A Model-free Method for Real-time High Precision Carrier Phase Observation

Tianyi Zhang, Qiao Meng, Quantao Yu, Jifei Tang, and Wei Liu
Institution of RF-&OE-ICs, Southeast University, Sipailou 2, Nanjing 210096, China

Abstract— Carrier phase observation is a high accuracy ranging method through tracking the phase of carrier signal and could be used in various applications. Carrier signal of the target in motion is a typical nonstationary signal and its model is hard to acquire. This paper describes a model-free approach of obtaining the total phase and the instantaneous frequency of the carrier signal in real-time with high resolution. The total phase is calculated and denoised using Savitzky-Golay filter to reducing measure noise. Meanwhile, the instantaneous frequency could be obtained by deriving the smoothed total phase which can also be implemented by a FIR filter like Savitzky-Golay filter. Simulation shows the performance of the proposed method.

1. INTRODUCTION

Carrier Phase Observation is a radio ranging method with high precision. It is based on measuring the difference of phase between the carrier signal received by receiver and the local reference signal [1]. Carrier phase observation could be used in various applications including radar ranging, satellite orbits determination, high precise relative GPS positioning and mobile communication location system. This method requires the full carrier cycles and the fractional cycle, which is called the total phase. Different approaches have been proposed to solve the initial integer ambiguity [2].

The total phase can be obtain by estimating frequency and phase from digitized carrier signal which has been converted to baseband signal. The problem is the carrier signal is a typical nonstationary signal and the model of carrier signal is difficult to determine due to the uncertain state of the target's motion. Since the distance is certainly a continuous smooth function of time in most situations, it can be well approximated by a polynomial within a finite duration interval, according to the Weierstrass approximation theorem. This class of signal is called the polynomial phase signal (PPS) and several approaches have been proposed for parameter estimation, such as polynomial Wigner-Ville distribution (PWVD) [3, 4], discrete polynomial-phase transform (DPT) [5], polynomial time-frequency distributions [6], etc. The computation of these algorithm increase rapidly when the required resolution of parameter estimation is high, hence the real-time observation is difficult to achieve.

This paper proposes a model-free method to obtain the total phase with high resolution in real-time. Parameters are estimated by de-chirp algorithm and chirp-Z transform in analysis window with short duration. The total phase is denoised by least squares (LS) polynomial fitting and the instantaneous frequency (IF) is got by computing the first derivative of the total phase. The rest of the correspondence is organized as follows. In Section 2, the total phase and the instantaneous frequency are introduced. The parameter estimation algorithm and smoothing filter is described and discussed in Section 3. In the Section 4, performance of the method is examined by simulation, followed by the conclusions in the Section 5.

2. TOTAL PHASE AND INSTANTANEOUS FREQUENCY

Carrier phase observation requires the estimator to keep track of the difference of phase in order to obtain the total phase including the full cycles and the fractional cycle. The average frequency is able to determine the full cycles as well, according to its definition. Therefore, the estimator can performed at intervals to relieve the pressure of computation.

Suppose the carrier signal is:

$$s\left(t\right), \qquad t \in [t_0 - T, t_0) \tag{1}$$

where T is the process interval called the integral time and t_0 is the output time. The total phase at t_0 can be expressed as:

$$\phi\left(t_0\right) = floor(\overline{f}\left(t_0\right) * T) + \varphi\left(t_0\right) \tag{2}$$

The average frequency of the signal during this time is $\overline{f}\left(t_0\right)$ and the fractional cycle at t_0 is $\varphi(t_0)$.

The velocity of target is a critical parameter for positioning and orbit determine which could be obtained by the instantaneous frequency. The instantaneous frequency of a monocomponent signal is defined as [7]:

$$f_i(t_0) = \frac{1}{2\pi}\frac{d}{dt}\phi\left(t\right)|_{t=t_0} \tag{3}$$

where $\phi(t)$ is the total phase. This expression indicates the instantaneous frequency is the first derivative of the phase.

