Fast Level Set Based Method for High Contrast Microwave Imaging

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Abstract— Microwave imaging is a non-ionizing modality and has applicability in many areas. Many numerical and experimental studies on microwave imaging have been performed in the past several years, with common impediments being the inability to recover high dielectric contrasts and slow speed of computation. We propose a method based on level set formulation to recover the shape of imaged targets even for the high-contrast case, when prior information on permittivity of the object is known. Our method significantly differs from the previous work as we evaluate the inverse problem without solving the adjoint field and consider the variational level set for regularizing the level set evolution. Hence, the proposed method is very fast. The applicability of the method is demonstrated on synthetically generated data for a two-dimensional (2D) scenario. The reconstructed images indicate that the method can produce accurate object localization and shape identification even for high contrast objects.

1. INTRODUCTION

In Microwave imaging, electromagnetic waves are transmitted, scattered and received in an investigation domain and the electrical properties such as permittivity and conductivity are estimated at each discretized element in the domain. Microwave imaging has applications in through-the-wall imaging, non destructive testing, and medical imaging. The imaging problem is usually performed by formulating it using the volume integral equation. The integral equation represents the relationship between complex permittivitty and the electric field inside and outside of the inhomogeneous imaged domain. It is non-linear in general since the electric field inside the object, which appears inside the integral, is a function of medium's properties. Common practice to solve for the unknown object properties is to linearize the integral equation by iteratively solving for the unknown complex permittivity and the field. This linearizing approach performs well when the object is not too large compared to the wavelength or the contrast (complex permittivity of the object with reference to that of the background) is not too high. However, its performance on contrast recovery degrades rapidly as the contrast goes higher. The primary reasons are non linearity of the problem and presence of multiple solutions (that is, nonuniqueness).

To overcome the limitation of the linearized approach, many approaches using global optimization methods are proposed. They are usually very slow and their applicability to 3-dimensional problems is particularly implausible. Approaches also have been suggested where prior information about the unknowns can be leveraged. Having accurate priors about dielectric properties, the microwave imaging problem essentially becomes locating the object and estimating the shape of the object. In other words, it can be thought of as a shape segmentation problem. Although it is understood that the prior knowledge about contrasts in the imaged domain may not be very accurate or easily available, we will make the assumption that the contrast is known to within some uncertainty. This is so that we can decouple the contrast and the shape retrieval problems, address the latter, and defer the accurate solution of the former to a separate solution step. We also note that even if the contrast values are known, conventional microwave imaging techniques such as the Born/Distorted Born Iterative Methods, or the Contrast Source Inversion method, still do not converge.

The level set method, one of the best shape estimation techniques in image processing, is a candidate that we wish to consider for this work. Although level set based shape recovery approaches for microwave imaging have been proposed before [4], they are very slow in general. For example, the approach used in [7] takes around 4–7 hrs on a single 64-bit AMD Opteron 246 processor running at 2 GHz to recover a 2D shape profile of 10 cm radius. The main computational burden is to estimate the field for given parameters at each iteration. The field computation has to be performed at every iteration, when the adjoint field based approach is used. We propose a method which is not based on the adjoint field evaluation.

2. THEORETICAL BACKGROUND

Microwave imaging is an electromagnetic inverse scattering problem. We assume a z-independent two dimensional (2D) scatterer, and the electric field polarized along the z-direction, corresponding to a transverse magnetic (TM) polarization. For a 2D scattering problem, the scalar electric field volume integral equation for a heterogeneous, isotropic, non-magnetic medium, enclosed in a region D is given by [1]

$$E(r) = E_i(r) + k_b^2 \int_D G(r, r')\chi(r')E(r')dv', \tag{1}$$

where $E(r)$ and $E_i(r)$ are total and incident fields, respectively, k_b is the wavenumber of the background medium with lossless permittivity, $G(r, r') = \frac{i}{4} H_0^{(2)}(k\|r - r'\|)$ is the Green's function, $\chi(r) = [\epsilon_r(r')/\epsilon_b - 1] + i[(\sigma(r') - \sigma_b)/(\epsilon_b\epsilon_0\omega)]$ is the dielectric contrast in terms of the permittivity and conductivity contrast, where the subscript r denotes the relative permittivity, and b denotes the background. Equation (1) represents the solution of the wave equation and it is used to determine the fields given the complex dielectric contrast. It is also called the forward solution.

Defining the scattered field as $E_s(r) = E(r) - E_i(r)$, with measurement points r on a surface S that encloses D in (1), we get

$$E_s(r) = k_0^2 \int_D G(r, r')\chi(r')E(r')dv' \quad r \in S,\, r' \in D. \tag{2}$$

Equation (2) can be used to estimate the dielectric contrast of an unknown medium, given some observations of the scattered field. Let L define the linear operator represented by $L\{\cdot\} = k_0^2 \int_D G(r, r')\cdot dv'$, then Eq. (2) is written using short-hand notation as $E_s = L\{\chi,\, E\}$. It is also sometimes called the data equation.

