAT-1 data measured and the Mie scattering RT model computed results. From the plotted results shown above, a new RT theoretical mode based on EPA can be extended further to simulate scatterers with non-spherical shapes.

4. CONCLUSION

In this paper, it has shown that the proposed method of incorporating EPA for the calculation of scattering from a scatterer can be combined with dense medium array correction factor and RT formulation for the calculation of radar scattering returns from a layer of embedded scatterers. With more powerful and faster computer technology, the new RT theoretical model that incorporates EPA can be further used to study the scattering of a layer of scatterers with irregular shapes.

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