3. MODEL-FREE PHASE ESTIMATION METHOD

3.1. Parameter Estimation

Windowed Fourier transform is a class of classic time-frequency analysis algorithm. A short window called observation window is used to localize spectrum in time. The critical issue of Fourier analysis is the signal in observation window is regarded as a stationary signal, which causes unpredictable errors during phase estimation. Therefore, de-chirp algorithm is performed instead. De-chirp algorithm, or called discrete polynomial transform (DPT) [8], is proposed for parameter estimation of linear frequency modulation (LFM) signal, which is a typical nonstationary signal. The advantage of De-chirp algorithm is the estimation of the frequency rate is converted to estimate the frequency of a sinusoidal signal. Discrete Fourier transform (DFT) is the classical algorithm for parameter estimation of discrete sinusoidal signal [9], while fast Fourier transform (FFT) is able to reduce the computation. The problem of DFT is that the resolution in frequency plane is limited by the length of data. Chirp-Z transform is applied to solve this problem since it can be computed rapidly with FFT as well. This estimator could be implemented in hardware conveniently as FFT can be realized through IP cores. The average frequency is calculated by frequency rate and initial frequency. Then the total phase could be got using the expression in (2).

3.2. Total Phase Smoothing

The measure noise is unacceptably large due to the length limitation of observation window. Polynomial fitting is able to reduce the measure noise since the total phase could be approximated by a polynomial. Meanwhile, the first derivative of phase is obtained conveniently by the polynomial coefficients. The polynomial fitting using the least square method is performed locally on a set of the total phase results, which is chosen by a slide window with fixed length. Data in the window is used to find the coefficients of polynomial fitting and output the polynomial value using these coefficients. As the window drops one point at the left and picks up a new point at the right, only the central result is reminded each time.

The proposed process of smoothing can be implemented by a special filter called Savitzky-Golay filter [10] or Taylor filter [11]. This kind of filter is a FIR filter, whose coefficients is readily obtained once the window length m and fitting degree p is determined. The first derivative of the phase could be got by this filter through changing the coefficients. The computation of polynomial fitting is considerably reduced and the architecture of FIR can be effectively realized in hardware.

Due to the uncertainty of signal model, the degree has to be selected cautiously. Simulation using a PPS of order 6 is performed to find the optimal value of p. Figure 1 depicts the performance

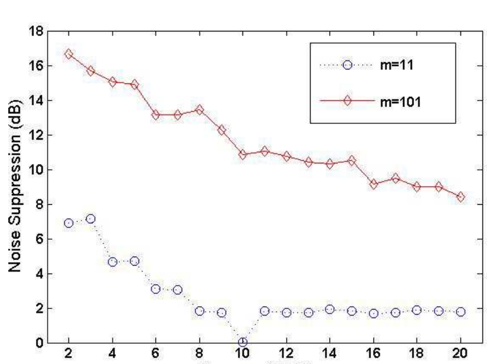

Figure 1: Performance of smoothing with different degrees.

of the smoothing when $2 \leq p \leq 20$. It also indicates that the degree of fitting should not exceed 3 though the order of PPS is higher than that.

4. SIMULATION

The performance of the method is tested by numerical simulation using PPS with additive white Gauss noise (AWGN). The precision of estimation is evaluated by root mean square error (RMSE). The simulation signal is expressed as:

$$s(n) = b_0 \sum_{i=0}^{6} \left[a_i \left(\frac{n}{f_s} \right)^i \right] + w(n), \qquad n = 0, 1, 2, \ldots, L \tag{4}$$