3. APPROACH

3.1. Level Set Formulation

Equation (2) is a non-linear problem in general, since the electric field, $E(r')$, is itself a function of the dielectric contrast. The idea is to use (1) or (2) to estimate the contrast and the field such that the error $\|E_s - L\{\chi,\, E\}\|$ is minimized. Permittivity and conductivity are well documented for many materials. Utilizing this information as a prior in the estimation problem, the image reconstruction becomes as a shape and location estimation problem. Once again we note that this is an intermediate and convenient assumption, and that ultimately we need to solve for the contrast as well. But we will assume the contrast is known as a stepping stone towards solving the high-contrast shape function inverse problem. Level set methods are widely used in the image processing field especially for segmentation, registration and denoising. Here, we exploit the level set method for image reconstruction. For image reconstruction, the cost function $J(\chi)$ can be defined without using any regularization as,

$$J(\chi) = \|E_s - L\{\chi,\, E\}\|. \tag{3}$$

The unknowns χ can be parameterized using a level set function ϕ as follows:

$$\chi(r) = \chi_i U(\phi(r)) + \chi_e(1 - U(\phi(r))). \tag{4}$$

where χ_i and χ_e are prior information on the object and background contrast respectively and $U(r)$ is the standard Heaviside function. We note that $\phi > 0$ ($\phi < 0$) represents the region occupied by the object (background) so $\phi = 0$ traces the boundary of the object. The object can also be non-connected.

The minimization of the cost function on the premise of the gradient flow can be written in the form of the evolution equation of the level set function using Euler-Lagrange equation as

$$\frac{\partial \phi}{\partial t} + \frac{\partial J}{\partial \phi} = 0. \tag{5}$$

Formally differentiating J and the first order Taylor series approximation yields the Gateaux derivate of functional $J(\phi)$ as

$$\frac{\partial J}{\partial \phi} = \mathrm{Re}\left\{[R'(\chi)]^{+} R(\chi)\right\}(\chi_i - \chi_e)\delta(\phi). \tag{6}$$

Here, Re indicates real part of the argument, R is the residue ($R = E_s - L\{\chi,\, E\}$) for the given estimation of E using ϕ, $^{+}$ indicates adjoint operation and R' denotes the derivative operation. From now on we use the notation A for the operator L, when the field E is known or estimated. Generally in Electromagnetics, the adjoint operations have been performed by defining the adjoint fields and solving the electromagnetic scattering problem. Solving the adjoint problem is computationally as intense as solving a forward problem and it has to be solved at every iteration. Though the adjoint and the forward problem can be solved together, they account for the majority of the computation time in the inverse problem. Having the prior information about the contrast, the computational time can drastically be reduced by applying linearized operations. Linearized operations have two key features. The adjoint operation for a linear system is the combination of the transpose operation and the complex conjugate operation, and estimation of the best reconstruction for the given field is similar to iterating the level set evolution until it converges. Hence,

$$\frac{\partial J}{\partial \phi} = \mathrm{Re}\left\{A^{+}R(\chi)\right\}(\chi_i - \chi_e)\delta(\phi). \tag{7}$$

Level set evolution typically develops irregularities during its evolution, which eventually destroys the stability of the level set evolution. One of the remedies is re-initialization, which has been extensively used to avoid irregularities and has ensured desirable results [2]. However, we consider the variational level set formulation [6], which does not require re-initialization, but instead the signed distance function is used as a regularization.

We propose a cost function for image reconstruction, which minimizes the residue energy and non uniform distance energy. The new defined cost function or equivalently an energy function $\mathcal{E}(\phi)$ is written as

$$\mathcal{E}(\phi) = \mu R_p(\phi) + J(\phi), \tag{8}$$

where $R_p(\phi)$ is the regularization term, $\mu > 0$ is a constant, $J(\phi)$ is the external energy function similar to (3). The regularization is called the internal energy of the function ϕ since it is function of ϕ only. The external energy is the residue rather than the edge indicator function as in [6]. The evolutionary approach balances between these two energies. The regularization term is defined similarly to [6] by

$$R_p(\phi) = \int p(|\nabla\phi|)dx, \tag{9}$$

where p is a potential energy defined by

$$p(s) = \begin{cases} \frac{1}{(2\pi)^2}(1-\cos(2\pi s)) & \text{if} \quad s < 1 \\ \frac{1}{2}(s-1)^2 & \text{if} \quad s \geq 1. \end{cases} \tag{10}$$

The potential energy has to be minimum at $s = 1$ to maintain the signed distance uniform and at $s = 0$ to keep the derivative of $R_p(\phi)$ away from $-\infty$. For details, refer to [6].