The parameters of the signal are $b_0 = 1$, $a_0 = 0.945975$, $a_1 = -1500.156$, $a_2 = 302.136$, $a_3 = 10068.894$, $a_4 = 60045.874$, $a_5 = 1256530.654$, $a_6 = 78413000.168$. Sample rate is $f_s = 5000\,\text{Hz}$, L is 15000 and the length of observation window is 50. The parameters of smoothing filter are $m = 11$ and $p = 3$. The signal-to-noise rate (SNR) is defined as:

$$SNR = 10 \log_{10} \frac{b_0^2}{\sigma_w^2} \tag{5}$$

where σ_w^2 is the variance of $w(n)$. Each Monte Carlo simulation is based on 100 realizations and the SNR is SNR = 8 dB. The results shows the RMSE of total phase after smoothing is 3.006°, which means the precision of ranging is about $1\lambda/120$ (λ is the wavelength of carrier signal). Using the L1 channel of GPS signal as an example, the accuracy is about 0.159 cm. The RMSE of instantaneous frequency is 276.087mHz.

The RMSE of the total phase has been plotted versus SNR in Figure 2. Note that the sample rate is changed to 50 kHz therefore the observation window is 500. Figure 2 shows the RMSE of the phase before and after smoothing under different SNR. The Cramér-Rao lower bound (CRLB) is displayed as well. This method exhibits a threshold at around −4 dB.

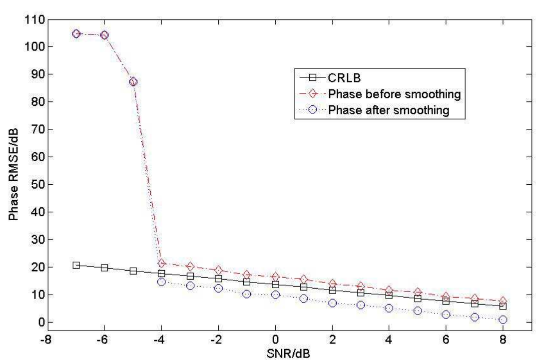

Figure 2: Errors of the total phase, relative to CRLB, as function of SNR.

5. CONCLUSIONS

This paper describes a model-free real-time method to estimating the total phase and the instantaneous frequency with high precision. Fractional phase and average frequency of the carrier signal are estimated in analysis window by de-chirp algorithm. The total phase is smoothed by Savitzky-Golay filter while the instantaneous frequency is obtained by a similar filter. The main modules of the proposed approach are FFT and FIR, which are suitable for implementing in hardware.

REFERENCES

1. Blewitt, G., "Basics of the GPS technique: Observation equations," *Geodetic Applications of GPS*, 10–54, 1997.
2. Han, S., "Carrier phase-based long-range GPS kinematic positioning," University of New South Wales, 1997.

3. Wang, P., H. Li,, and B. Himed, "Instantaneous frequency estimation of polynomial phase signals using local polynomial Wigner-Ville distribution," *2010 International Conference on Electromagnetics in Advanced Applications, ICEAA)*, 184–187, IEEE, September 2010.
4. Barkat, B. and B. Boashash, "Design of higher order polynomial Wigner-Ville distributions," 1999.
5. Peleg, S. and B. Friedlander, "The discrete polynomial-phase transform," *IEEE Transactions on Signal Processing*, Vol. 43, No. 8, 1901–1914, 1995.
6. Bi, G., Y. Ju, and X. Li, "Fast algorithms for polynomial time-frequency transforms of real-valued sequences," *IEEE Transactions on Signal Processing*, Vol. 56, No. 5, 1905–1915, 2008.
7. Boashash, B., *Time Frequency Signal Analysis and Processing: A Comprehensive Reference*, 2003.
8. Peleg, S. and B. Porat, "Linear FM signal parameter estimation from discrete-time observations," *IEEE Transactions on Aerospace and Electronic Systems*, Viol. 27, No. 4, 607–616, 1991.
9. Rife, D. C. B. P. and R. R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Transactions on Information Theory*, Vol. 20, No. 5, 591–598, 1974.
10. Savitzky, A., and M. J. E. Golay, "Smoothing and differentiation of data by simplified least squares procedures," *Analytical Chemistry*, Vol. 36, No. 8, 1627–1639, 1964.
11. Meng, Q. and J. Lu, "A filter designed based-on Talyor series model with least square method," 4, Journal of Southeast University, 1995,