The Gateaux derivative of the functional $R_p(\phi)$ is

$$\frac{\partial R_p}{\partial \phi} = -\mathrm{div}\left(\frac{p'(|\nabla\phi|)}{|\nabla\phi|}\nabla\phi\right). \tag{11}$$

Linearity of the Gateaux derivative and, from (11), the level set evolution equation can be expressed as

$$\frac{\partial \phi}{\partial t} = \mu\mathrm{div}\left(\frac{p'(|\nabla\phi|)}{|\nabla\phi|}\nabla\phi\right) - \mathrm{Re}\left\{A^{+}R(\chi)\right\}(\chi_i - \chi_e)\delta(\phi). \tag{12}$$

3.2. Numerical Implementation

For solving the forward electromagnetic scattering problem, we use a two-dimensional, vector element based finite element method (FEM) [5]. In this implementation of the FEM, we employ first-order Whitney edge elements as basis functions for the total electric field, and a first-order absorbing boundary condition to terminate the computational domain.

For solving the evolution of the level set function, we use the central difference scheme to approximate all of the spatial partial derivatives, whereas we use the forward difference scheme to approximate the temporal partial derivative. The approximation of Eq. (12) by the above difference scheme can be simply written as

$$\frac{\phi_{i+1} - \phi_i}{\Delta t} = L(\phi_i), \tag{13}$$

where $L(\phi_i)$ is the approximation of the right hand side of Eq. (12). Δt is artificial time, and it represents the evolution iterations. We choose μ and Δt such that the Courant-Friedrichs-Lewy(CFL) condition $\mu\Delta t < 1/4$ is satisfied.

4. RESULTS

We performed several experiments to validate the proposed method. In the reported numerical results, the scattered field data have been obtained by using a numerical simulator based on FEM. The simulation has been performed using the following configuration. The T/R antennas are located 20° apart from each other on the circumference of the circular domain of radius 1.5λ. The operating freespace wavelength is 14.28 cm and the domain is meshed at $\lambda/100$ to maintain error in field calculation within a nominal bound. The total number of measurement is 324. In the image reconstruction process, the domain D was meshed at $\lambda_g/50$ for the forward solver (FEM), where λ_g is the wavelength in the object medium. The domain D used for estimating the complex permittivity (for (2)) was a circular domain of radius 1λ (free space wavelength). The domain was meshed at $\lambda/21$. The discretization gives image size of 41×41 pixels.

The results shown here are generated by calling the forward solver only 12 times, and 80 level set iterations for each call of the forward solver. Comparing with the existing methods [3, 7] where the forward solver has to be called on the order of 1000 times, it reduces the number of the call to forward solver by a factor of 100. In terms of the computation time, the average runtime for each forward solver is 25 seconds and the average runtime for each call of level set is 35 seconds on a Linux desktop with a Xeon processor and 24 GB RAM so the reconstructed image is generated on an average in 12 mins.

The background medium is considered to be as air/vacuum. The starting point for vector χ is the minimum energy solution $(A^T y)$. The level set evolution equation parameters are set as $\Delta t = 0.15$ and $\mu = 0.12/\Delta t$. The reported values are by means of preliminary trial and error evaluations.

The first experiment is on objects with high contrast. We considered a circular cylinder having radius 0.4λ, located at the center of simulation domain. We performed two simulations. In the first simulation, the permittivity of cylinder is 6 and in the second simulation, it is complex and is $2 - 1.5i$. The respective reconstruction images are shown in Fig. 1. The reconstruction domain is

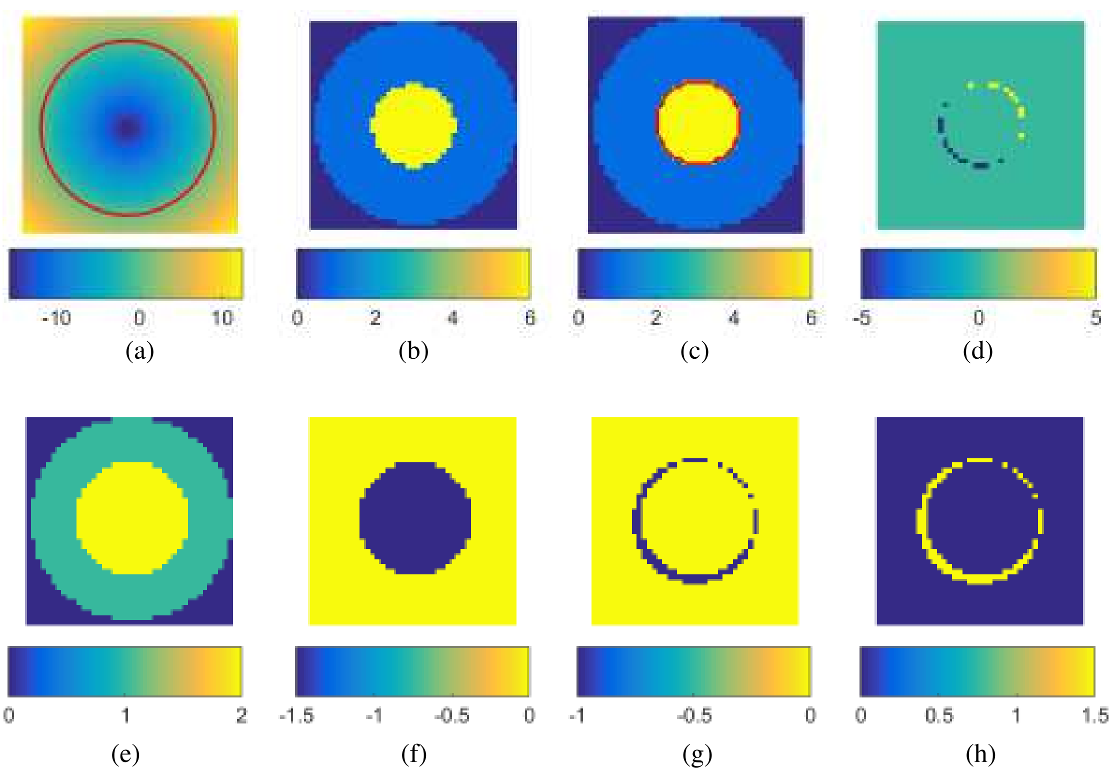

Figure 1: (a) Initialization for a level set function. The red contour illustrates the zeroth level, i.e., object boundary. (b) Actual configuration and lossless object. (c) Reconstructed image using the converged level set (depicted by red contour). (d) Error in reconstruction. (e), (f), (g), (h) Actual configuration of the object with loss; (e) real part and (f) imaginary part. (g), (h) Error in reconstruction. Colorbar at bottom represents permittivity.

circular so that outside of the largest circle, the region is void. The method estimates the shape correctly within the accuracy of the numerical error.

To evaluate the efficiency of different shapes, we considered different cases, which consist of non-connected objects and sharp boundary. The geometry of the problem and various cases are shown in Fig. 2. We started the minimization with the initialization of the level set function the same as for objects in Fig. 1. The reconstructed results are shown in Fig. 2, where the red contours depict the object boundary and they are in good match with the true objects.

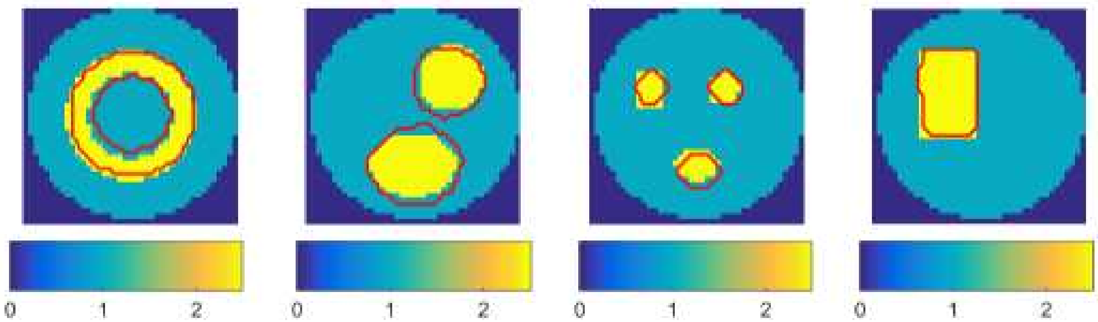

Figure 2: Testing set: images of different objects. True objects are shown in yellow. The converged level sets(boundary of the reconstructed objects) are overlaid as a red contour. Colorbar at bottom represents permittivity.

5. CONCLUSION

We presented the fast level set method without solving the adjoint field for microwave imaging. The prior information on permittivity allows us to recover high contrast using the level set shape based method. Our approach is computationally less intense than other approaches because it reduces the number of forward solver calls by about two orders of magnitude. The reconstructed images for various shapes and high contrast had an excellent shape recovery, localization and error to within a single pixel. Future work includes evaluating the performance of the method on actual data, incorporating objects with discrete permittivity in the level set framework and extending it to 3-dimensional framework.

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